Multirate Digital Signal Processing
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Lecture – 32 (Part-2)
Multi-rate DSP Framework for Multi-Carrier Modulation

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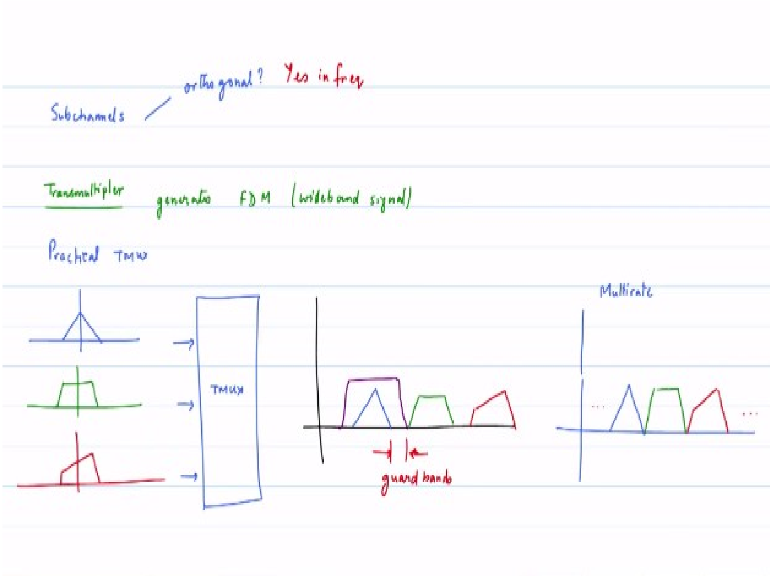

Okay, now before we proceed further, I need to refresh your memory in terms of the trans multiplexer; a transmultiplexer is one that generates a frequency division multiplexed signal, generates a frequency division multiplexed that means, typically a wideband signal now, practical trans multiplexers, this is what they would do so, if I were to give a set of 3 signals, one of them of this type, a second signal which is same bandwidth but different spectrum.

And a third signal which has something different again bandwidth, so the practical trans multiplexer, the trans multiplexer that takes these signals, so these signals coming in to a trans multiplexer; practical trans multiplexer will generate for me a signal that looks like this, okay again these are conceptual, so there would be the blue signal, the green signal and then the red signal, okay, key question; why were there gaps?

It is a practical trans multiplexer, so I need to be able to filter out and recover these signals, so the only way I can do that is if I can have a filter that is flat and then has got some roll-off and so that it does not distort my signal but again at the; so this gap is actually inevitable, we call it the guard band; the guard band is one that helps us in apply practical filters, so we said that we are talking about a practical trans multiplexer, these are guard bands.

Guard bands are wasted spectrum because that is not useful so therefore, it reduces the overall efficiency of your transmission scheme but you cannot avoid it because that is part of the; now on the other hand we talked, we saw another trans multiplexer that this is the practical trans multiplexer, we saw another, there is a multi-rate concept called trans multiplexing where you did the synthesis filter bank.

And there we said I am just going to up sample, I am going to create the following multiplex signal, it is going to be; it is going to look like this why because I did not use any excess bandwidth, okay dot, dot, dot on both sides that is a trans multiplexer that does not have guard bands okay but again, we did not have not yet gone into the process of actually designing these filters.

So, it is very, very important that we sort of retune ourselves saying yes, there are some practical constraints but at the same time my multi rate concept says if I have m; 3 channels which have got bandwidth B, I should not be using more than 3B that is the only way I am maintaining the optimality of the signal, okay. So, this is one concept I just want you to keep in mind.

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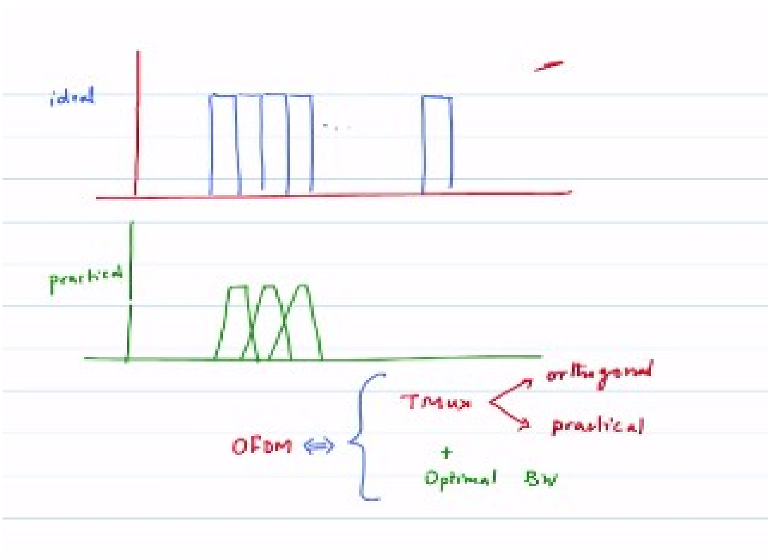

And another related concept that is also important so today, what we are doing for water filling is this, am I right? You are dividing the channel into sub bands or your signal into sub bands and you are maintaining; you are assuming that the signals are orthogonal and but what will it be in practice; in practice, there is going to be some amount of roll-off, so which means that there will be overlap between sub channels okay.

So, this is going to be the practical case, this is the practical case, this is the ideal case now, we know that we can achieve even if the 2 waveforms are overlapping in frequency there are certain conditions that you can impose on them to maintain orthogonality, so therefore we can generate orthogonality but this is very important, so a way to think of the OFDM is first of all it is some form of trans multiplexing, right.

Then you have to maintain orthogonality and you want to achieve with practical filters that is also a very important consideration, so a trans multiplexer that maintains orthogonality that has practical filters and a very important one, optimal bandwidth, okay. Now, what you are saying is; hey, I have to maintain orthogonality which means there is going to be overlap and in the process do not give me guard bands, I do not want to waste time in guard bands.

And if I can achieve all of this, then we talked; talking about OFDM, now you see why OFDM is of so much of interest for the following reason, I can achieve capacity, I can achieve any wasted, I can achieve the with the practical filters I can achieve it, I can achieve it without asking for too much of excess bandwidth, so again OFDM becomes a very interesting issue for us however, one catch is there what about Equalization.

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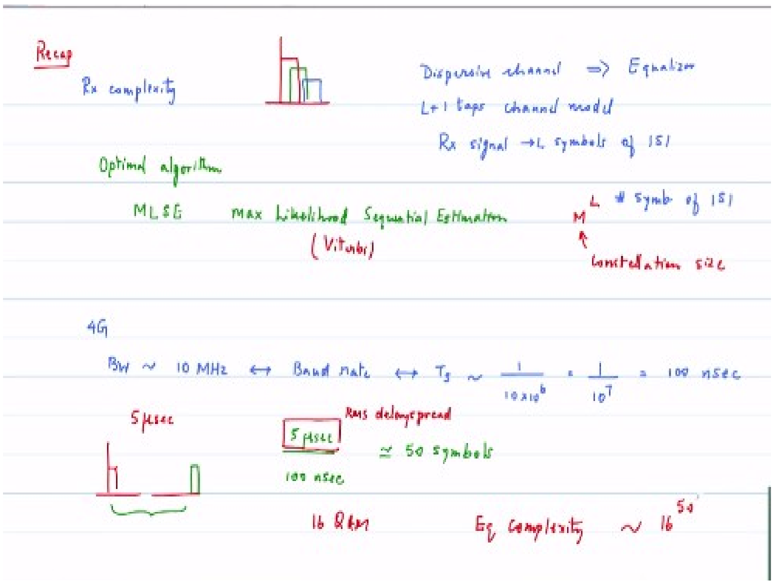

And you kind of said you know all wideband everything all of a sudden you kind of did not really address the equalization issue, no, no we actually have addressed it actually, we yesterday's this is a recap, so this is; this slide is just from reuse from yesterday's talk, so we said that typically in a practical channel, this 5 microseconds is actually a very, very practical number, many measured wireless channels have got RMS delay spread roughly of the order of 5 milliseconds okay.

So that is why we are using that number and if I use a 10 megahertz signal that means my baud rate is approximately a 10 million which means, my symbol rate is 100 nanoseconds, came out to be 50 symbols of ISI that means, the transmitted symbol now may show up after 50 symbols have gone by, so which means that at any given time I have overlap of 50 symbols which means ISI of 50 symbols which means the complexity is going to be 16 power 50, okay.

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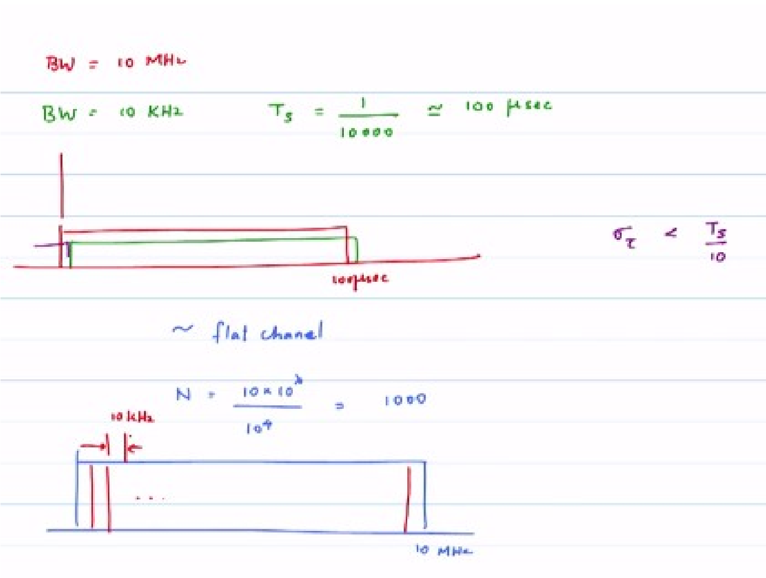

Here, comes the most important element now the previous case what was the bandwidth; 10 megahertz okay, now ask the question, supposing my bandwidth had been 10 kilohertz instead of 10 megahertz, what would be the symbol duration? It would be 1 over the baud rate would be approximately 10,000, so symbol duration would be 1 over 10,000 okay, so approximately you are talking about something in the vicinity of 100 microseconds.

Sketch it for me what does this look like okay, so here is a symbol that is 100 microseconds long okay, 100 microseconds; this is 100 microseconds and echo is coming with a delay of 5 microseconds okay, here is the echo that is it so, what does it look like to the signal, some small disturbance is there but it is not from something which is 100 symbols away it is my own symbol, which is just delayed.

So, how much inter symbol interference will you get, so you the previous symbol let us say was transmitted as purple, you will get some amount of combination from the previous symbol that is your inter symbol, very, very small, so this is a case where ISI would be more or less can be ignored, you can treat it like noise and say well you know it is some small impairment, I am just going to treat it like noise.

And as we mentioned yesterday, the RMS delay spread; if your RMS delay spread, sigma tau is < 1/10th of a symbol duration, right 1 or 0.1 times the symbol duration then you can more or less say that my channel is considered flat, so this is a case where you could almost treat it as a flat channel, so this particular scenario would be a flat channel now, notice that this 5 microsecond RMS delay spread actually looked like a terribly dispersive channel for a 10 megahertz signal.

But for a 10 kilohertz signal, it was not a problem at all so, the goal is choose your N, number of sub channels such that you get 10 * 10 power 6 divided by 10 power 4 which basically means that roughly of the order of 1000, you divide your bandwidth into 1000, then all of a sudden your time dispersion becomes non-issue your ability to achieve capacity becomes all of a sudden you know you could working with a 10 megahertz channel.

It is very difficult to do equalization, no possibility of doing achieving channel capacity you know all of a sudden you are saying hey, I now have the ability, so what we have done is you took a wideband signal 10 megahertz, you divided it into sub channels, each of these sub channels were 10 kilohertz wide, so this was 10 kilohertz and then you completely transformed the problem into a domain for which we have a lot of tools okay.

So, this is why we are so interested in OFDM, this is why see otherwise from a multi rate point of view, it is very easy to introduce OFDM, just take the DFT that is it but why are you interested, why is it so important for us to study OFDM in this context now, supposing this where the case how would you do the equalization okay, so you can almost ignore equalizer but if there is some dispersion how would you do the equalizer.

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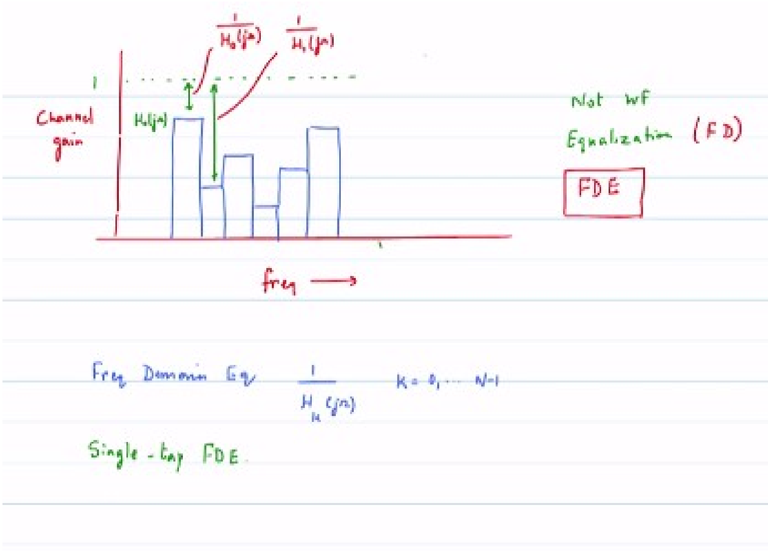

Another very important question for you to answer me, so in other words, I have achieved the following okay, I have divided my broadband signal, wideband signal into sub channels and these sub channels some of them are seeing good channel gains some are not seeing very good channel gains okay, so this is the scenario that I am seeing now, this is the channel gains okay so let me write it down here.

So, we have what we have shown here is channel gain, after I have divided into sub channels channel gain and this is the; this is frequency on this axis okay, now as long as the channel gain is not flat across the various entire bandwidth of the signal that means, I do have frequency selective fading that means, I must do equalization, how do I do equalization for an OFDM or this multi carrier type of signal.

Very easy, we say that okay pick some reference point, okay and whatever is the distance between that you just boost it up, just boost that sub channel up okay, so typically what is the reference line, I take the reference line as 1 that means, how much; by how much should I boost if this level is H0 of j omega magnitude that is the level of the first channel, then I must boost this by 1 divided by H0 of j omega that will bring me up in terms of magnitude and phase.

So, actually do not need to use the magnitude, let us use the actual so if that level it is a complex number if I do 1 over H0, so for the second one what should I do; I will do 1 over H1 of j omega, so you just take the frequency response and take the reciprocal of it all those channels will get boosted up and you will see a flat line okay. Now, is this water filling, I see yes and no which way is it.

No, this is not power, so what am I doing here; I am equalizing I am sort of balancing the channel gain, so it looks like water filling do not mistake it for water filling, so this is not water filling, what is it? It is actually equalization, it has another name probably have always thought of equalization in terms of time domain, this is actually frequency domain equalization, so this is called FDE.

So, when we say that these multi carrier signals like OFDM have very simple equalization, what do we mean; calculate the channel gain, take the reciprocal of the channel gain for those channels then we are through we get that so, when we say very simple equalizer, a frequency domain equalizer just requires you to compute 1 over Hj or let me use some other I do not want to use this H of Hk of j omega, k = 0 to n – 1, okay.

There are n sub channels, I am going to equalize them in this fashion and this is also, it is referred to as a single tap FDE because each channel gets only one tap right. There is no, you do not have to have a filter, single tap FDE, so this is the beauty of OFDM or the; the system that we are studying and that is why it is so important for us from a communications perspective.

It helped me achieve capacity, it solved to some extent the equalizer complexity problem, so now let us see what are the key elements of this concept, how do we take it forward and how do we exploit it to the maximum possible.

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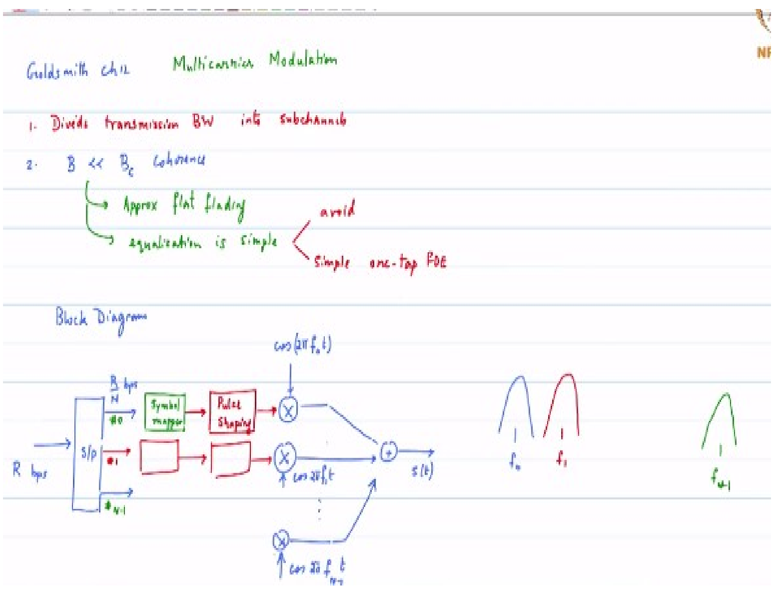

So, the material that we are going to be now studying is actually found in Goldsmith chapter 12 would definitely encourage you to read that okay, it is under the heading multi carrier modulation not necessarily did not call it as OFDM but we are going to develop it into a OFDM chapter okay, multi carrier modulation MCM okay, so if you go and open Goldsmith chapter 12 here is what you are going to see the first statement, okay.

The first statement is that divide your transmission bandwidth, this is the first statement that you will see and I hope you will go and read that into sub channels, okay and you know why and you know why you are dividing the sub channels no problem, okay that is well understood okay, the second statement that you will find is that these sub channels bandwidth are chosen such that they are much less than coherence bandwidth.

And you know why, because you want the frequency gain to be constant, so the statement will be that if you choose it according to this, you will have approximately flat fading, ah, I know exactly why this statement is there and because you already know the background of what has already been discussed here, so basically this will be a scenario where you will have approximately flat fading, okay.

This also says that it is equalization, equalization is simple okay why because you know that you can do a single tap frequency domain equalizer, so either you can avoid equalizer that means, you will have some residual ISI to contend with or you can have a simple one tap FDE okay, both of which are very useful for us to understand, okay. Now, there is a block diagram that will be given in in Goldsmith's book and this is the; what the block diagram is.

Of course, if you saw this first you may not have necessarily linked it to the other concepts that we have talked about but otherwise this is what you will find so, the block diagram for multi-carrier modulation is the following where I have a channel or a data source which is coming at are R bits per second, I am going to do a parallel to a serial to parallel converter, I will come up with N parallel channels, each of which is containing R over N bps.

So, this is channel number 0, channel number N - 1okay, now each of these channels let us quickly trace what the path of them is going to be, you will group them in these bits into either 2 bits or 3 bit sets and then map it onto a symbol, so the first stage will be a symbol map, you will take the bits and then you will map it to either a QPSK 16 qam, 64 qam, whatever you want, then the next stage is you have to prepare it for transmission.

You will do pulse shaping, pulse shaping is to ensure that you have a compact bandwidth, so pulse shaping a raised cosine any of those types of pulse shaping is permitted that is done now, you have to translate it into carrier frequency okay, so the multi carrier modulation says first channel I am going to translate it to a frequency f0; 2 pi f0 * t, okay and then add it up. The second branch; branch number 1 would go through a symbol mapper so this is number 1.

Symbol mapper will go through a pulse shaper and then will come out and then you would have a second carrier which would be cosine 2pi f1 * t; 2pi f1 t that is my second carrier and then the last one will be cosine 2pi f n - 1 * t that will dot, dot, dot so basically you have got all of those and this and (()) (20:53) and modulated to different center frequencies, so this is the notion where you are trying to generate something of this type, okay.

This is at a center frequency f0, the second one is at a center frequency f1 and the last one is at a center frequency f n – 1, this is what we are trying to generate, okay, it is an FDM signal where these frequencies are chosen, so that orthogonality can be preserved this was how multi-carrier modulation was envisioned, okay this is now again, this has no reference to the multi rate parts, it does not have anything to do with trans multiplexer but just say this is how I want to transmit these signals.

Now, the question is who were the first one to come up with this concept and why, the first people to come up with this concept was military, okay why would they want to do this type of a transmission? Okay, first thing supposing I did not have to use all these frequency bands, okay my data is sufficient that it will go on one of these bands, so at any given time I will keep changing my center frequency, you cannot jam my signal, understood, number 1.

So because it looks like my signal is suddenly at f0, you try to jam there I would have already have moved to this one, so this is a very, very attractive scheme for that. The other one is you can scramble your signals, right you can the; let us say so if I was interested in tapping one of these, these signals are constantly getting scrambled, so signal number 0 goes on this carrier for some time, it goes on this carrier and then switches.

So, basically your information, so you are transmitting on multiple channels but these the data is getting mixed up, so at the receiver if you try to; even if you managed to decode the bits you do not know how to put them together because only you know the transmitter how you have scrambled the signals across, so the military had several reasons for doing it and of course, they would have to do a practical trans multiplexer, so they would have these frequency bands.

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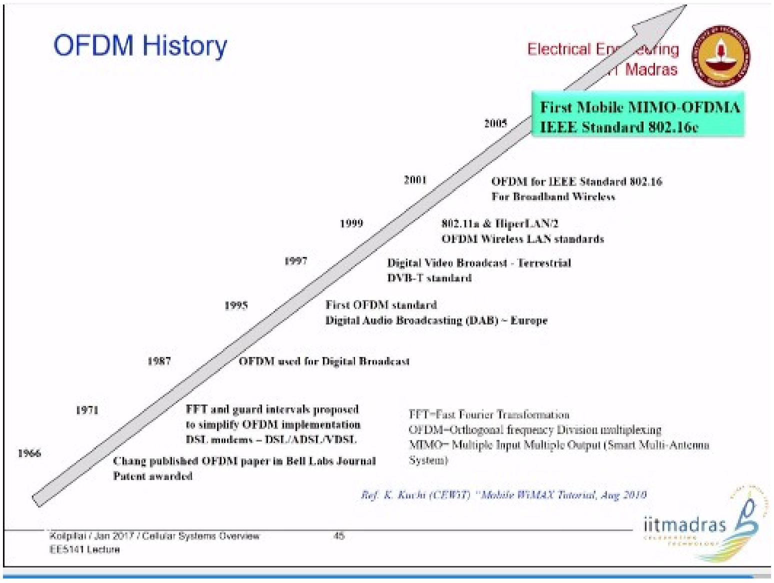

Then of course over a period of time, there was a another development that happened and let me just show you one slide and then we will stop with that okay, so here is a slide which says this is how multi carrier modulation evolved over time, military use it in the early 50's, very early in 1966, they saw commercial potential for this and it was a telephone company called Bell telephone system okay.

Now, why did they see value in this particular system and in fact if you can see all the development all the way from 1960's all the way to 2000, almost 40 years it all came from people who are using either telephone or a television cable for transmission and let me end with that statement as to why they were interested and why they so the scenario for a telephone line was this.

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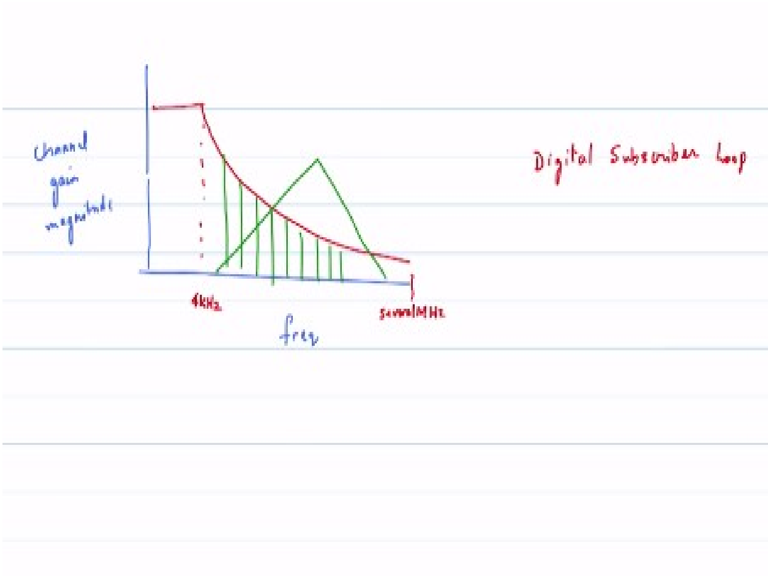

This is the channel gain magnitude, okay magnitude of the channel gain as a function of frequency okay, the telephone line had a response that looks something like this, it had reasonably flat response to around 4 kilohertz and that is why all voice transmission happens in 0 to 4 kilohertz, okay it does not have very good transmission around DC, so you do not extend it all the way there.

So, it starts from low frequency all the way to 4 kilohertz and after this it just sort of goes off, so what people did was well you know the gain is falls off very fast or basically at high frequencies there is really no use, so they always thought that the telephone line by the way this goes to probably several megahertz okay, so several megahertz the response goes up to several megahertz; 2 megahertz, 3 megahertz.

So, the question is basically, you ignore the bandwidth between 4 kilohertz and 2 megahertz for that matter because the portion that you are transmitting has got good gain you forgot about, then people said hey you know what this bandwidth is actually quite big, can we use it for transmitting data okay, leave the 4 kilohertz for voice, this one, so this is where these modems came about, this digital subscriber loop modems.

Digital subscriber loop was a telephone line but they were used to transmit data, they left the 4 kilohertz and then, so DSL modems came because they recognized that the bandwidth of the channel was much larger but the problem is you transmit any signal through this, it will get very heavily distorted because the channel gain is but the good thing is it is not changing, it is more or less somewhat constant and you can, so digital subscriber loop.

So, I say that okay, leave this portion I want to transmit something on this part okay, so if I transmit a signal that's like this it is getting heavily distorted and equalization becomes complex because I have to now design a very complex equalizer, then these people said hey you know what do not do this, transmit in small narrow bands and then what do you do, do water filling, do single tap equalization get the signal out and get.

So, this is why they were interested in, so telephone cables, TV cables all of them saw that hey, there is a portion that is being used and then there is a huge portion that is not being used and the reason is not being used is because it does not have a constant gain or a flat gain but no problem I know how to achieve capacity for those and we can actually use that information and today, you can transmit several megabits of information almost 100 of megabits per second on using this what was considered as wasted bandwidth of the telephone cable.

So that is why the telephone people became interested and that is why you see for almost 40 years of development were nothing to do with Wireless at all, it was people who are trying to get the maximum out of the telephone channel basically, when there is non-uniform gain and then the wireless people came and said hey you know what that is useful for me as well because I have a system where the channel the gain is not constant, fluctuating.

No problem, whatever technology you have developed, I can take it and then apply that became OFDM, we will pick it up from here in the next lecture, thank you by this tomorrow, we will meet thank you.