Multirate Digital Signal Processing

Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture – 30 (Part-1)
Capacity of Wireless Channels – CSIR

Okay good morning. Let us begin. Today, we will pick up from where we left off in lecture 28. So lecture 29 was the review lecture.

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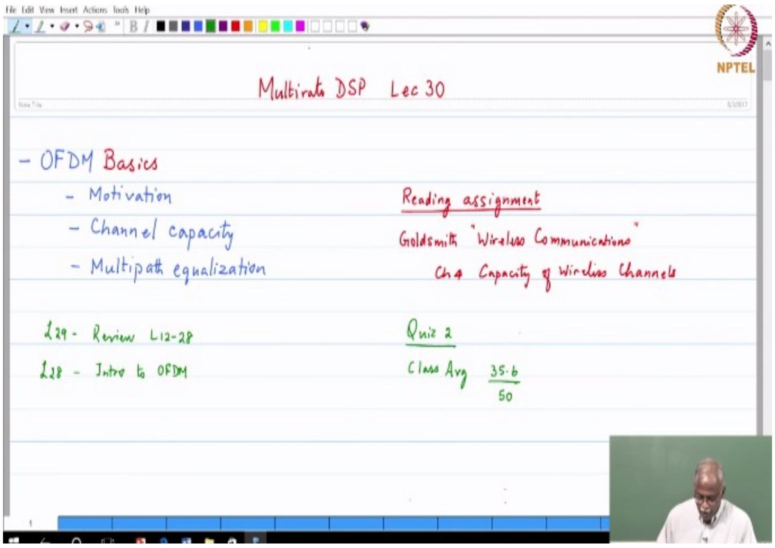

So lecture 29 was the one where we had a review of lectures 12 to 28 and lecture 28 was what we had last covered. So we pick up from the introduction to OFDM that was given in lecture 28. So that is our starting point for today. Today's material does include capacity of wireless channels. So one of the excellent resources for this is chapter 4 of Andrea Goldsmith's book : Wireless Communication. So I would definitely encourage you to have a look at that.

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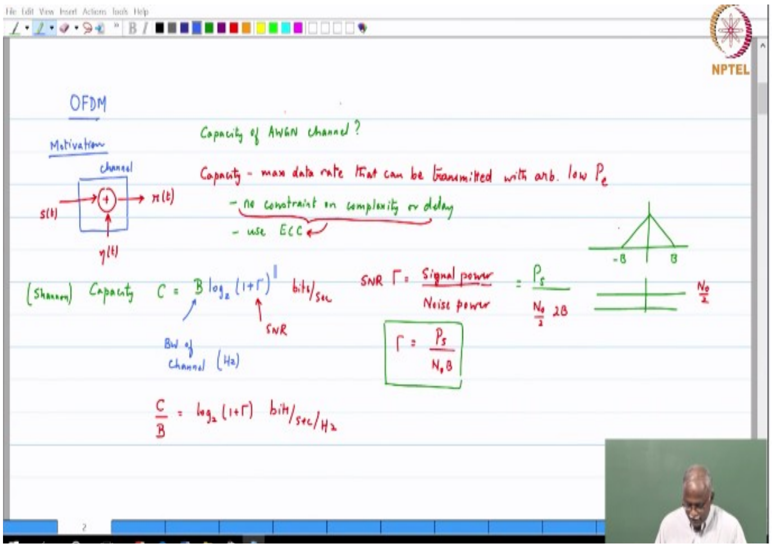

So by way of refreshing what we had looked at in the last lecture, we said that Shannon capacity is given by B times logarithm base 2 of 1+SNR and we defined SNR as a signal power by noise power, noise power being the noise spectral density times the bandwidth of the signal that we are transmitting. So given that if you have a channel where the impairment is additive white Gaussian noise then we know how to calculate the capacity of the channel. So that was the first result that was established.

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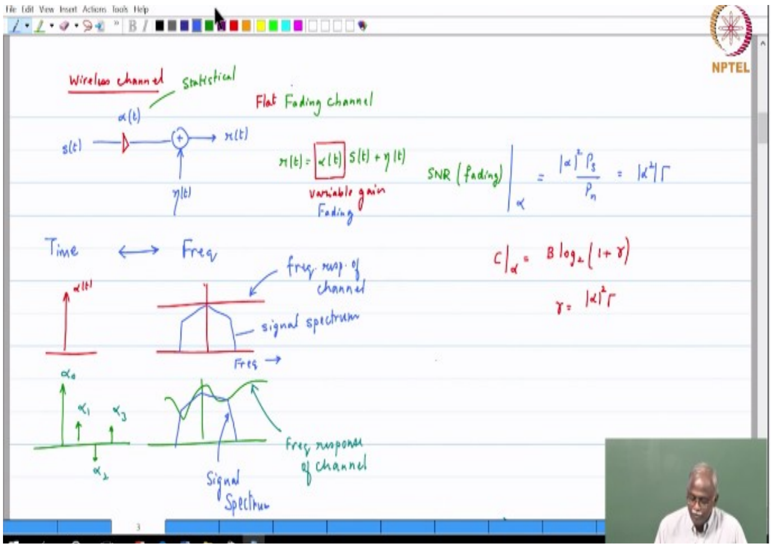

The second element that was highlighted was that we are dealing with wireless channels. These are not static channels; these are constantly changing because either the user is moving or the environment is changing. So we have to introduce variable gain term called alpha okay. So alpha is a function of time depends on where you are in terms of the location. So the received signal now has a modified equation.

So the received signal r of t is=alpha of t times s of t+eta of t. Notice that this is the new term, this is, you can think of it as some kind of a variable gain term okay. Think of it as a variable gain but maybe the correct way to look at it is the effect of fading okay. So gain means it is something signal becomes larger. In the wireless channel, very rarely the signal becomes larger than what you transmitted.

So basically it actually can have a dip in terms of the amplitude. So actually it is a fading coefficient. So think of it as a parameter that you have to be careful in terms of monitoring because it could make your signal level go down okay. The second, the next element which we talked about in the context of the wireless channel was whether the signal has got time dispersion. So let me just draw a figure which will help clarify this point.

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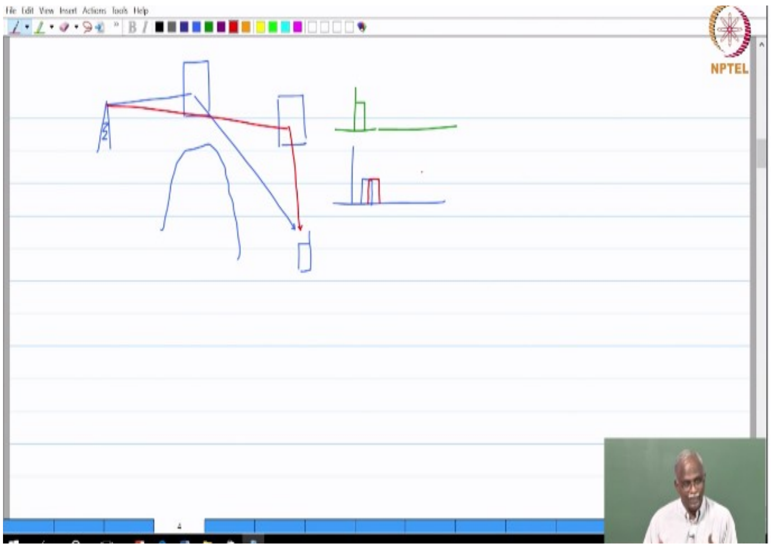

So assume that you are transmitting from a base station and you are basically picking up the signal in a mobile. So assume that there are buildings and objects that will act as reflectors and for some reason there is no line of sight path. So let us assume that there is some sort of an obstruction which obstructs the line of sight path. So the signal basically gets to the mobile, non-line of sight, so that is path number 1.

Second path is okay, so basically these 2 signals if they have different path lengths, then if this is my transmitted signal, again I am just sort of highlighting it. This is just for illustrative purposes. If I transmitted a pulse, I would receive it on the blue path with some delay of time, that would come delayed and the red path probably a little further delayed in time okay. So basically you will see that there is multiple copies that are present and they actually could cause the signal impairment.

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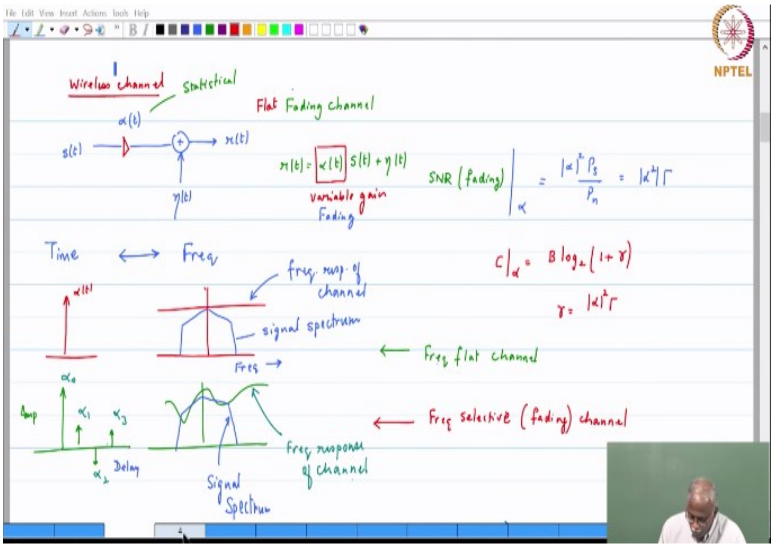

Now let me go back to the figure that we drew last time. If I had only one path that means I would have only one alpha. So which means that in time my channel response looks like an impulse, if I look at it in the frequency response, it is flat. So this is where we get the terminology frequency flat channels. So this would be this if you this would correspond to, this particular one would correspond to a frequency flat channel okay.

So these frequency characteristics of this channel is that all frequencies experience the same gain, so therefore there is no distortion of the signal. So if the blue line was a signal spectrum that is what you would receive at the transmitter as well. Now the second case where there are multiple copies like what we have drawn and they are coming at different delays. So let us say that for illustrative purposes there are 4 copies of the signal.

They are all coming delayed in time. Now yes this is what the channel impulse response looks like in time. So what is your y axis? This is amplitude. The x axis is delay okay. It is also measured in terms of time; milliseconds or microseconds. So this is what it looks like and the frequency response. Now because there are multiple taps, it is no longer flat, you get sort of a frequency variation.

What you do not want is to have a null because that means that signal will not propagate at all but there may be some variations which we do not control. Basically, we have no control over what alpha 1, alpha 2, alpha 3 are and what their delays are. So basically if the frequency response of the channel is not constant then we call it as a frequency selective channel okay and very often we also add the term fading along with its frequency selective fading.

Why did the channel become frequency selective? Because it was dispersive and then it does not say constant and it is because of the fading effect. So frequency selective fading channel or a flat fading channel okay. So these are the 2 scenarios. If this happens and the blue line is your signal spectrum you can see that your signal is going to be distorted and the receiver will have to undo the effects of this channel. So this is one element that we looked at okay.

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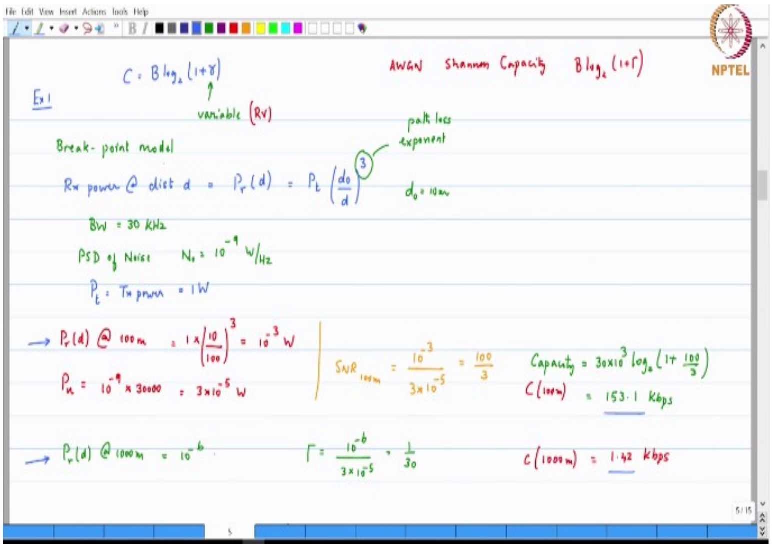

Now going back to the context of the capacity. So in an AWGN channel, we define Shannon capacity as the bandwidth times logarithm base 2 1+gamma. Now we are in a fading channel, wireless channel, so therefore I cannot talk about a fixed gamma. I talk about a lower case gamma where it is a variable okay and in fact this will be a random variable okay. We will come back to that.

Right now it is just that it is not a fixed quantity okay. So now I would like to highlight what is the impact of the SNR going up and down. So for that we took a particular example, we looked at the two data points. What happens when you look at the SNR at a distance of 100 meters and then what happens when you look at the SNR at a distance of 1 kilometer 1000 meters?

So basically we looked at 2 examples. We saw that the SNR's are different and it turns out that in the case of at 100 meters, we get 153.1 kilobits per second. At 1 kilometer, it goes down to 1.4 kilobits per second. So again the impact of SNR is something very important. So we need to keep that in mind. So now translate this to the fading channel okay. Basically, the fading channel says SNR is not constant, it is going up and down which means the capacity is going up and down.

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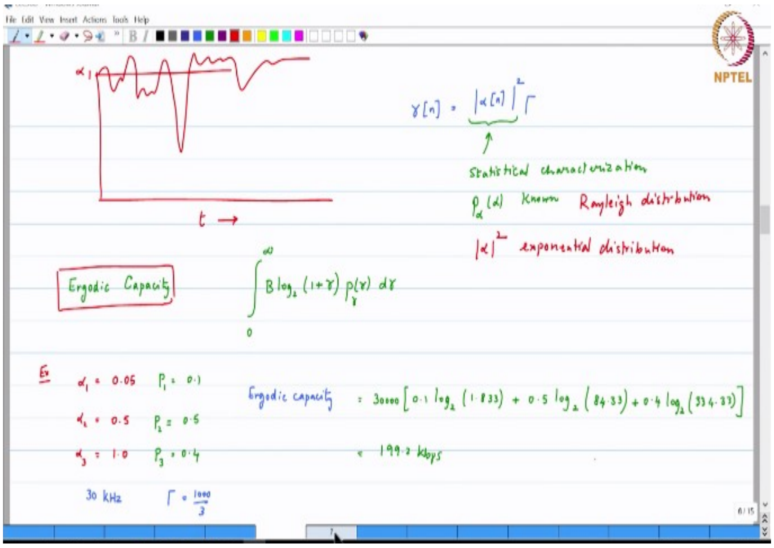

So we need to now talk about a meaningful reference point for the, so what we then talked, what we then said is if you know the probability distribution of the SNR okay, of the SNR then what we can say is that we can now talk about a probabilistic capacity, it is B times logarithm base 2 1+gamma but that is only for that particular value of gamma and that is defined by the PDF right. This p gamma of gamma is the PDF of the SNR.

So we said that okay, so that is the direction in which we are going, it is no longer a fixed number. It is an ergodic capacity or some number that is statistical. So therefore we are going to explore this a little bit more. So we took a specific example. We said that let us take a channel where there are 3 possible SNRs. By the way, alpha 1 is the amplitude coefficient, SNR means it will be alpha squared, so we have to take that.

So basically the instantaneous SNR will be alpha squared times gamma okay. So the SNR is alpha 1 0.05 times with the probability of 0.1, alpha 2 with the probability of 0.5 and alpha 3 being 1 with the probability of 0.4 okay. So we then looked at the different values and I asked you to just quickly verify that you did get an ergodic capacity which came out to be 199.2 kilobits per second. I hope you were able to do that.

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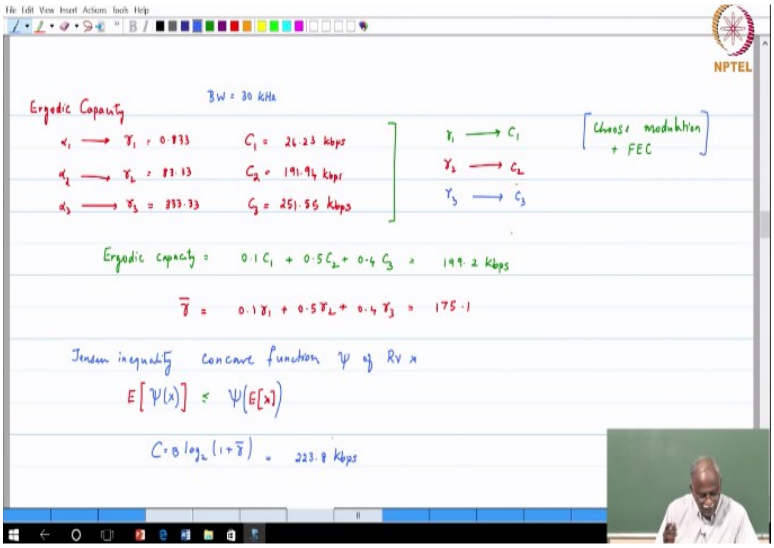

Let me just add to that with the following additional information, so basically we are looking at the capacity, ergodic capacity. That is our task in this particular problem. So we said that there are 3 SNRs, so there is alpha 1 which will lead to an SNR gamma 1. You should have gotten the gamma 1 to be 0.833; alpha 2 will be leading to gamma 2 which was 83.33, alpha 3 leading to a gamma 3 which was 333.33.

Now this corresponded to a Shannon capacity C1 of 26.23 kbps okay. Again, you would have done this as intermediate steps. I am just summarizing the final values, 191.94 kbps and C3 being 251.55 kbps correct. So notice that all of this is the capacity for just a 30 kilohertz channel. The bandwidth is 30 kilohertz but Shannon's limits, Shannon's theorem says you can pump several times the bandwidth.

So that means you are doing a modulation that is not binary but some higher-level modulation. So that is how you will get bandwidth of, so ergodic capacity. Ergodic capacity which was what we calculated in the last slide. Ergodic capacity will be 0.1 times C1, that is the probability of, +0.5 times C2+0.4 times C3 which is what comes out to be 199.2 kbps okay. Please do verify it and now there is a very interesting parallel observation.

What is gamma bar? So basically there was 3 possible SNR's. The average SNR is given by 0.1 times gamma 1+0.5 times gamma 2+0.4 times gamma 3 that comes out to be 175.1

somewhere in between these 3 values. Now there is a very important result in coding theory which, information theory, which you may have studied in digital communications. I will just quote it without result.

It is called the Jensen inequality. The Jensen inequality says, it basically deals with concave functions. If you have a concave function, if I have a concave function, psi of the random variable X okay. So which means that I will write it as psi of X that is the, it is a concave function and if I were to calculate its expected value, this will be<or= expected, the psi of the expected value, psi of the expected value of X.

Just use the correct type of brackets. Psi blue is for this function within bracket expected value of X okay. So basically the capacity function which is basically a logarithmic function behaves like is basically a concave function. So this ergodic capacity which is the expected value of the capacity if I were to now compare it with the capacity of the average SNR. So basically I would get something which is psi of expected value of X.

So we are looking at the capacity which is given by logarithm base 2 of 1+, of course you have to use the bandwidth also, equal to bandwidth times logarithm. This is gamma bar okay, so which means that if it was an AWGN channel with this average SNR, would be better or worse than my fading channel okay? So if you compute this, it comes out, let me just make sure I got the correct number.

Yes, it comes out to be 223.8 kilobits per second okay. So observation, very key observation that comes here. These bad SNR's are really are killing my capacity. So that is a key observation because if I can smooth out basically take the average SNR, actually I get a much better number okay. So notice that the capacity using average SNR is actually much higher than what I can achieve with the ergodic capacity.

So that is observation number 1. So I am really we have to be very careful with those channel conditions when the SNR goes low because that is which is that is going to affect our ability to achieve the capacity of the channel okay. So this is an important observation. Now I want to sort of take this forward and ask you to think about or respond to this question. Now how will I achieve this capacity?

So basically I have this scenario right, I have this scenario, sometimes it is gamma 1, sometimes is gamma 2, sometimes is gamma 3. So how do I achieve this given that I have an SNR gamma 1, how do I achieve capacity C1? It is by saying I recognize that the channel is not very good, so I apply the appropriate modulation. So I have to choose the modulation and the channel coding.

Remember Shannon capacity includes channel coding. I call it FEC, error correction, forward error correction. The combination of these two will help me achieve gamma 1 okay. So let me say, maybe it is binary modulation with some coding achieves this capacity. Now I move, then from gamma 1 to gamma 2 to achieving capacity C2, same bandwidth 30 kilohertz. Now if I want to achieve higher capacity, I must change the modulation, I must change the channel coding to match the channel conditions right.

So obviously, and likewise there is a third channel which is gamma 3 to capacity C3. So this choice of modulation and FEC are very important in us being able to actually achieve the capacity that we are interested in okay, so keep that picture also in your mind okay. So now I want to take this forward. Now who knows the channel condition? Does the transmitter know what the channel conditions are?

In general, no. You just transmit, depends on the channel conditions the signal will come with an impairment at the receiver. So the receiver knows the channel conditions.

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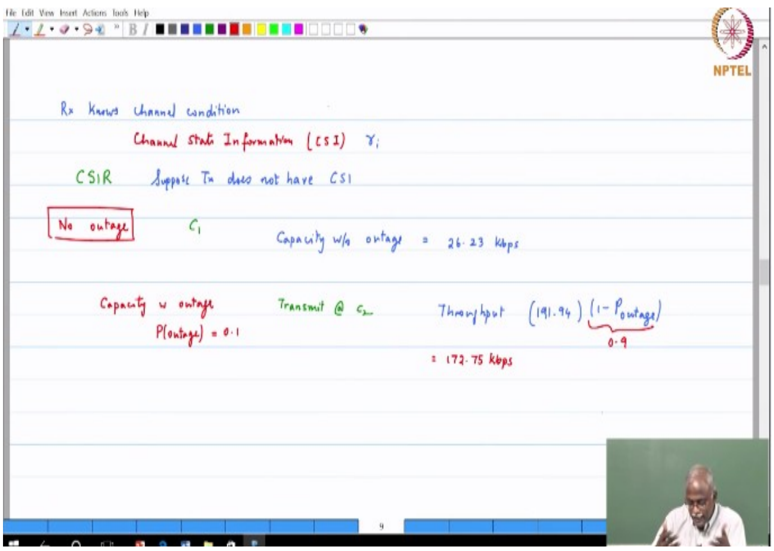

So always the Rx knows the channel conditions okay. So we usually refer to this as channel state information CSI okay, and by default the scenario that we are looking at is CSIR; that means channel state information known to the receiver and this is always true because the receiver will receive the signal and it will estimate but transmitter it does not have information.

Suppose transmitter does not have information, suppose transmitter does not have the, basically what are we talking about channel state information, it is gamma i right. They want to know, you want to know what is the instantaneous SNR, supposing Tx does not have CSI information, does not have CSI. What can you do? So basically now the problem being posed is; I want to achieve the best possible scenario but I should not have outage.

So I am going to impose the condition no outage okay. What do you mean by outage? I transmit something but it is received at the receiver with errors okay. So if I go back and look at the scenario that we are looking at, now which of these is the most robust transmission schemes. Is it the 1, 2 or 3? Which is the most robust? 1 is the most robust because you are transmitting with the minimum constellation size with maximum error protection.

That is the robust size because you are likely to make errors because SNR is bad, whereas the one that is optimized for capacity, getting the maximum information across is gamma3 but if my transmitter does not know this information then I do not have an option because you are telling me I cannot say the packet did not get to me because if I transmit gamma3 mode when it is actually gamma1 channel conditions, what will happen?

Errors will occur, I will not, so basically no outage means that the only option I have is to stick with the whatever I can get for C1 because that is the most robust mode and I do not know where this gamma1, gamma2 or gamma3. So this is a very important statement, so which basically says that if I insist on no outage, when the transmitter does not have information, capacity without outage, if you insist on this then nothing much I can do.

I can only give you C1; this is what you can get out of a channel that potentially could give you 200 kilobits per second you are going to get only 26.23. Then, we sort of get a little bit modified question; we say okay I am willing to tolerate some outage okay. So capacity with outage, that starts to get a little interesting, capacity with outage okay. We say that okay let us say gamma1 occurs, outage is unavoidable.

I am going to sort of taken out; outage means that packet will not get through. So then what can you transmit at, at what rate can you transmit? You must get through at gamma2 and gamma3, so which means you can aim for C2 because C2 is designed such that it will go to at gamma2, of course if the channel is gamma3 which is better, of course this transmission will go through error-free.

So capacity outage, with probability of outage=0.1, that means when gamma1 occurs there is going to be errors but I am going to take that hit. So that means under this condition, I can transmit at the rate C2, transmit at C2 so which means that I will get a much higher value 191.94 kilobits per second, but that is not my throughput, my throughput has to be, throughput is 191.94 kilobits per second*1-probability of outage right.

Only when you are not in outage, you can achieve that condition. So with this is actually 1-0.1, which is 0.9. So effectively what we have achieved is 172.75 kilobits per second, much better okay. So of course if I get a little greedy and say can I do can I go to C2, you will find that the amount of outage is actually more and your actual throughput actually comes down, you can, probably an interesting one to just take a look at that and see what actually happens with that okay.

So this is the overall scenario that yes if I do not have knowledge of the channel information at the transmitter, I do have to live with some amount of outage. The time that actually what is getting wasted here is there are times when it is gamma1 channel conditions during which time you have transmitted anyway because you did not know about it and that information got lost, so you wasted some power.