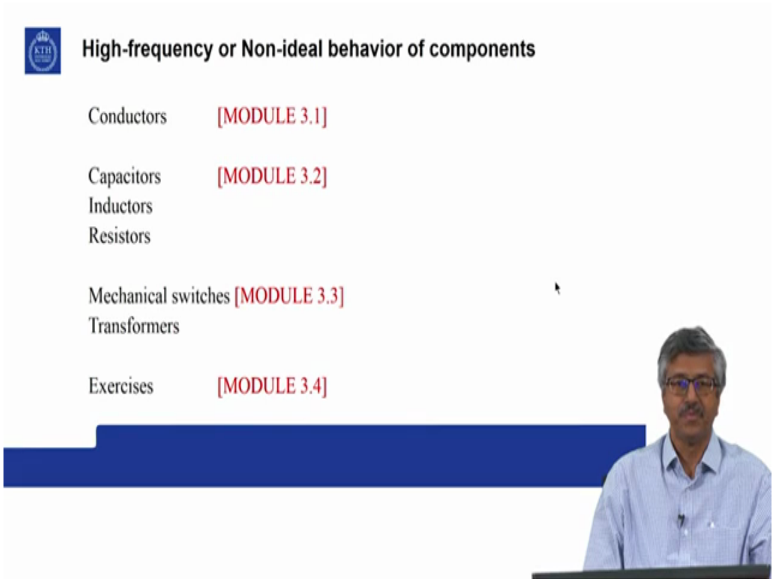
Electromagnetic Compatibility, EMC
Prof. Rajeev Thottappillil
KTH Royal Institute of Technology
High-frequency behavior of electrical components - Conductors

(Refer Slide Time: 0:24)

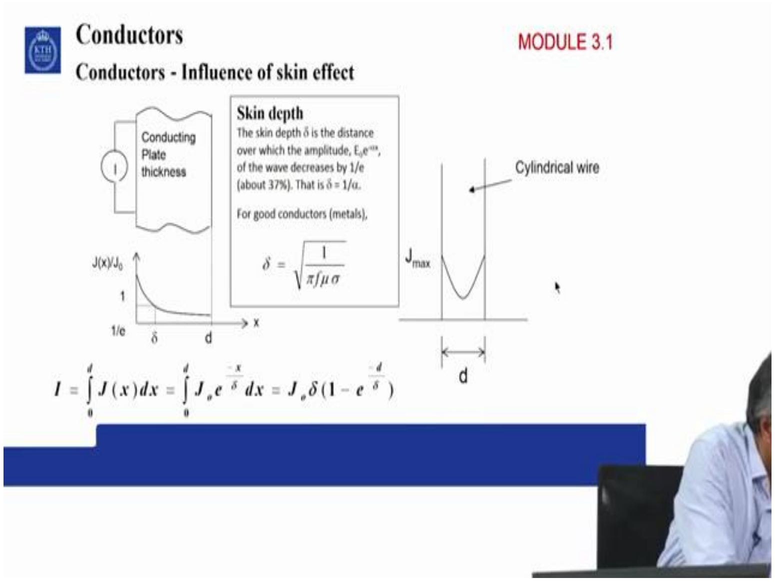


Welcome back again. Now, we enter into chapter 3, high-frequency behavior of electrical components. This chapter is divided into four modules. First, we will look into the behavior of conductors, plain conductors in the form of wires, it can be of different cross-sections, round wire or a rectangular wire. It does not matter what it is. Or it can be plain sheets. Then, we look into some of the electronic components that are called capacitors, inductors and resistors.

These are the basic passive electronic components. And of course, these electronic components also has got wires in it in the form of component leads, where you solder these components into printed circuit boards. And you get a combination of conductors and these ideal capacitors, inductors and resistors. Then we look into mechanical switches which are supposed to be switching off the current, on the current. So ideal mechanical switch commute between zero voltage and certain finite voltage in zero time but of course there are no such ideal mechanical switches.

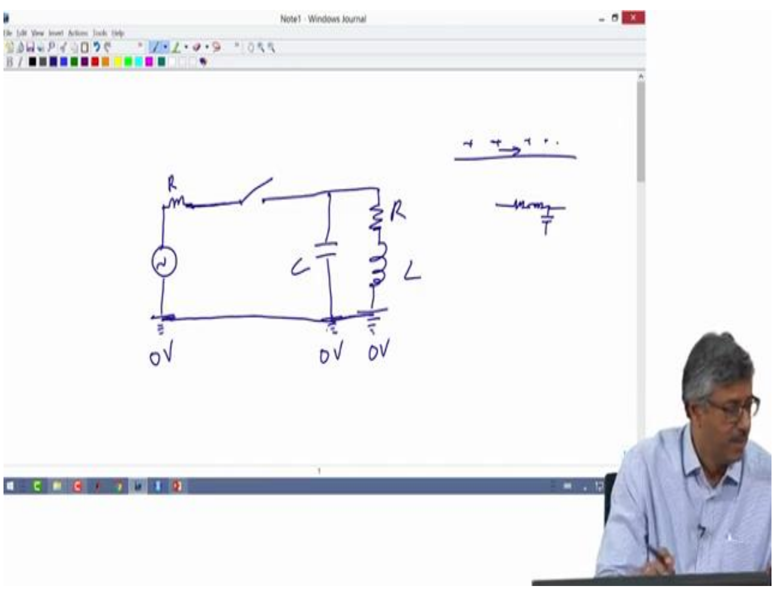
Then the last module is some numerical exercises. We have an idealized version of these components in our mind, but we can see that these components are not behaving as we think it would under high frequencies. When the frequency is increasing, lot of things are changing the behavior of conductors and capacitors, inductors and resistors, also switches or transformers. We will look into those.

(Refer Slide Time: 2:25)



Now before talking about conductors, I will try to illustrate what I mean to achieve in this chapter.

(Refer Slide Time: 2:45)

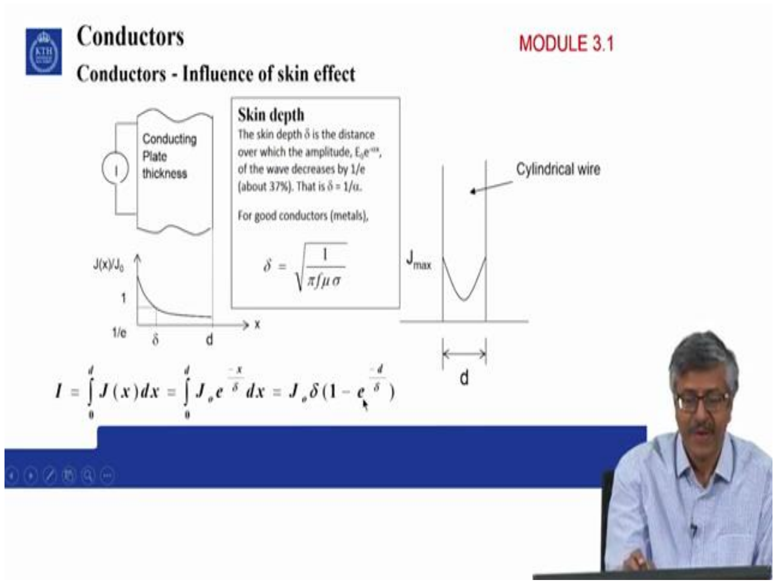


Take the case of source. It has got some internal impedance. Let us represent it by R. So the source is connected to a load by a switch. Then you have certain resistance and inductance for the loads. So, the source has a reference, so this is the symbol for the ground. This is grounded over here. Or we can have capacitors across, again it is grounded or connected to reference. So this is a kind of a circuit. This is R, L and C. So, in this circuit we think of certain behavior for R, certain behavior for L and C. And by this (ground) we means that they are all at zero volts, same voltage.

But when you do this in practice, you have some sort of a track connecting all these grounds together and that is made of a conductor. Similarly, all these components are connected together by conductors. So these conductors carry certain currents and it can have certain voltages in it. So due to that, it is producing some electric fields and some magnetic fields. So associated with electric and magnetic fields and the currents and since these are not ideal conductors, you can have some resistance, some inductors and some capacitance like that.

So what we call as a plain conductor of zero impedance is actually an impedance network under high frequencies, when frequency is increasing, not for DC. For DC, you can have some resistance of course. Then the capacitors also are not ideal, so what your real circuit under high frequencies is much more complex than that we see over here. We can have potential difference between this point and this point because of the impedance of this conductor. So it is these things that will be, that we are coming into by describing the non-ideal behavior of each of these components. So the concept developed here will be used in chapter 4 while talking about cross-talk and other interference coupling mechanism.

(Refer Slide Time: 5:47)



So, let us go to the conductors. Especially we will introduce an important concept called skin effect. It kind of idealize or it kind of state that under high frequencies the current density across the conductor is not uniform. For example, if you have a super-conductor, since you cannot have any field penetration inside the ideal conductor, all the currents will be confined to the surface, because it is super-conducting, conductivity infinite theoretically. So you cannot have an electric and magnetic fields inside the conductor, so there cannot be any current inside or potential difference inside. So all the currents are confined to the skin.

But there are no real super-conductors or the conductors that we use are not super-conductors and have finite conductivity like copper, iron or aluminum. With finite conductivity and under DC, the current distribution across the cross-section is uniform. So if you take a round conductor, and if it carries a DC current, the current distribution is uniform. So if we represent J_{max} like this, so it will be a constant current distribution like a straight line, not a curved line that is shown over here.

So, in a plate of thickness d, again it will be a straight line, not a curve like this. But what happens when conductors are subjected to high frequency, when a high frequency current is passing through this, then the magnetic fields tend to crowd or the currents tend to crowd along the surface of the conductor. So we have maximum current density along the surface and the current density is reducing as you go deeper into the conductor.

So then you can define a condition like this, the skin depth delta is distance over which the amplitude, $E_0\, e^{-\alpha x}$, of the wave decreases by 1/e. That is, alpha is the propagation constant that we have seen in chapter 2. If the wave amplitude is decreasing by 1/e, that is 1/e is about 37 percent. Then that distance is called the skin depth. So if we substitute in this, you can see that skin depth is nothing but 1/α, the attenuation constant.

So skin depth is 1/attenuation constant (1/α). And for good conductors we have defined, we know the attenuation constant and skin depth is obtained as $\sqrt{\frac{1}{\pi f \mu \sigma}}$, where f is the frequency, μ is the magnetic permeability and σ is the conductivity of that metal. So this is the relation for skin depth. So, inspection of this will show immediately some interesting properties of skin depth.

For example, skin depth is decreasing with increase in frequency. That is, as frequency increases more and more, currents are crowding along the surface. So this is cylindrical wire, round in cross-section, so this is vertical is current density. So as the frequency is increasing, this becomes steeper even. And as the frequency is increasing even further, you may come to a state in which most of the current is confined to the surface.

And the inner part of the conductor is not carrying any current. So this is the reason why we have stranded conductors often when we carry high frequency currents. Because stranded conductors are more efficient than a single solid conductors because we do not use all the cross-sectional area of the conductor for carrying the current. Now if you integrate for the current across 0 to d, the thickness and the current density, then you can get this expression:

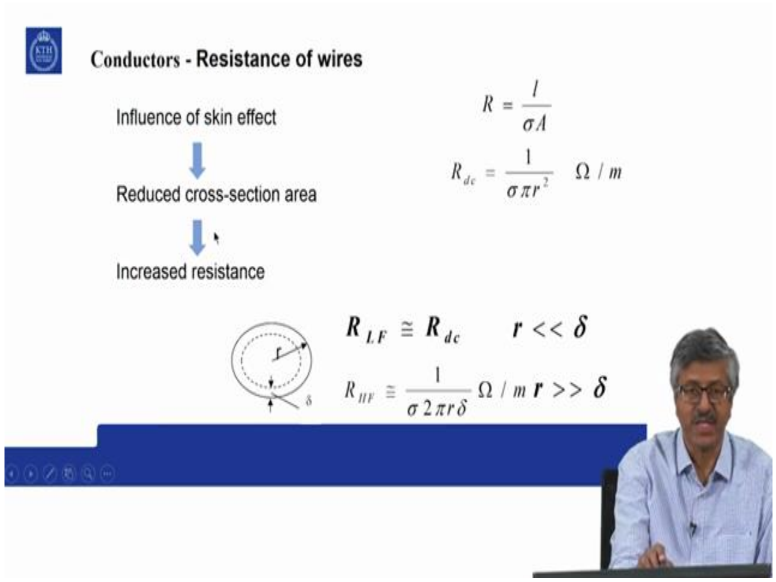
$$\int_0^d J_0 e^{\frac{-x}{\delta}}\, dx$$

J_0 is the initial current density, x is the distance along this axis. Then you will get

$$J_0 \delta (1 - e^{\frac{-x}{\delta}})$$

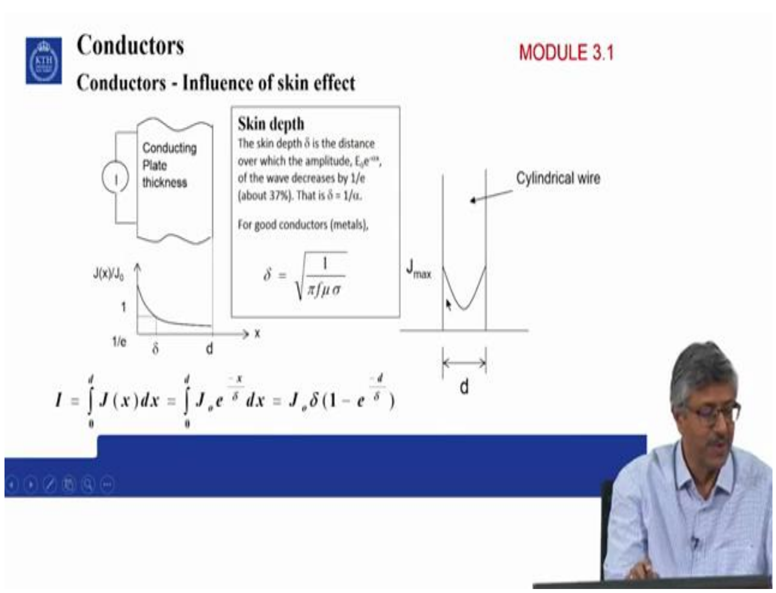
So this would be the expression for the total current.

(Refer Slide Time: 11:40)



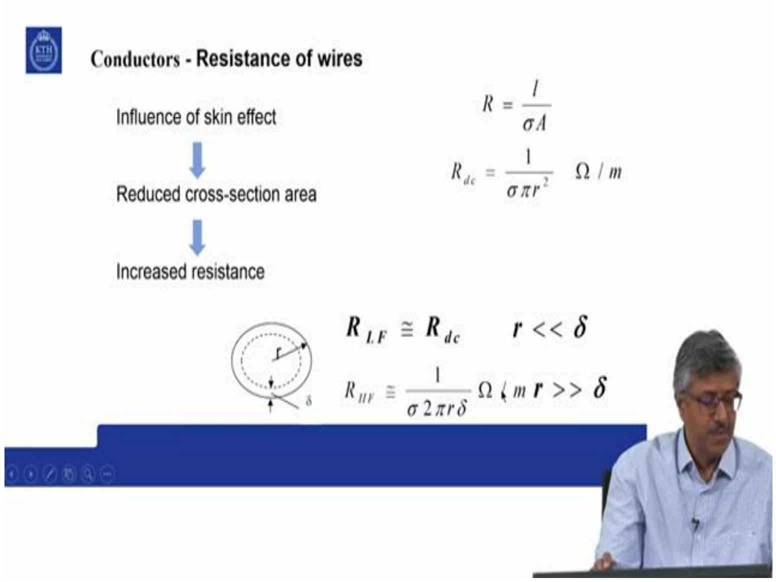
And you can take a special case of round conductors and do a calculation for high frequency resistance. $R = \frac{l}{\sigma A}$, σ is the conductivity, A is the cross-section. So this is the formula for the resistance. And per meter DC resistance will become $\frac{1}{\sigma \pi r^2}$. That will be the per meter DC resistance. Now the influence of skin effect is coming in this way, reduced cross-sectional area.

(Refer Slide Time: 12:41)



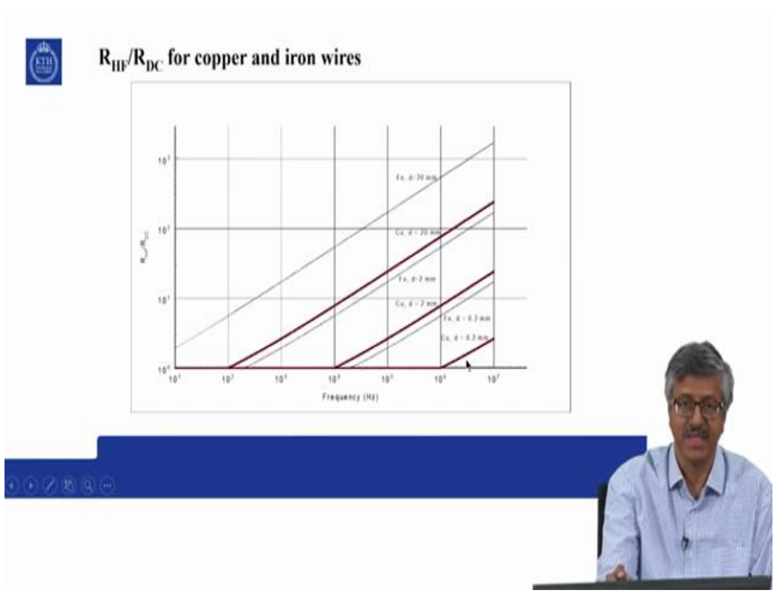
Because as you go much higher in frequency here, this δ is much smaller, much smaller compared to d. So this becomes almost negligible and almost all the current is confined to the surface. J_0 δ will become the total current. So, they are confining to one skin depth δ.

(Refer Slide Time: 13:10)



So that is what we see over here. So low frequency resistance is approximately equal to DC resistance when R << δ. When the radius is far greater than skin depth, and if it is confining to one skin depth δ, then high frequency resistance can approximated as $\frac{1}{\sigma 2\pi r\delta}$, then area of this small section over here, small washer shaped section here. That is, $2\pi r\delta$ so that will be this area. So that will be the high frequency resistance.

(Refer Slide Time: 14:01)

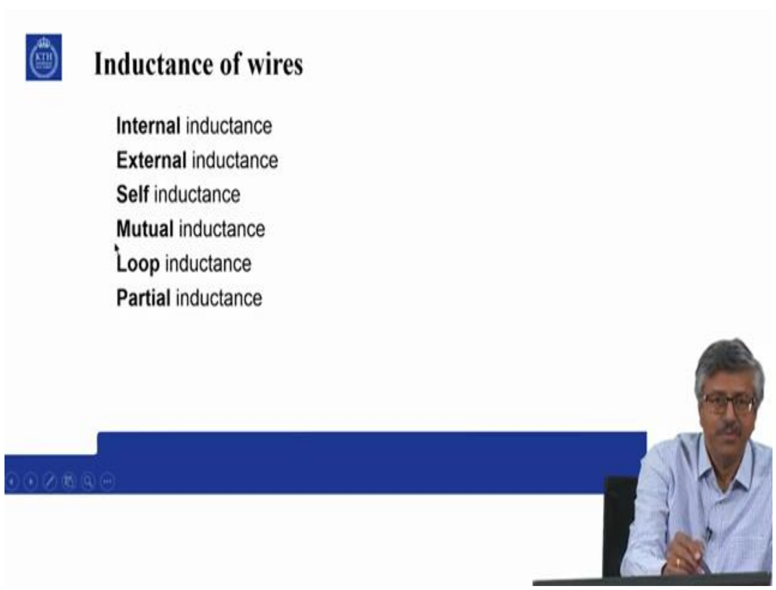


Now we can see, we can do a comparison of high frequency resistance to DC resistance for copper and iron wires. So high frequency resistance is much higher than DC resistance and if you take the ratio, you will see that the effect of skin depth is already coming at 10 Hz for very thick iron wire of 2 cm in diameter or 20 mm in diameter. Even at such low frequency you have some effect of skin depth. So for copper already at 100 Hz of 2 cm diameter copper, you will have effect of skin depth.

Say for example, at 10 KHz, already iron will have a high frequency resistance that is around 30 or 40 times the DC resistance. So skin depth make a big difference in the resistance. For thin wires, the effect of skin depth happens at, much higher frequencies. Say if you want to, if you decide that you need a surface area for the conductor, that is effectively equal to say 20 mm for copper of diameter, you can achieve the same cross-sectional area of 20 mm with a diameter.

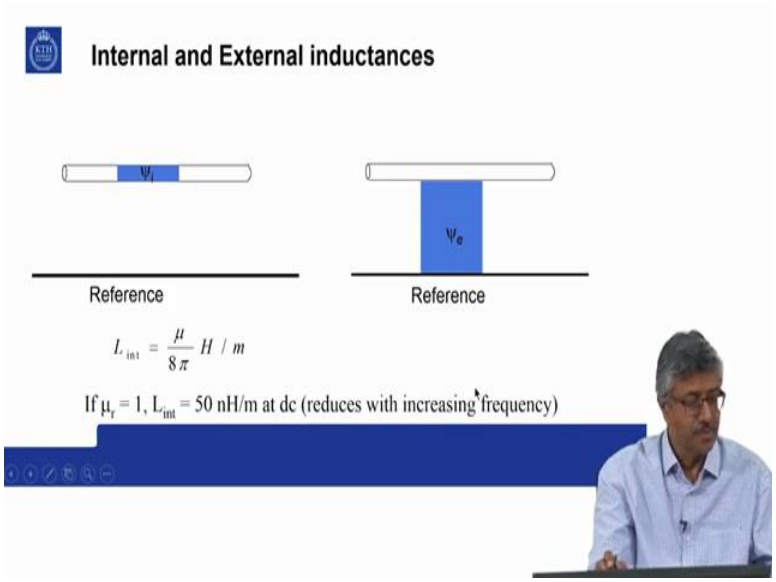
So πr^2, so that same area, cross-sectional area you can achieve by stacking several conductors, small conductors together. That is, standard construction. Then you will have to face the probably most skin effect only at 1 MHz or so instead of already at 100 Hz. So this is a strong motivation for using standard conductors. Of course, another reason is that the mechanical one. Standard conductors are very flexible compared to solid conductors.

(Refer Slide Time: 16:42)



Now another property of wires are inductance. Everywhere we have inductance, anything that carries a current or have certain magnetic field around it will have some inductance. If you look at the literature, inductance concept is more confusing than resistance or even capacitance. Because, often inductance is in association with return circuit and often that is not very clear. For example, you have terms like internal inductance, external inductance, self-inductance, mutual inductance, loop inductance, partial inductance. So there are several terms like this. And what are these? What are the differences between these various types of inductance? So the next few slides we will devote time for this.

(Refer Slide Time: 17:47)

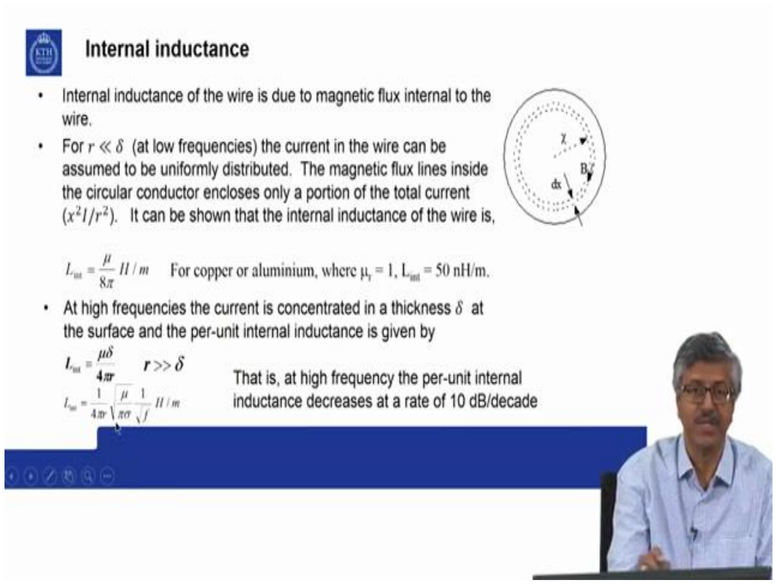


So here we describe internal and external inductances. Say suppose we have a reference, and some current is carried by this conductor and these currents fill the inside of this conductor, so there is flux inside. You just ignore all the flux outside now. So the flux inside linking with that current itself, that you can define an internal inductance. Inductance is defined as flux linkage divided by the current causing it. So that is called the internal inductance. The flux is confined to the internal part of the conductor.

For DC and for low frequencies, you can find that internal inductance is nothing but $\frac{\mu}{8\pi}$, H/m. That will be the internal inductance. And this will be for copper conductors, internal inductance will be, $\frac{\mu}{8\pi}$ will be around 50 nH/m. Internal inductance reduces with increase in frequency because, as you increase the frequency, you do not have that much flux linking with it because the currents and fluxes are more confined to the periphery.

So you can set up equations and find that inductance will be reducing with increase in frequency. So internal inductance is not of a problem really for most conductors because external inductance is much larger. So external inductance is defined as the flux due to the current in this linking with this routed circuit or reference so this flux divided by the current, or rate of change of this flux divided by rate of change of this current. So that is external inductance. So this will be much larger compared to the internal inductance.

(Refer Slide Time: 20:22)

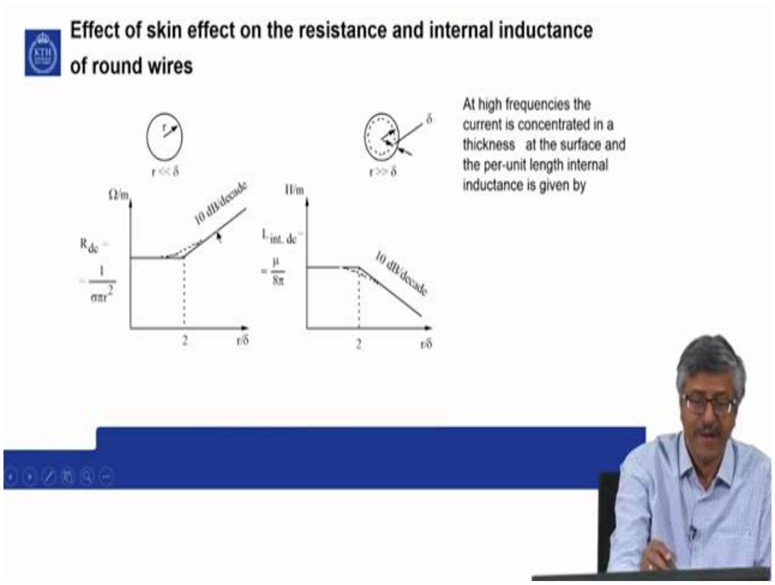


So some more details of finding the internal inductance is given here. So this is radius r and this is x, let us say arbitrary value and you get the total current by integrating from x is equal to 0 to r. All the current densities available here, so if you take a ring at a distance x having a flux B like this, you see that for low frequencies the current in the wire can be assumed to be uniformly distributed. And the magnetic flux lines inside the circular conductor encloses only a portion of the total current and that portion is proportional to πx^2, this area. Whereas total current is proportional to the area of πr^2.

So you can take this ratio and this is the portion of the magnetic flux inside, linking with this current. Now this we have seen, so we get $\frac{\mu}{8\pi}$ if you do the integration. And at high frequencies the current is concentrated in a thickness of δ at the surface. And per unit length inductance is given by $l_{int} = \frac{\mu\delta}{4\pi r}$. This also you can set up from this figure and integrating with appropriate limit you will get this value for r>>δ.

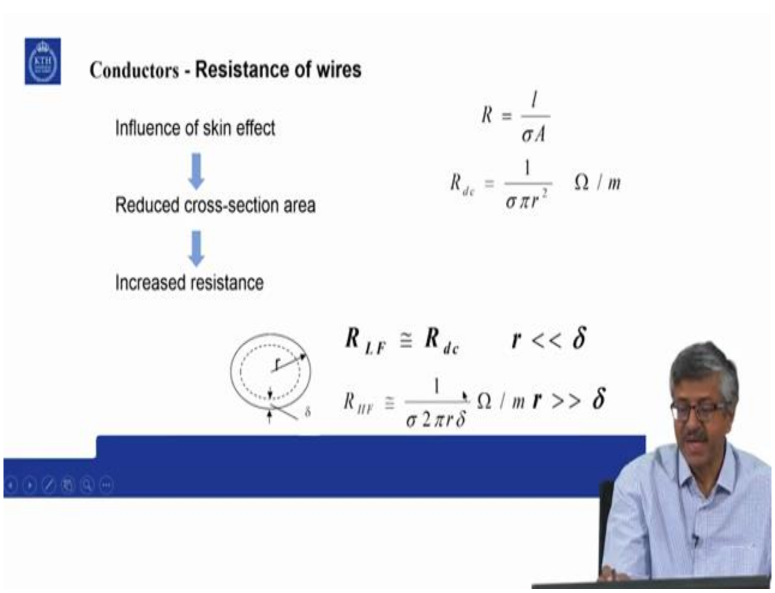
So internal inductance, explicitly writing it out δ, you see that it is reducing in proportion to the root of frequency. So you can say that high frequency the per-unit length internal inductance decreases at a rate of 10 dB/decade. In the previous class you have seen what it meant by, a dB. Then because even resistance you can express in terms of, or impedance you can express in terms of dB. So 20 log $1/\sqrt{f}$. So 20 into 1 by 2 is 10. So per 10 times this in frequency, you have 10 dB increase in inductance. So that is the meaning of this equation.

(Refer Slide Time: 23:31)



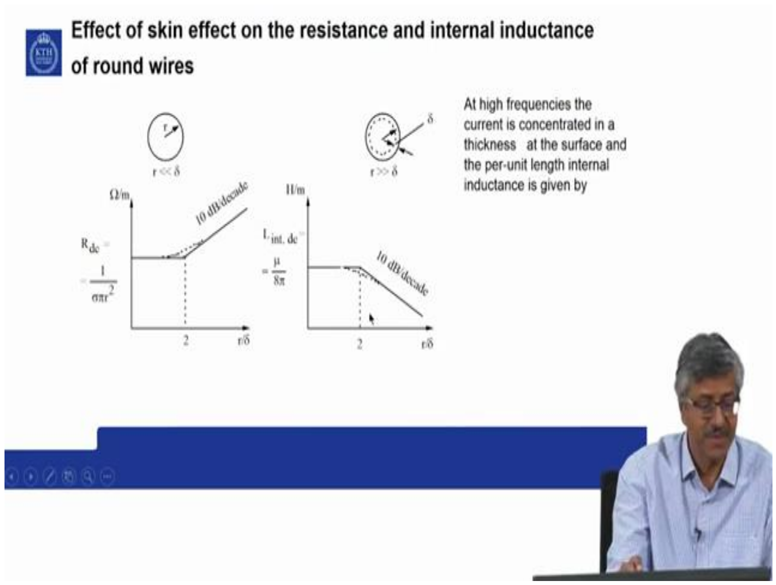
Now effect of skin effect on the resistance and internal inductance of round wires: we have seen this that it is increasing 10dB per decade. So it is shown in the form of a curve here. The basic expressions you have seen.

(Refer Slide Time: 23:56)



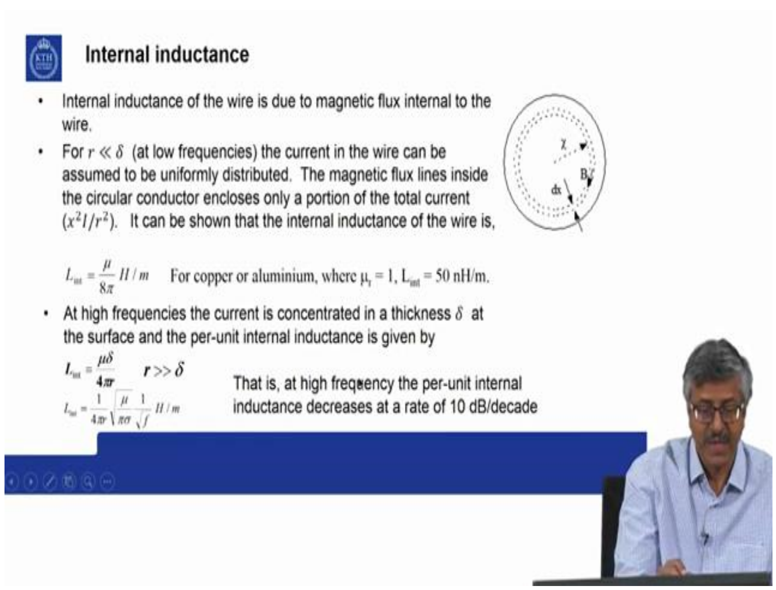
This is the expression, 1 /δ. And $\delta \propto 1/\sqrt{f}$, so root of is coming on top, so 10 dB per decade increase.

(Refer Slide Time: 24:14)



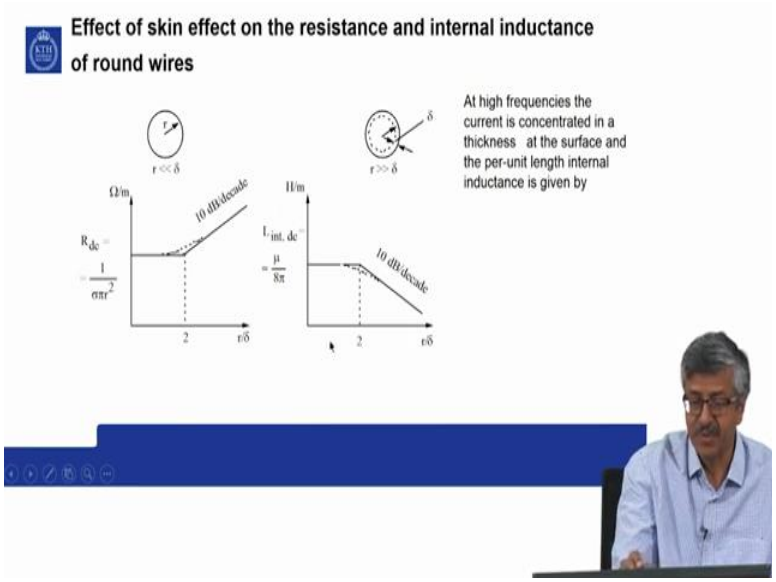
And that is what we are seeing in this picture. So this is return in terms of the ratio r/δ. Then this is certain dB /decade decrease, that expression we have seen before.

(Refer Slide Time: 24:42)



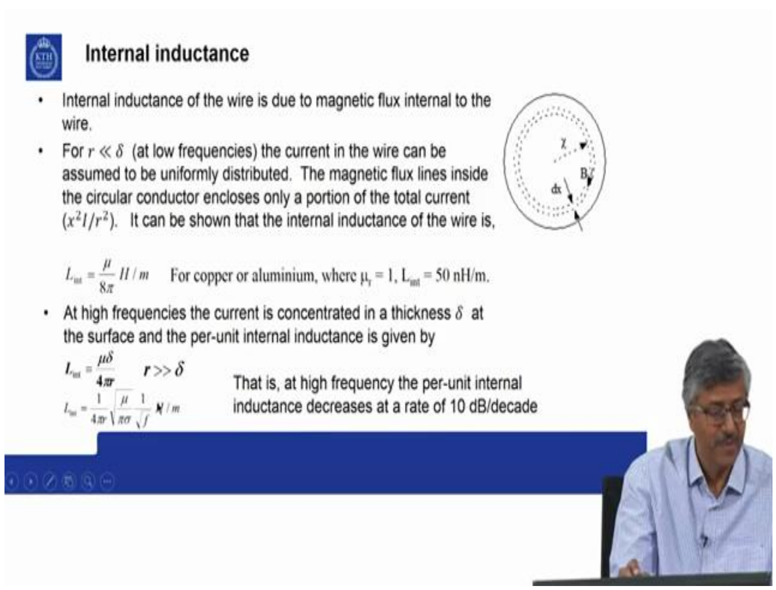
This expression.

(Refer Slide Time: 24:51)



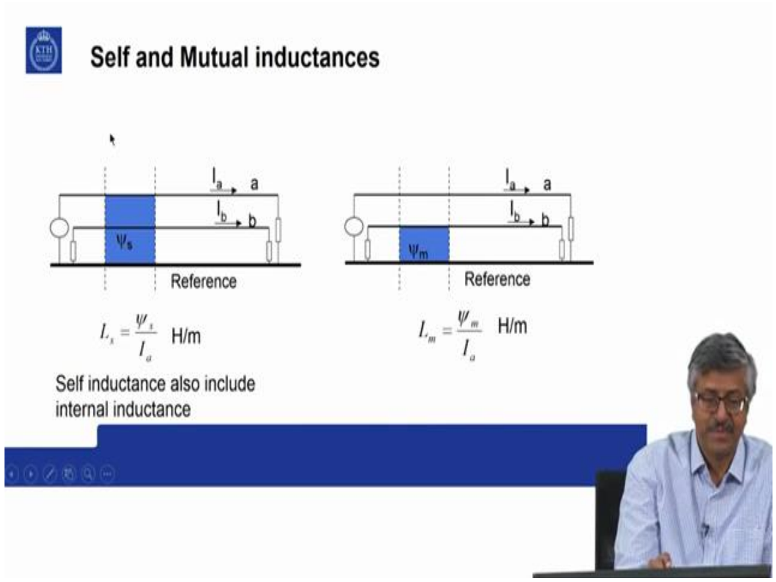
You take r/δ, you plot across it.

(Refer Slide Time: 24:56)



So that is coming, this is high frequency, f is coming here.

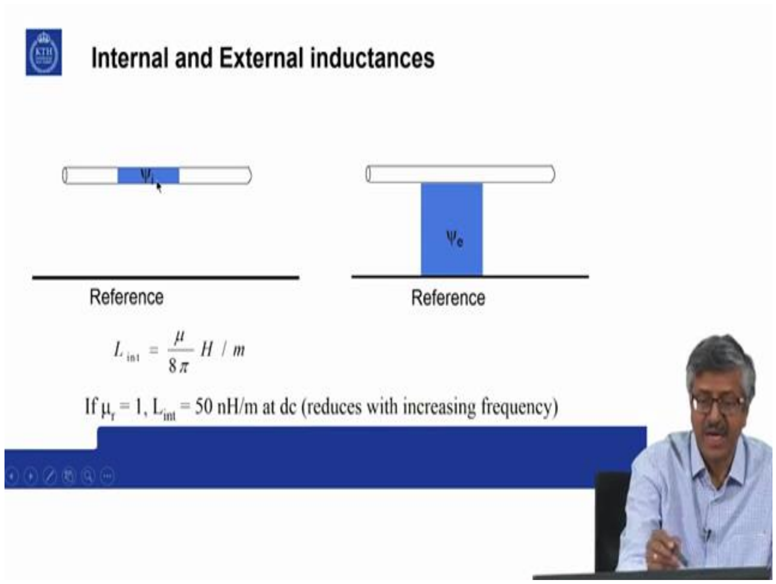
(Refer Slide Time: 25:15)



Now self and mutual inductances, self-inductance is defined like this say for example, you have some flux φ_s, that is marked over here and you have certain current I_a here. So you take this circuits, ‘a’ and reference here like this. You take all the flux set up by I_a between these two wires per meter. And that flux derived by the current I_a, that is defined as self-inductance. So self-inductance will also include the internal inductance of this wire. Then distance of the wires also will be included in the self-inductance.

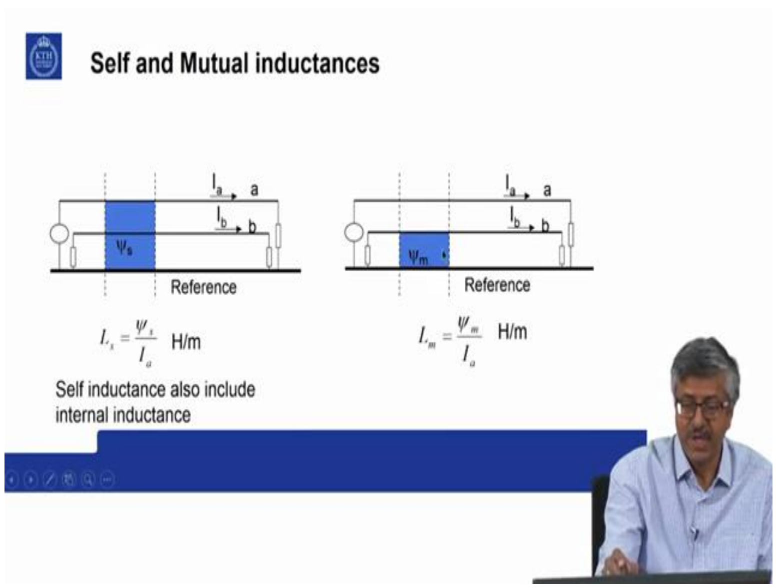
Now in the mutual inductance, you take this current, you see how much it will, how much of the flux lines, flux will couple with the circuit b. So that one, that flux, mutual flux divided by this current is called mutual inductance, H/m. So, self and mutual inductance is different from internal and external inductance, so there is important differences, so do not confuse between them. Internal inductance is part of the self-inductance.

(Refer Slide Time: 27:16)



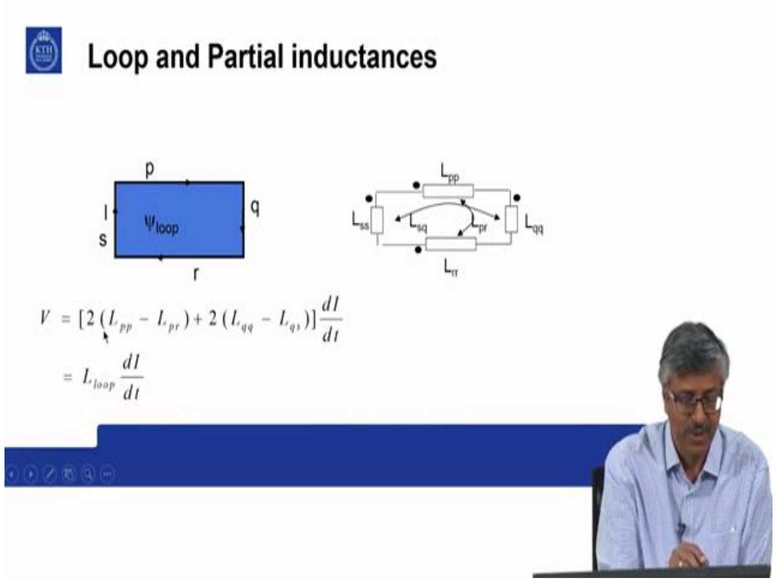
So here we have seen the external inductance. So this external inductance of this wire plus this internal inductance together will form the self-inductance.

(Refer Slide Time: 27:29)



And this is mutual inductance, interaction of the current in one circuit with the other circuit. So this is mutual inductance.

(Refer Slide Time: 27:49)

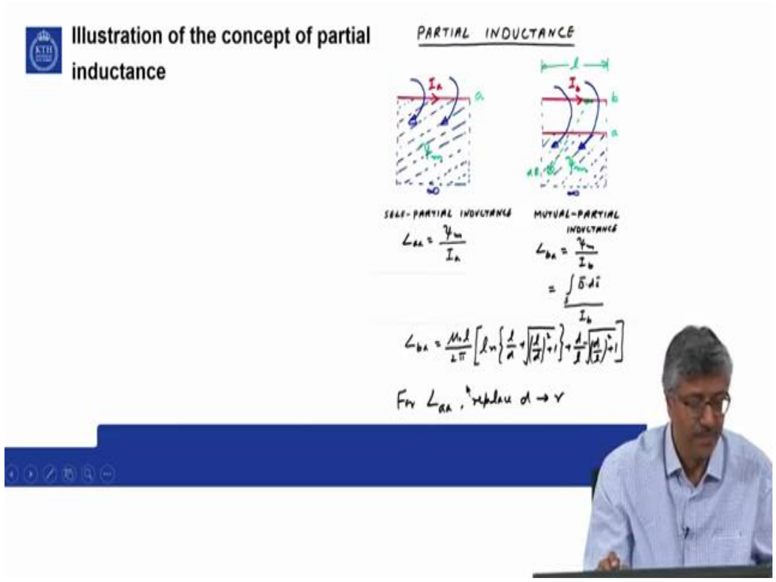


So loop and partial inductances, so these concepts are not very common but sometimes they are used. Say for example, if you have a loop like this with sides p, q, r, s, carrying some current because of loop inductance, so you take the circulating current and flux confined by this loop. So the ratio is defined as the inductance, but sometimes the loop can be of regular shapes or in zigzag fashion. So it is very difficult to calculate exact area and exact ratio of area to current.

So in that case you can assume that this is having an inductance, this portion, say side p and partial inductance and this portion is assumed to have its own inductance. This portion has the inductance. So you are assigning to each piece of this wire that are orienting differently, one inductance value. So how you define this inductance value that we will see in the next slide. But if you can define like this, you can calculate the total voltage drop across as $V = L_{loop}\frac{dI}{dt}$.

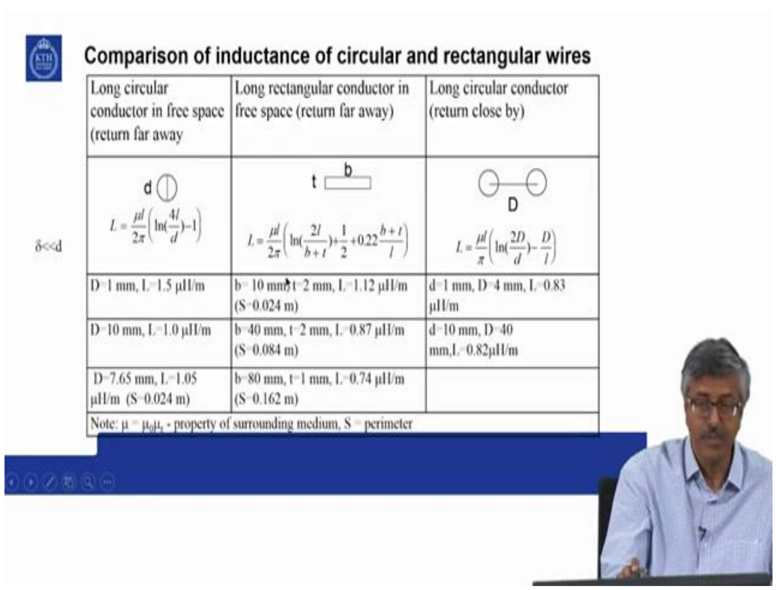
But loop inductance is nothing but, now here the current is entering, so the positive one, so here the current is entering from the opposite side. So L_{pp}, two times L_{pp}, then L_{pr} is between this and r. L_{qq} is this, L_{qs} is between this and this. So L_{pp} and L_{rr} are the same, so that is why you have the factor 2 over here because in either way it is the same one. So you get $V = L_{loop}\frac{dI}{dt}$, so you can write it in this way. Now how do you find L_{pp}, L_{qq} or L_{ss} and L_{rr} which are the same?

(Refer Slide Time: 30:29)



So they are defined like that. So if have a current carrying conductor I_a, and some flux φ_m and assume that you have imaginary return conductor at infinity and if you can set up this expression, L_{aa} equal to φ_m / I_a, and do a calculation, that is $\int B.ds \big/ I_a$ or $\int B.ds \big/ I_b$ for mutual inductance, then you get an expression like this. So for self-partial inductance, you write b same as a and replace d by r, diameter by r. Then you get for self-partial inductance, so this you can get from some of the electromagnetic textbooks, these type of expressions.

(Refer Slide Time: 31:39)

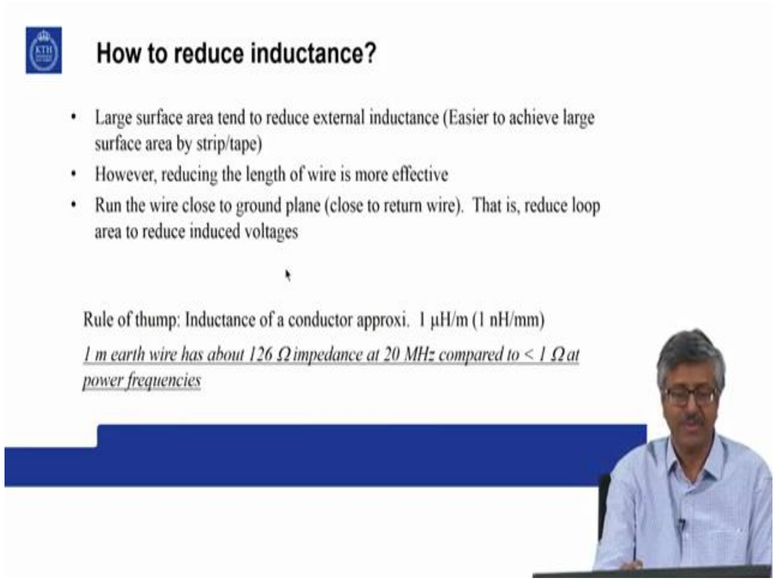


Now another comparison of inductance of circular and rectangular wires. For example, you have a long circular conductor in free space, so return is far away. And in the second case, you have a long rectangular conductor in free space, and return is far away and long circular conductor and return close by. So again from handbooks you can get these expressions for inductance value. So let us do some numerical calculations on this value and see what kind of inferences one can get.

For example, in the circular conductor, D equal to 1 mm, very thin wire, you get L equal to 1.5 μH/m. As a thumb rule, often you say that inductance of a wire is 1 μH/m length because the variation is logarithmic depending upon whether return conductor is close by or far. And if diameter is 10 mm, inductance becomes, because we have more surface area and inductance becomes 1 μH/m. But if you bring the return from infinity to close by, then you see that for d equal to 1 mm and return 4 m away, inductance can be reduced from 1.5 μH/m to 0.83 μH/m.

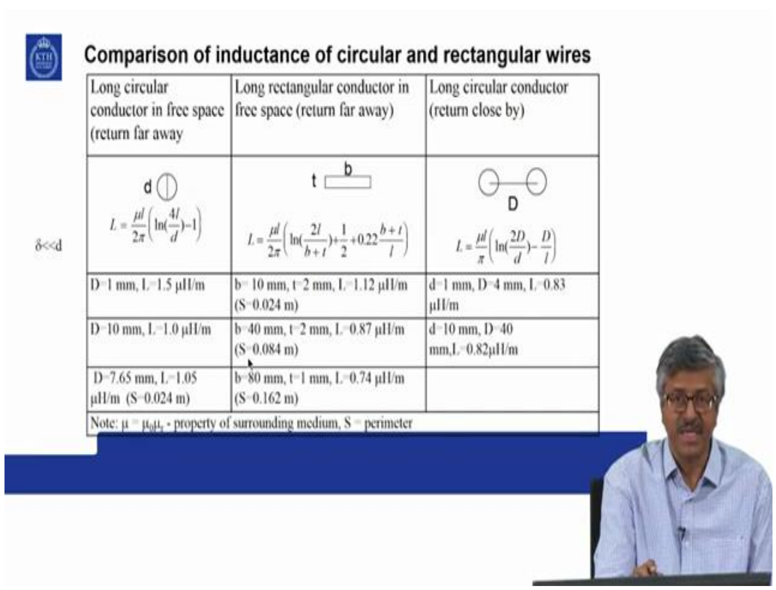
You can have a reduction by bringing close the wires together. It makes sense also because when you bring close the wires together, the flux linking with that enclosed area is also small. Now for the same thing with the rectangular conductor, here this is the expression for inductance and certain thickness, S=0.024, you get surface area and inductance is 1.12. And to achieve that you need a wire of diameter of 7.65 mm, that is almost 1 cm, it is very thick rod. You need that for achieving L equal to 1.05 whereas already close to that value you can achieve with thin strip which is very easy to realize.

(Refer Slide Time: 34:51)



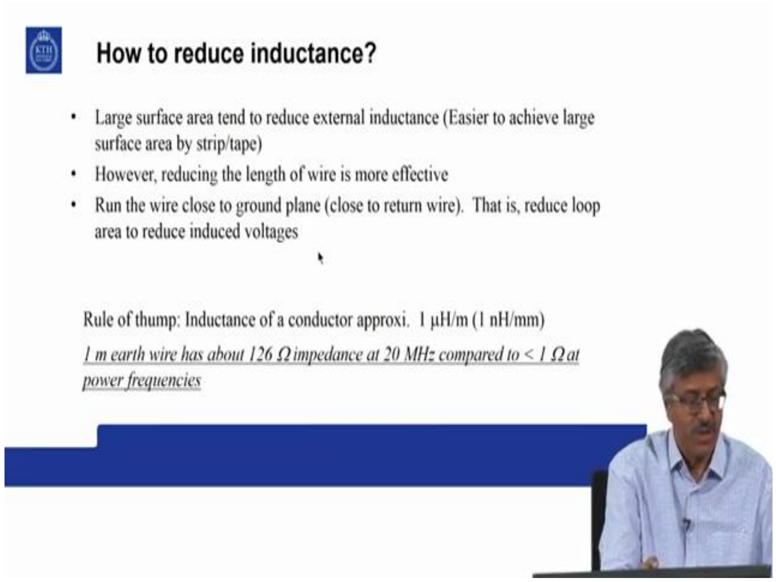
Then how to reduce inductance? As we have seen before, large surface area tend to reduce external inductance and it is easier to achieve large surface area by strips or tapes rather than round conductors. However, reducing the length of the wire is more effective.

(Refer Slide Time: 35:16)



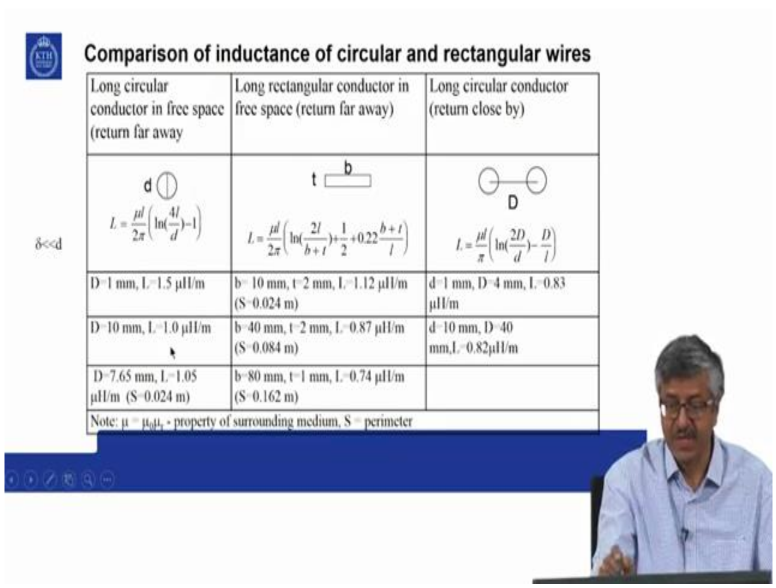
So here you have seen that by increasing the diameter to 10 or increasing the surface area, you achieve a reduction of only 0.5 µH/m. But instead of 1 m wire if you can use 0.5 m wire, already your inductance become 0.75, half of 1.5. So reducing the length of the wire is more effective in reducing the total inductance than increasing the thickness.

(Refer Slide Time: 35:56)



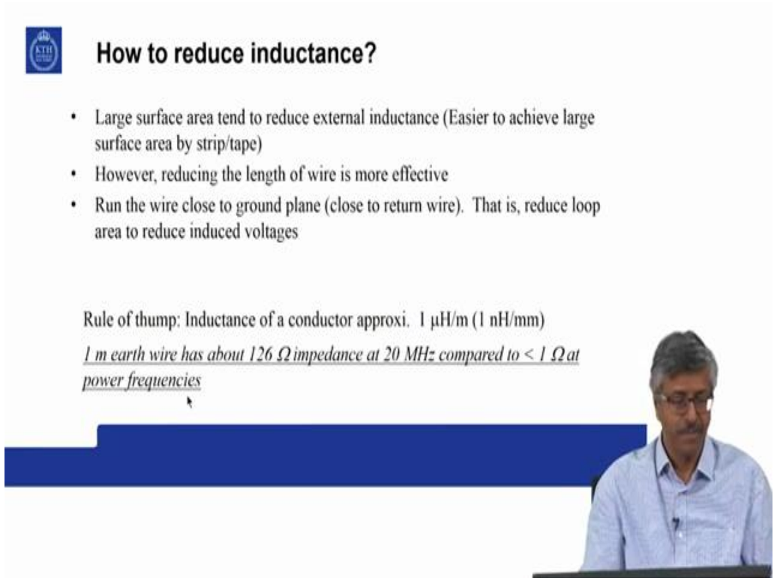
Then another measure is run the wire close to the ground plane, close to return wire, that is reduce loop area to reduce induced voltages.

(Refer Slide Time: 36:16)



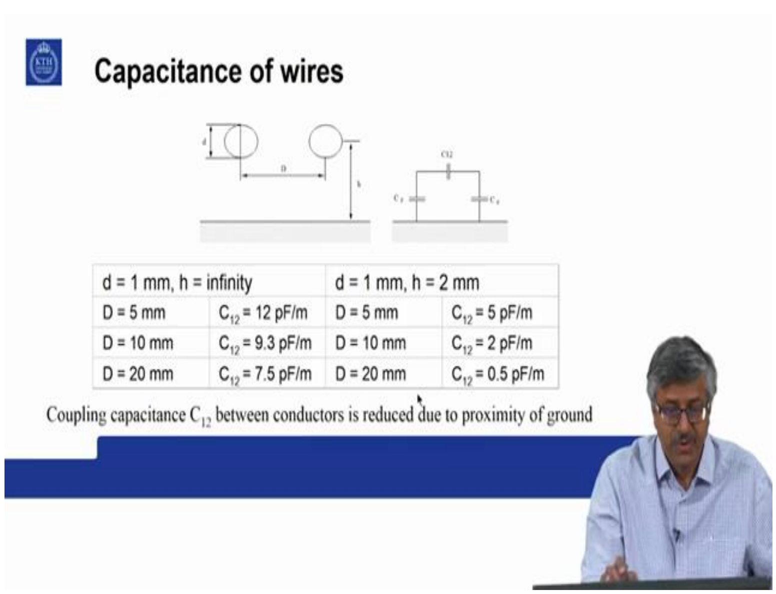
So this we have seen it comparing this picture with this one and the calculations below.

(Refer Slide Time: 36:22)



So rule of thumb, inductance of a conductor is approximately 1 µH/m or 1 nH/mm. Every mm reduction will reduce at least 1 nH from inductance. 1 meter earth wire has about 126 ohm impedance at 20 mega-hertz compared with less than 1 ohm at power frequencies. So the importance of inductance cannot be neglected while finding the impedance under transient conditions because it can have a tremendous effect.

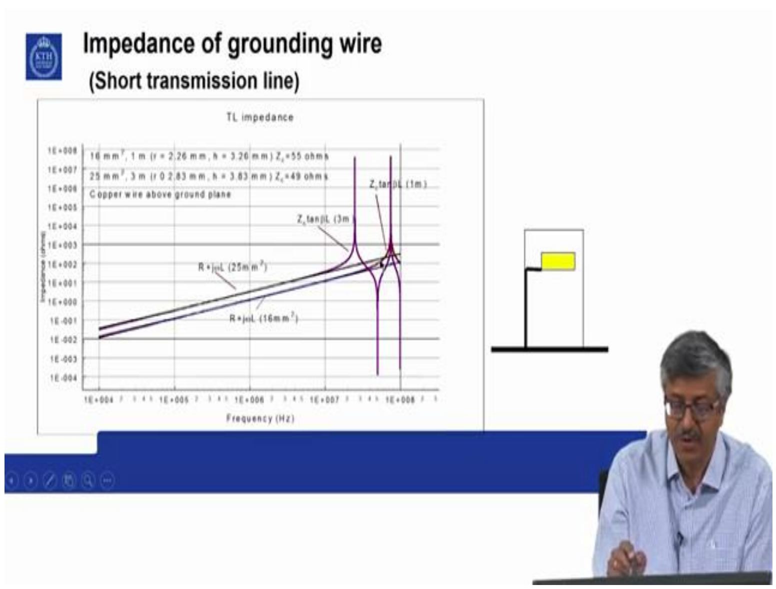
(Refer Slide Time: 37:04)



Now capacitance of wires. You have two conductors of cross-section, two parallel conductors, suppression distance D, cross-section is d and height is h. Now you have a capacitance, if they are carrying voltages, you have effect of capacitance between these two conductors. You have capacitance between these conductors and to the ground, so that circuit is shown here. C_{12} is the mutual capacitance, C_r is the reference capacitance or capacitance to reference. Now if diameter is 1 mm and h is infinity and if, so you do not have the influence of the proximity of the ground, it is in free space.

So when d equal to 5 mm, capacitance between them is 12 pF/m. Now here you are reducing now a ground plane very close to these two conductors, so just 2 mm away, immediately you can see that the capacitance value reduces from 12 pF/m to 5 pF/m. And if it is, you increase the distance from 9.3 to 2, then 20 mm, 7.5 to 0.5 pF/m. So you can say that ground plane has an influence in this mutual capacitance. So in general ground plane decreases this mutual capacitance between the two conductors. So this is called the coupling capacitance.

(Refer Slide Time: 39:20)



Impedance of grounding wire, this impedance of wires are very important whenever we consider grounding. Say for example, if you have an equipment in an equipment rack, suppose you are asked this question, you have two wires, you have 16 mm squared 1 m wire, a 25 mm square 3 m wire length. So how you will use those two, these? One may be tempted sometimes to say that okay, 25 mm square wire has lot of thickness, so it can reduce the inductance and other things.

But at the same time it has got larger length, 3m. So one can guess that okay, 16 mm square wire may be better. But let us do a calculation, see if it is really indeed true. So this is frequency and this is internal impedance. You calculate R + JωL for 25 mm square, 3m wire. So it comes over here. Then around this, around this value now 200 MHz, Z tan(βL), that is the impedance, it peaks.

This is the reference peak over there. And at around 500 MHz, there is a trough. So likewise it goes. Whereas this reference behavior happens in the 16 mm square wire, much later, only towards 700 or 600 MHz it is coming (because it is only 1 m in length). So, in this case surely you will be using 16 mm square, 1 meter wire for earthing, so that you have much larger range of frequency for operation. So this ends this particular module.