Multirate Digital Signal Processing
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Lecture – 33 (Part-2)
Recap of Multirate DSP Concepts for Building OFDM

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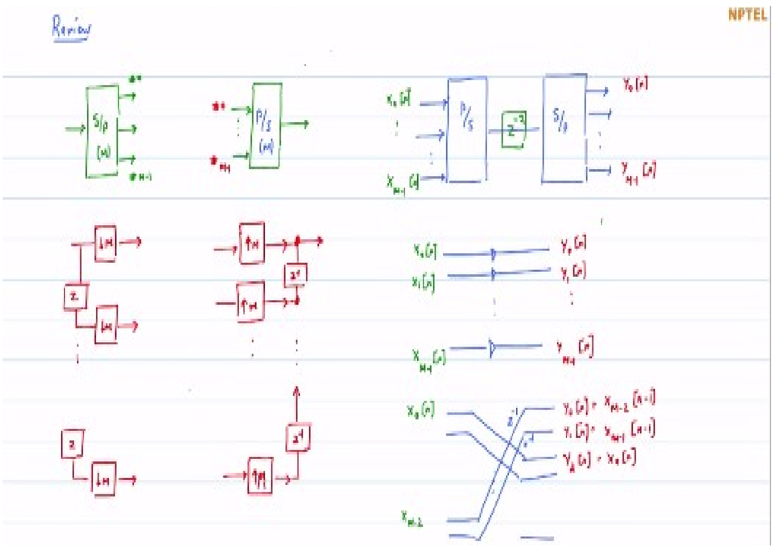

Okay, now I want to also refresh your memory on some other results that we have done, so this is going to be a quick review; review with a very specific purpose and hopefully, this is going to be the building block for some of the things that we are going to be discussing today. So, if you remember, we wrote about a block which had the following notation, it was a parallel to serial converter of size M, okay.

Now, do you remember how this was done, parallel to serial conversion, do you recall? Again, basically it was a down sampler, okay by M, that was the top branch and then we passed it through an advance operator, right because we want x, if x0 is on the first branch, we want x of 1 on the other branch, which means that it must have passed through an advanced operator, downsample by a factor of M dot, dot, dot last till, advance operator and then you passed it through a down sample by a factor of M.

So, this was the serial to; I do that wrong, it is a, this is a serial to parallel converter, I am sorry this is not parallel to serial, this is a serial to parallel converter okay, yes and then we also had another block which was the parallel to serial converter, so which to which you would feed in multiple blocks, this would have multiple blocks, this is output number 0, this would be output number M – 1.

So, likewise you have the input number 0, input number M – 1, this is a parallel to serial convertor, also of dimension M and this one we said had the following structure where you would have the up sampler, in order to get the sequencing correct, you would have up samplers in each of these branches but these would be combined using a delay, if you remember that was the of course, you can do it, the parallel to; serial to parallel conversion using a delay operation but then the data samples will get swapped around.

So, basically this is the structure that we have, okay up sample by M, going through a chain of delays and then getting combined okay, so I will just put dot, dot, dot, this is a, and adder, all of these are adding nodes okay, so that was the parallel to serial; parallel to serial conversion okay. Now, there was a result that we wrote down if you remember, which was sort of, seemed like a somewhat of a trivial result.

The result was; if I have a serial to parallel converter of dimension M that means, I will get these parallel lines and then I pass this through a parallel no, it should do the other way. I am sorry, I need to do the parallel to serial converter, okay, I have many of these parallel to serial converter, and then followed by a serial to parallel converter and if these are denoted as X0 of n all the way to X M - 1 of n and the outputs are denoted as Y0 of n to Y M - 1 of n.

This was trivially mapped in the following way, it was all parallel lines with a gain of 1 okay, so this was the X0 of n, X1 of n, X M - 1 of n, mapping to the output so basically, there was nothing happening in the processing, the input got translated to the output and this was a, seemed like a very trivial type of result okay. Now, can we revisit this problem because that is what is going to be interesting to us?

When we actually did the representation of the serial to parallel and the parallel to serial, we did not really study any applications, the applications are now coming in here. Now, keep this structure in mind basically, how the signal is combined in a parallel to serial converter and the serial to parallel. Now, if I introduced a delay of 2 units of time z - 2 okay, can you help me map what these input output relationships are going to be?

So, basically there are M inputs exactly like before, okay and there are M outputs, not clear that they will be just a straight mapping like before, let us just visualize it, there is a branch, take the upper branch, the topmost stream, the uppermost stream goes through a up sampling by a factor of M, then hits this delay, z power - 2 and then comes to the branch on the outside with the down sampling by a factor of M that will be 0, because that we know.

Because if there is any nonzero delay between the up sample and the down sampler that will get killed, so will this signal ever have a chance of coming to the output? Let us see if we come through the other; any of the other branches, if it comes to the next branch, it's up sampled by a factor of M, there is a delay z power - 2 and then the next branch has got z that means, overall transfer function is z power -1 will get killed.

But if you drop down one more channel, z power - 2 in the channel z squared in the chain will come out so actually, the X0 of n actually comes out on a different branch, it comes out on this branch, okay. So, this is X0 of n, Y2 of n = X0 of n okay, so and you can easily verify that the X1 of n will come out through Y3 and so on until we hit X of M – 2, okay. The last one is X of M - 2 okay, now X of M - 2 means that it was the last but one branch, it is got M - 2 delays in the chain that it will go through.

Through the channel, what did it get; another 2 units of delay, so it got z power - M as the; now which of these branches will it come out through; the first one because z power - M is allowed, that means non zero but when I do the down sampling, I will get 1 unit of delay, so this one will be Y; we use the red colour, this one will be Y0 of n is actually = X of M - 2 do not forget the delay, there is an n – 1.

So, basically there is a connection between this one and this but we have to mark a delay on that branch okay, when you simplify and similarly you can verify that the last branch also comes out with a delay okay, so Y1 of n = X of M - 1 n – 1, so a very interesting thing happened, it kind of took a cyclic shift, all of them sort of got a cyclic shift but when they came back up those 2 branches had a delay element to it okay.

So, this is something for you to keep in mind because we are going to build on this concept and just as we used this concept earlier, and we will come back to use it in today's class. We also had another concept that we have leverage, like to use it for today's discussion as well okay.

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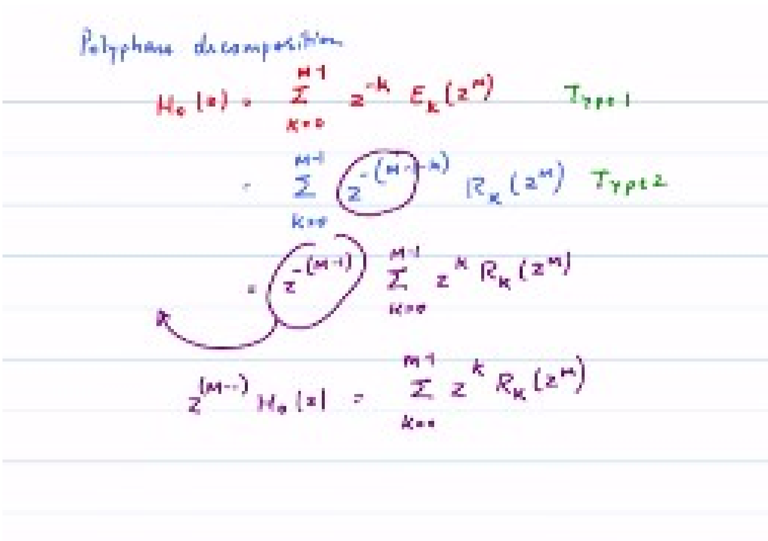

So, the other one was polyphase decomposition, remember there were 2 types of polyphase decomposition and almost very rarely we came across any application of the type 2 decomposition, so let me just refresh your memory on that because today we will be using the type 2; summation k = 0 to M - 1 z power - k Ek z power M, this is type 1, type 2 is summation k = 0 to M - 1 z power - M – 1 - k Rk z power M, this is type 2.

Now, one observation that we did not emphasise earlier because it was not necessarily the time for it, is to simplify this one, take this z power M - 1 outside the summation, what will it become; z power - M - 1 summation k = 0 to M -1 z power + k, there is minus of minus; z power + k Rk z power M, so this is type 2 also basically, I just have simplified that. So, actually another if you take one more step you take the z power - 1 to the other side, then what we get is z power M + 1, sorry z power M - 1 not + 1; z power M - 1 times H0 of z = summation k = 0 to M – 1.

This is z power k; z power k Rk z power M, okay I am just, nothing clever or tricky here just basically, rewrote the type 2 polyphase decomposition okay, now for a very specific reason, this looks like a non-causal representation, it is actually not non-causal it is strictly, basically what we did was you took a filter H0, you kind of shifted it and rewrote it so that you get these representation.

Now, why did we even take the effort to do this primarily, if you want to go back and look at this structure, if you want polyphase components which have a structure without delays but see otherwise, what you would do is; you end up with only the type 1, all of them will have delay, so if I want a structure where there is the provision for me to move the down sampler around but I want to have powers of z, right, delay, advance element; advance elements then that is why, so anything which requires this type of a representation you will actually, we will actually utilize the type 2 polyphase decomposition. So, again keep that picture in your mind, we will come back to link it to the, when we need it in a short while, we will do that.