Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology- Madras

Lecture – 04 (Part-2)
Reconstruction Filter – Part2

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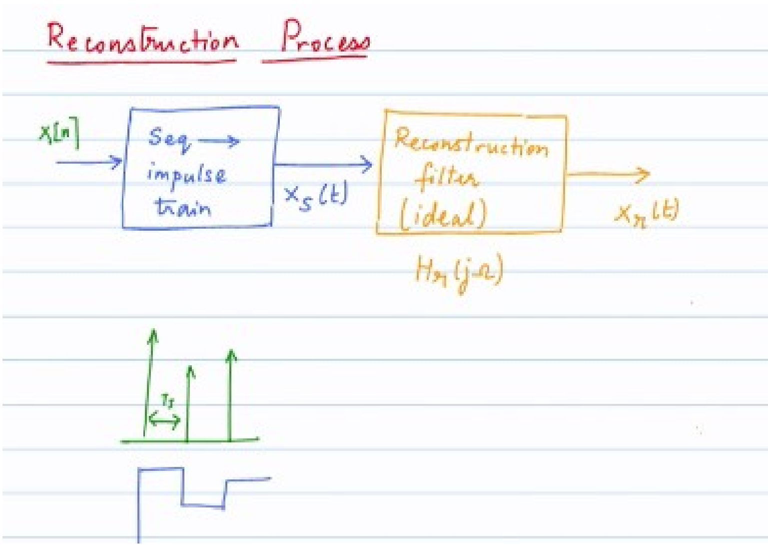

So, the reconstruction process, I have an impulse train which becomes Xs of t multiplied by the reconstruction filter Hr of j omega, a Xr of t.

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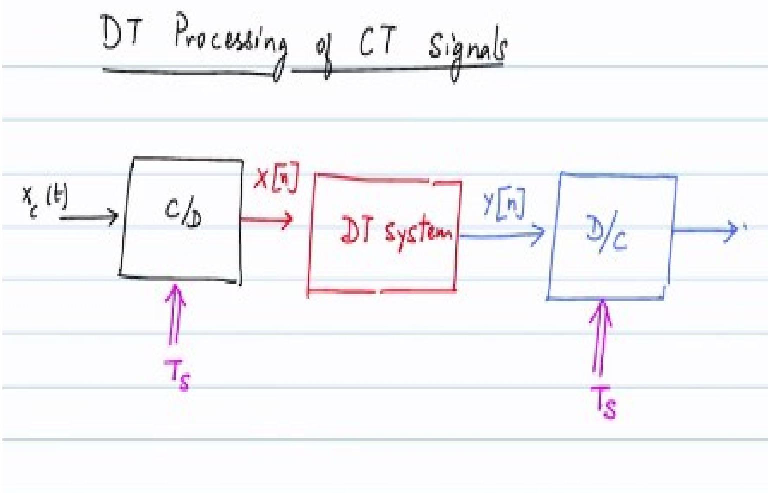

Now, I want to also at this stage itself, look at the some of the practical elements of the reconstruction process.

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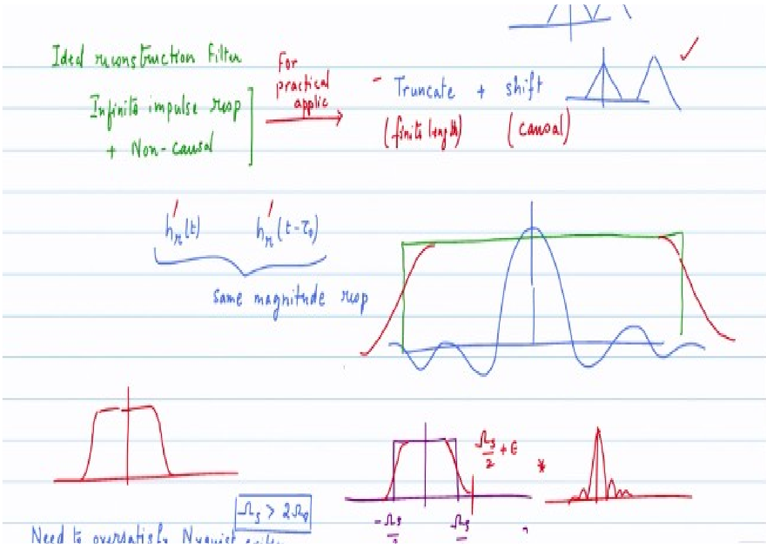

So, let us take a quick look at some of the aspects okay, so the ideal reconstruction filter that we have; we already noted the following properties, it is infinite impulse response, it is a sinc function that goes to infinity on both sides, so I call it as infinite impulse response and it is non-causal that it has an impulse response which is non-zero for $t < 0$, so this is not practical, I cannot use this in any practical reconstruction process.

So, I need to first of all change this from here, so for practical applications the one on the left will not do, I cannot use it, I have to do something different, so the first thing that I do is, I do not deal with infinite impulse responses, we have to truncate it once the amplitudes go below a certain value. So, the first thing that we will say is that we will truncate okay, truncate at what point? That needs to be decided but at least we will truncate to make it something that is manageable.

Then we of course, it will still be non-causal because it you are truncate it symmetrically assuming that you do not to destroy the symmetry properties, so then you have to shift, so the shift is; this is to make it finite length, truncate to make it finite length and the shifting is to make it causal okay, so those are the 2 things. Now, the another observation or property that we just want to observe is that if I have hr of t, okay and hr of t – tau0; some shifted version.

Then, they have these, both of these have the same magnitude response, okay both have the same magnitude response because the spectrum of this is just a e power j, a phase term when I take the magnitude that goes away, so both have the same magnitude response okay. Now, take it one step further if I truncate and, shift and truncate again I do not change the on sort of the truncated filter.

So, truncate; the shifting is something that I would say is not going to be an issue, I can always (()) (03:10)), and this term which we need to take care, so what I need to worry of what is the effect of truncation so, truncation essentially means that we have an underlying filter that is infinite impulse response, I will just draw it very quickly, it may not be very symmetric and other things.

But I want to truncate it, there are several ways in which we can truncate, we can truncate using a rectangular window; symmetric rectangular window or one of many, many windows which have got a smooth transition, smoother than so basically, you can have a window that that has a smooth transition, okay. So, ultimately the goal is to limit the impulse response of the reconstruction filter.

So, the corresponding frequency domain interpretation of what we have done; the ideal impulse response is an ideal reconstruction filter is a brick wall filter which goes from - omega s/2 to omega s/2, now the effect of truncation means that I am multiplying in the time domain, so which means I have to do a convolution in the frequency domain, so convolution in the frequency domain, these windows if you look at their response basically, they are low-pass in nature.

What you will find is that you will have some low-pass response for the window, okay now convolve these two; what you find is that you have a widening of the brick wall, so basically the low-pass property is retained, zero crossings are retained, the Nyquist property is retained because the windowing does not affect where the zero crossings occur, the spectrum becomes a little bit wider.

So, what we can achieve in practice is a reconstruction filter which has got the Nyquist properties; that is the corresponding zero crossings but has got a response that looks (()) (05:24) without on the sides okay, now notice that the cut-off point, this is omega s/2 + epsilon, it is slightly larger okay. Now, this is the main reason, why right from the beginning we were saying do not do exact Nyquist rate need to over satisfy the Nyquist criterion.

Because any time I need to do a reconstruction filter, the windowing based filter is probably the easiest way to do that, need to over satisfy the Nyquist criterion, why because the I do not want aliasing and basically I want to make sure that I can apply a proper filter without distorting the signal spectrum, so do not do the Nyquist sampling basically with Nyquist sampling means that do not do this type of sampling, this is not what we want.

But instead go for something where the Nyquist sampling is over satisfied and that will help us achieve with the practical filters, we can do the reconstruction, (()) (06:40) that is why we say that always we want to over satisfy the Nyquist criteria okay, so basically choose your sampling frequency to be greater than 2 times omega0, the band limited signal okay. So, again I hope (()) (06:55) what are some of the we need to keep (()) (06:58) and of course the key elements then.

"Professor – student conversation starts" I am sorry? Okay, okay so the very good question, the question is why am I trying to truncate and not try to actually use an ideal filter. See the, again we are going back and forth between what can be realizable and what can be not realizable, so this green line, basically a brick wall filter, even in the analog time I cannot realize it.

So, the way to visualize it is, why am I not able to realize it? Because that is a brick wall filter and one way to think about it is because this is a filter that will be in infinite in terms of and it is non-causal, so how do I get so, basically you move over from the continuous time to thinking about saying okay, if this is the property that I am, this is why it is not realizable basically, the brick wall even if I say use whatever order Chebyshev filter you will not get a response like this.

So, what is it that we can achieve, what you can achieve is something where the edges are not like a brick wall but some sort of tapering is occurring and that basically says you know to sort of link it to what we are familiar with if you had an impulse response which I then window then shifted, then I would get first of all a causal filter, which is realizable, the second one is it will be fine, it is basically what I am saying is the windowing plus the shifting gives me a realizable filter that can be again equated to either a Chebyshev filter or a Butterworth filter in the analog domain.

And say that okay whatever is the best that you can do and you will find that most of the times the best that we can do in terms of the analog filters will be of this form right. so it is (()) (09:05) the brick wall is not, and how do we make it to go from there to something that is realizable but a very important question because very often, we are trying to make that link and again in fact during the course of this lecture itself, we will come to one more such non realizable condition which we will have to change.

Because in the reconstruction process if you notice I assumed a block which is not realizable, how do I generate Dirac delta's? I cannot do that so, what is the next best that I can do instead of a Dirac delta, so this is again maybe it is a good, good, good that you raised the question. **"Professor – student conversation ends".**

So, basically what we are saying is this is what we need to generate right, corresponding to the impulses x of n, dirac deltas which are spaced by Ts, these are dirac deltas, now this again I; very difficult for me to generate, but what we can generate in practice is something which is of the following form, so what we call as a sample and hold, so you take a certain value and then say okay it is a rectangle approximation.

So, sample (()) (10:24) it, holds it till the next sample comes along, so the next sample comes along, so instead of a direct deltas of the proper scaling, I get a sample and hold circuit. Now, what is the impact of a sample and hold, so that means my reconstruction filter now has to change because now I have changed the continuous time signal, so what is the change in the reconstruction signal in order to make it realizable? So that we will answer that question.

But good that you asked because always important to know how do we go from something that is ideal to something that is realizable? Very, very important that is part of our the thing that we want to help and okay, so again; yes, **"Professor – student conversation starts"** no, no digital anti-aliasing filter you can specify it as a causal filter, so you of course, you can design digital filters that are non-causal and then do the shifting.

But there are several filters which again when we come to it maybe we can make a statement but let me just sort of make a statement and this in the last page of the (()) (11:37) **"Professor – student conversation ends".**

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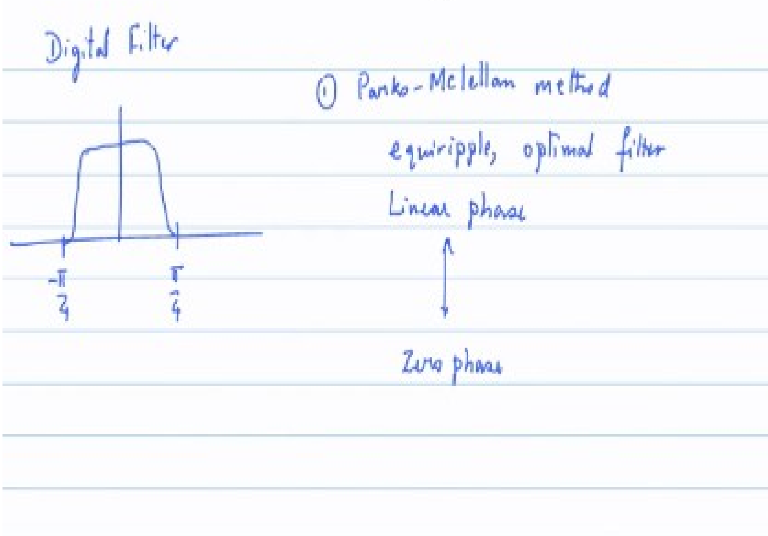

So, in the context of digital filters, I am glad that you asked, so a digital filter okay, so now we said that we have a digital filter which has a certain response and we can specify and the cut-off happened to be pi/4; - pi/4, now there are several methods, methods that (()) (12:07). Parks McClellan method which designs for us, if equi-ripple optimal filter, okay, Parks McClellan filter is also a linear phase filter.

Now again, I would like to refresh or recall to your mind, there is a relationship between linear phase and zero phase, zero phase requires symmetry in the time domain right, the Parks McClellan filters are centred around 0 and that are non-causal that is the zero phase version but because they are a finite in length you can always shift them to get a linear phase filter, the minute you shift a zero phase filter it will become a linear phase filter.

So, what you will get is it, so yes you are right if you want zero phase, it will be non-causal but if you are willing to take a linear phase filter which is perfectly okay because in our case the reconstruction process primarily has to remove the spectral copies, so which means that linear phase filter would do the job equally well so, FIR filter designed by one of the methods like the Parks McClellan method would satisfy the requirements for what we are looking at in terms of reconstruction.

But yes is there a zero phase version of the same filter yes, there is because if you look at it as a filter with a time domain symmetry around omega n = 0, then it will be zero phase and symmetric you shift it, it becomes linear phase with and causal, okay good, very good question. **"Professor- student conversation starts"** Yes, okay, very, very good question, so the question is when I do the truncation, right, let me see where did I; where do I have the truncation figure okay, here it is.

So, the question is how do I design the truncation such that I do not run into aliasing effects, so this is precisely where the over satisfying the Nyquist criteria will come into play, see even if it has a ringing effect let me see if I can highlight this okay, so here is a ringing effect but you notice that based on what filters we design, we can have the ringing to be minimized, so in fact there are filters which have got the following type of behaviour, so it goes like this and then pretty much stays down, okay.

So, there are certain windows that have got very, very low side lobes, now you would use something like this to make sure that you do not have ringing again, the reason I used a one with ringing is to show that you should not forget the fact that when I do truncation there will be some side lobes which have to worry about but there are filters which have very low, and so what will happen is this part will get widened, right.

Your, the main brick wall part will get widened but you are over satisfying the Nyquist criterion, so once you satisfy; over satisfy the Nyquist criterion, you can make sure that the windowing you know yes, it will have ringing but it will die down at some point and it dies down before the aliasing part begins, right very good question. The reconstruction filter is an analog filter but just like we did anti-aliasing where we did a combination of analog and digital, we will find that we can do a combination of analogue and digital as well.

Because what you can do is; you can do over sampling of the digital signal, so that your spectral copies now move further apart, correct when I do over sampling of a digital signal it will move the spectral copies further apart and then my analog filter just has to get rid of the this thing, so yes the last stage of the reconstruction has to be an analog filter, but it does not have to be a high-Q analog filter, if you have incorporated some of the multi rate techniques.

No, wait, wait, please ask your question again, correct, well, if you want to visualize it from an impulse response point of view, but you may not implement it as a convolution at all, the last stage maybe just an analog filter, so I am using this as a way by which to explain how do I go from a non-realizable filter to a realizable filter, where do I have to pay the price; the price will come in the fact that I cannot get a brick wall filter but I will get a filter with a finite transition, right that is all.

As long as we are comfortable with that this red line will be an analogue filter, right but in terms of the visualization we can think of it as okay, the way we achieved it was by changing the underlying property of the ideal reconstruction filter to match something that is so, because the windowing sort of gives us a vague explanation as to why the spectrum became rounded, you did not have a brick wall response.

Why did it become > omega s/2, again that is because of the windowing and the ringing that will come in because of windowing, so any practical filter will be of this type which means that yes I need to be over sampled with respect to a Nyquist rate, very good. **"Professor – student conversation ends"** Okay, I am not sure what is the time okay, so let me just sort of give you something for you to think about as you prepare for the next class.

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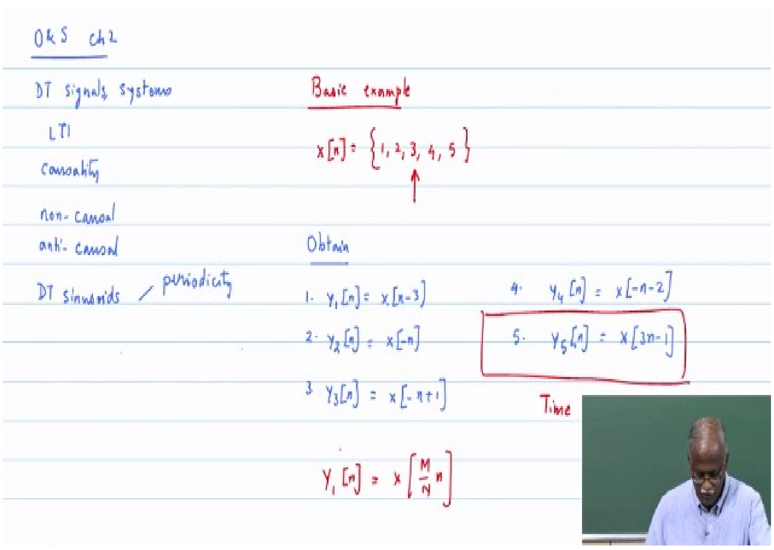

And we will just spend a minute to highlight what it is that we need so, basically I would like you to review Oppenheim and Schafer : Chapter 2, all the elements of discrete time signals of discrete time systems, the LTI properties, how does it manifest itself and also the notions of causality and when do we say something is non-causal, when do we say something is anti-causal again, these are just basic definitions but I would like you to be comfortable with that.

And I would like to look at several some simple examples and again, one very unique property of the discrete time signals is the discrete time sinusoids, okay again, when will there be periodicity? Not all discrete time sinusoids are periodic, so look at what is the conditions for periodicity and maybe as a simple test case what I would like you to do is; look at the following example, okay.

I will just write down the question and then request you to try it out so that you can we can then look at it in a subsequently okay, so if I have a sequence x of n which is 1, 2, 3, 4, 5 with an arrow under the 3 that means that is the origin, I would like you to obtain the following. Sequences that are obtained from this; 1; y1 of n is x of n - 3 basically that is a shifting of the sequence.

Second; y2 of n is x of - n, time reversal, y3 of n is x of - n + 1 that is shift and time reversal, fourth; y4 of n = x of - n -2, again shift and a time reversal, a little bit you have to be careful when you do the shifting and the last one is y5 of n, x of 3n -1 okay, now I am sure you are familiar with this I am sure you would have done lots of examples what we would very much like to focus on is the last case.

Because that is taking us into the multi rate domain because what we are fundamentally doing here is changing the sampling rate, we in Oppenheim and Schafer, it is called time scaling okay, so please look at time scaling very, very important, you can either increase the time scale or reduce the time scale and the most general form of time scaling is y1 of n = x of M/N * n, okay.

Now, the exact definitions of the time scaling are all present; are given in the book, this is the form that we are most interested in okay, so please make sure that we look at it I think we have lost power but let me stop here basically, look at time scaling, basic operations with a focus on time scaling because that is what I would like to pick up in the next class, thank you.