Electromagnetic Compatibility
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Module 2.1A
Electromagnetic principles - Faraday's, Ampere's, Gauss' equations

In this chapter we will review the principles of electromagnetic. You might have taken a course in electromagnetics and vector algebra before I suppose. So this would be only a review just to remind you those principles that you have learnt already.

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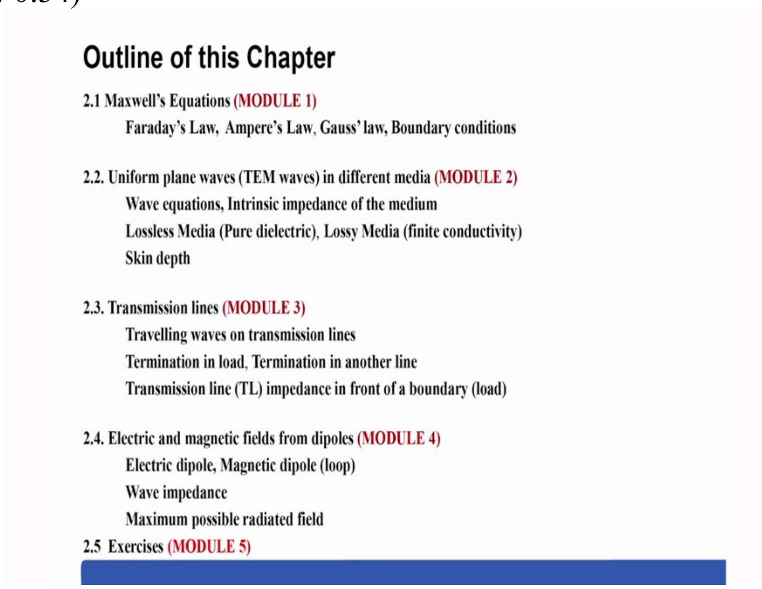

This is the outline of this chapter. It is divided into 5 modules. Each of these modules will be approximately 20 minutes to 30 minutes of lecture time. First in module one, I will consider Maxwell's equations. Maxwell's equations will be presented in the form of Faraday's law, Ampere's Law, Gauss's Law and boundary conditions. We will look at the derivation of these equations. This part is very much required, if you would like to know how electromagnetic fields, that is electric fields and magnetic fields are interacting with circuits. The circuits can be in the form of parallel lines or power conductors above the ground or it can be traces on a printer circuit board or it can be component leads.

We will consider uniform plane waves or transverse electromagnetic waves in different media. This is a special case of electromagnetic fields in which electric and magnetic field vectors are in a plane perpendicular to the direction of propagation. We will look into wave equations, we will consider intrinsic impedance of the medium, that is the ratio of electric and magnetic field perpendicular to the direction of propagation. We will look into the pure dielectric and the lossy media, the finite conductivity, especially we are interested in the behaviour of metals when plane waves are falling on the metals. Then we will look into the concept of skin depth, that is how much the electric and magnetic fields can penetrate a metal or a conducting media. This section is very much important when we discuss shielding, shielding of electronic circuits using metallic enclosures.

Then we will consider transmission lines in module 3. Here also, the solutions for travelling waves on transmission lines, we are actually considering quasi-TEM waves but we are presenting the equations in terms of circuit parameters like inductance, capacitance, et cetera. Especially we are interested in termination in load as well as termination in another line. That is when the impedance of the transmission line are different. We also will consider transmission line impedance in front of a boundary. This part is important whenever we need to consider transient analysis of transmission lines or grounding conductors.

In module 4, we will look into electric and magnetic fields from dipoles. We have cases in which component leads or tracks on a printer circuit boards are causing electromagnetic disturbances. So we need to calculate how much will be emitted from this. We will see that any of this can be modelled as combinations of small electric dipoles and small loops. We will find the equation for radiation field, specially the maximum radiation field. And we will also consider wave impedance. especially how wave impedance is different between electric and magnetic field when you are close to the source. Then in module 5, we will do several numerical examples.

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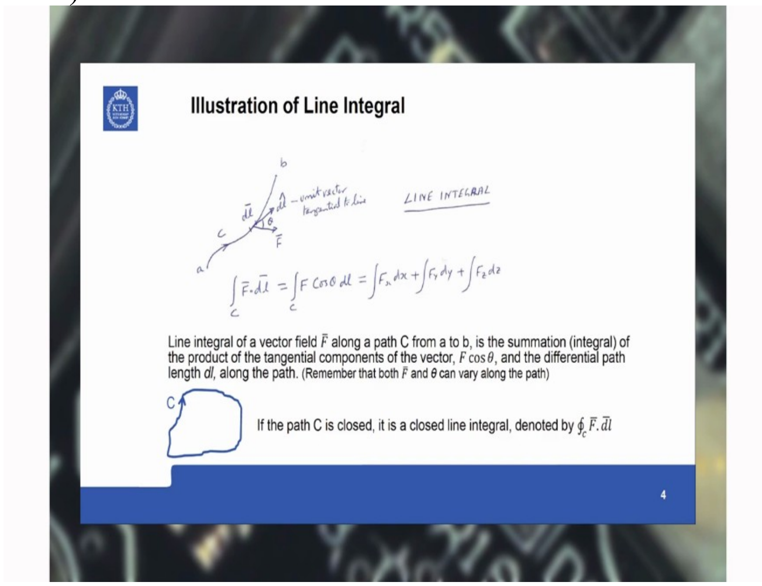

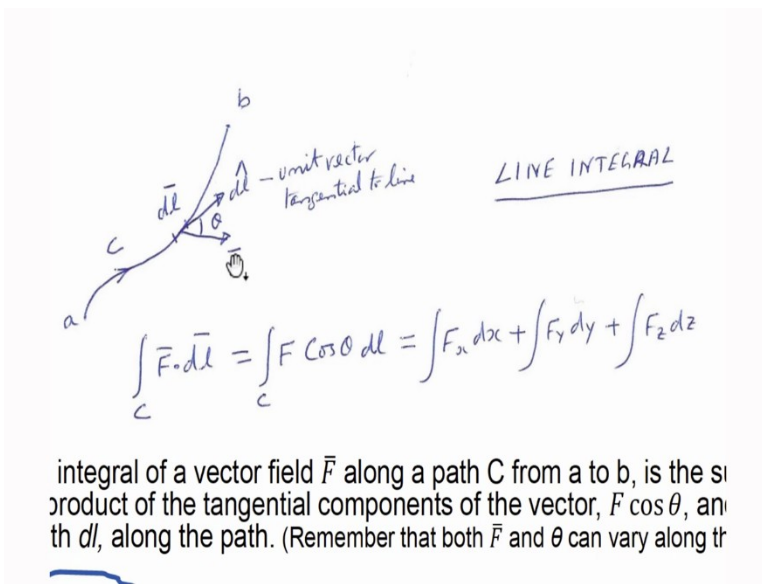

First we consider the illustration of line integral which will come in several of the Maxwell's equations. Imagine a vector field in space all around here. Here the bar above F indicate that it is a vector, that is it has not only a magnitude but a direction also. Consider a path A to B and a small section of this path which can be indicated as a vector dl, the direction of this path is tangential to the small section dl and that is indicated by a unit vector and a hat symbol is shown for a unit vector.

Now this vector F and unit vector dl form an angle theta. Now if you take the dot product of these 2 vectors, then we get the component of this vector along line F cos theta. Now let us see what is the meaning of the line integral indicated by a path C from A to B of the vector F. It is defined as line integral of a vector F along a path C from A to B is the summation, that is the integral, of the product of the tangential components of the vector, that is F cos theta and the differential path length dl along the path.

Now you have to remember that both F and theta can vary along the path. They are not, they need not be constant. That is why, you need to have this integration or summation. Now if the path C is closed, then it is called a closed line integral which is denoted by the integral symbol with a circle as well as a subscript $\oint F.dl$ so this is the closed line integral.

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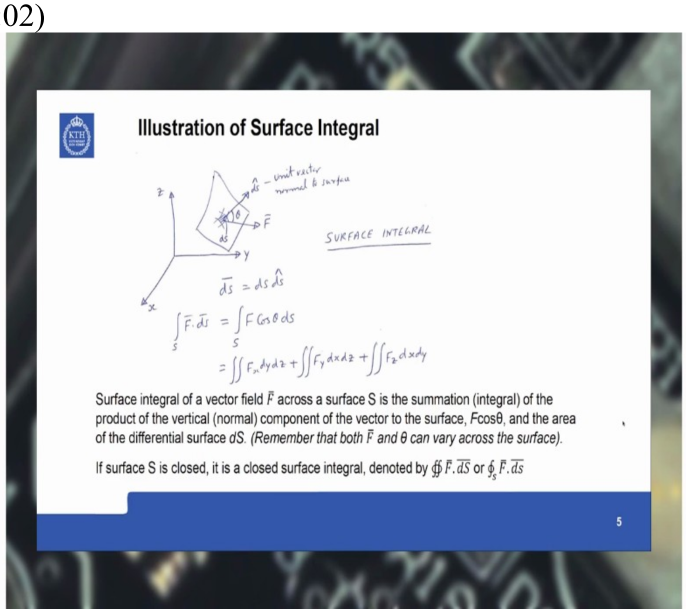

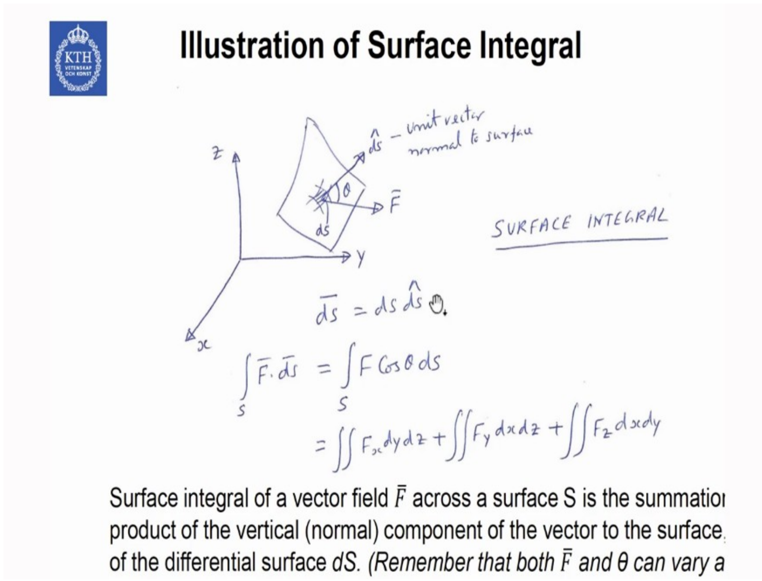

Now let us look at the meaning of the surface integral. This is the Cartesian coordinate system and there consider a small elemental surface area dS and dS can be represented as a vector and that is equal to the area of this small elemental area multiplied by the unit vector representing that area. And the unit vector is perpendicular to the small elemental area, perpendicular to that surface. Now there is a vector field F in space. Then dot product of vector F and the unit vector is F cos theta and the surface integral of F dot dS that is surface integral of a vector field F across a surface S is defined as the summation or integral of the product of the vertical component of the vector to the surface F cos theta and the area of the differential surface dS.

$$F\,.\,ds = \int_s F\cos(\theta)\,.\,ds = \iint F_x\,dydz = ¿ \iint F_y\,dxdz \iint F_z\,dxdy$$

$$\int_s ¿$$

So you are multiplying the component of the vector field F along with unit vector with the small elemental area, then summing it up all along the area. We need to do this summation or integral because both F and theta can vary across the surface. If surface S is closed, it is a closed source integral denoted by F dot dS and with this symbol, like this.

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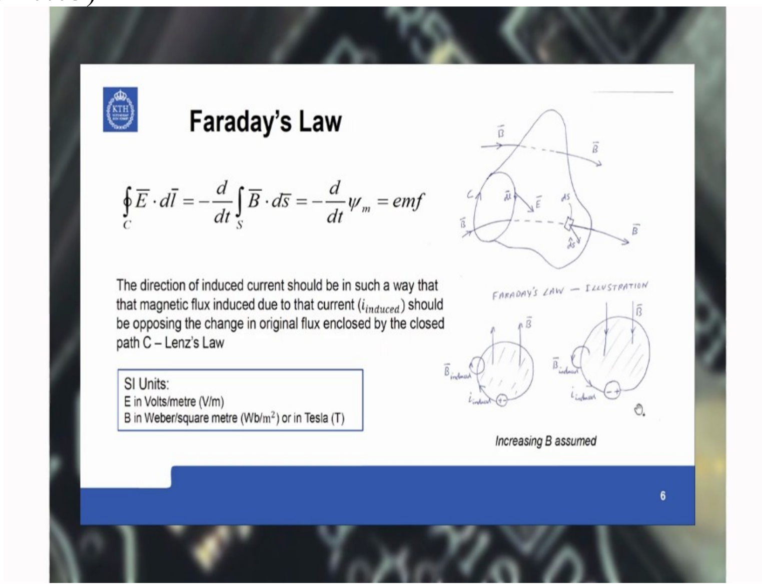

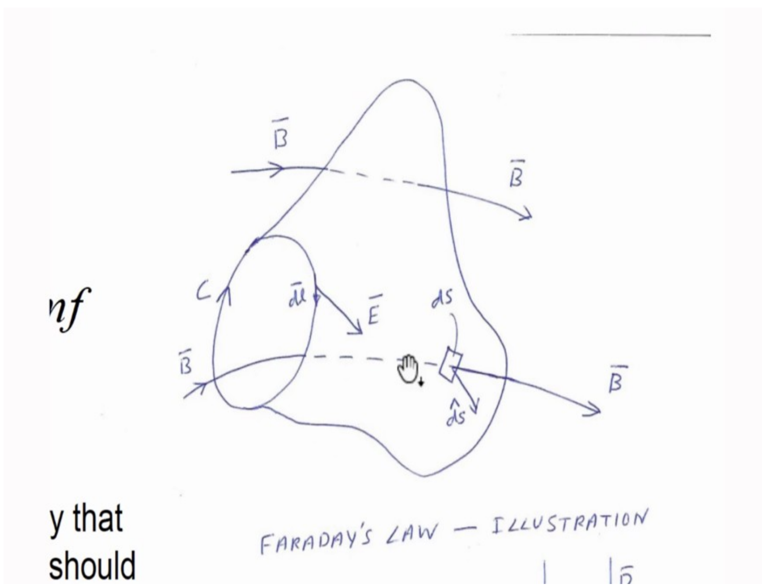

Now look at Faraday's law. Here, concentrate on this illustration. You can consider it as a closed plastic bag or a balloon where only one side is open. Now this is the open side. Now there is a magnetic flux density vector field B. Now there is an electric field vector field E, electric field intensity. Now what Faraday's law is stating is that if you sum up all the component of the E field along a closed path and multiply it by this small elemental length, that is if you take the line integral of E along the closed path, then that is equal to the surface integral of the magnetic flux density vector.

$$\oint_c E \, . \, dl = \frac{-d}{dt} \int_s B \, . \, ds = \frac{-d\psi_m}{dt} = emf$$

That is, if you look at a magnetic flux density B coming out of this small elemental area and if you multiply the normal component of that with the elemental area, you get the net flux coming out of this limited area. So you are summing up all those kind of fluxes coming out of the body of the plastic bag. Then you take the time derivative of that, a kind of flux coming out and that is also called EMF. So line integral of the electric field along a closed path is equal to the rate of change of total magnetic flux coming out of the area enclosed by this path.

Now the direction of the induced currents should be in such a way that the magnetic flux created due to that current induced should be opposing the change in the original flux enclosed by the closed path. So this is called Lenz's law. So we are using Lenz's law in finding out in which direction the EMF should be, the polarity of EMF.

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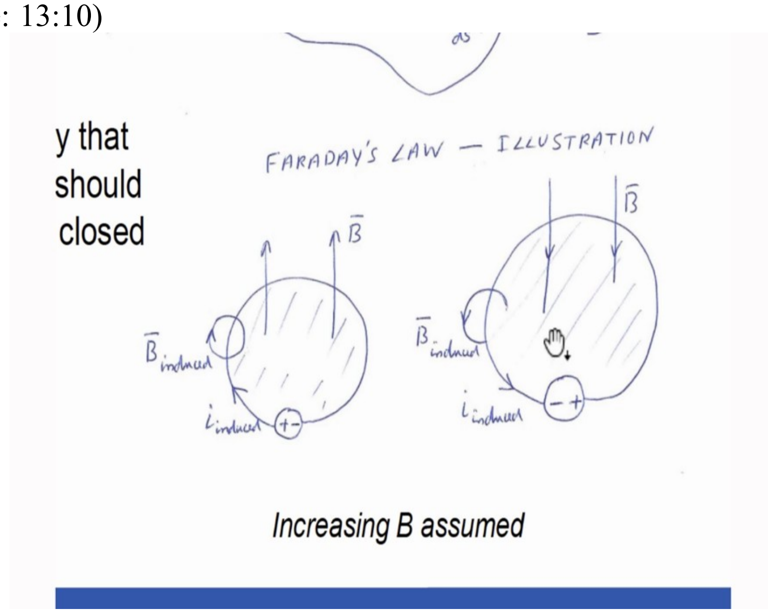

Now for that, consider this diagram, a simplified diagram where B is assumed to be increasing. Now this increasing B should be producing an EMF in such a direction such that the current produced should be producing an induced magnetic flux density or magnetic flux that is opposing this original flux. So using the right-hand rule, the magnetic flux has to be in this direction. So this is opposing the original flux. So this is the correct direction. So that is how you take care of this negative sign.

Now in this case, on the right, the increasing flux is in the other direction. So here, to oppose this flux, you have to have a polarity like this for the induced EMF. Then only it will produce a current in this direction and that induced flux will be opposing this original flux. So in the modelling of the magnetic field interaction, you will be using this principle.

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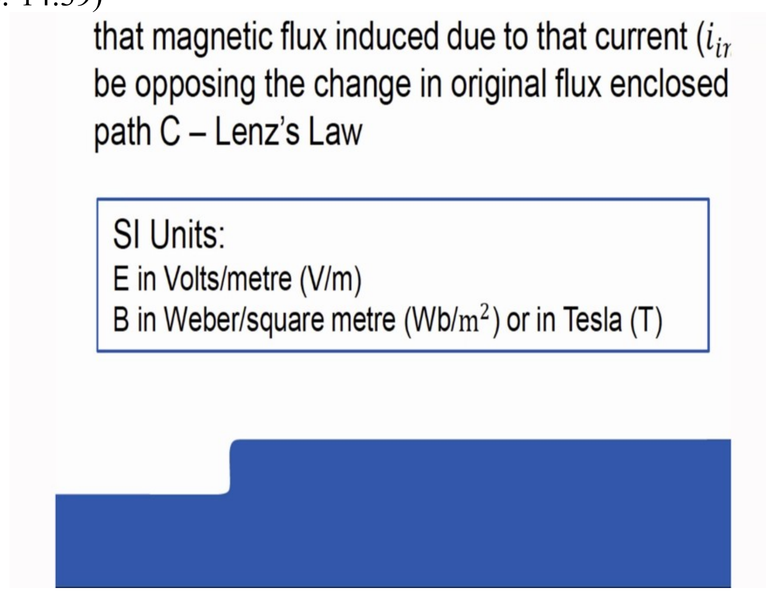

Now let us look at the units here. E is in volts per metre and B is in Weber per square metre. So Weber per square metre is also called Tesla. So these are the SI units.

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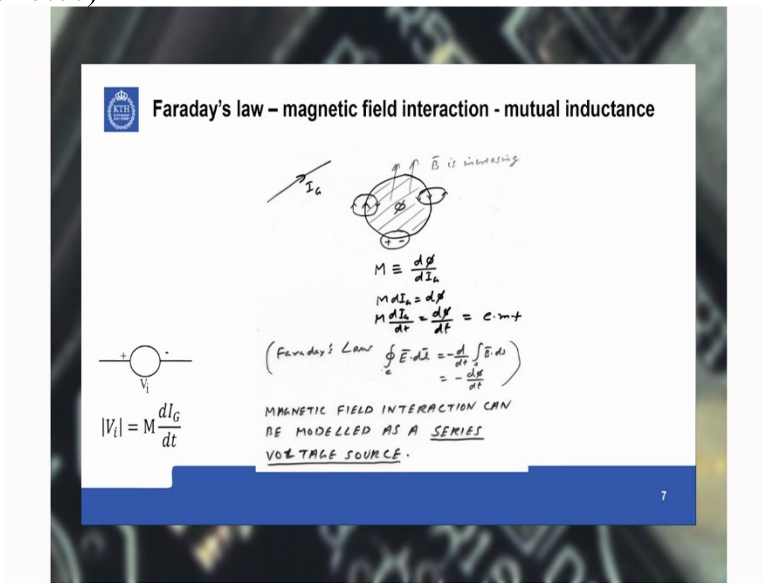

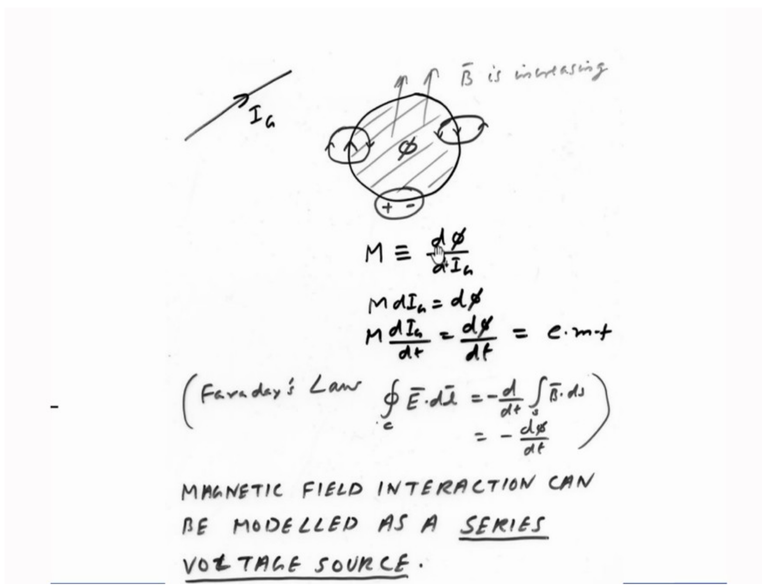

Now this is an illustration connecting the Faraday's law or magnetic field interaction with mutual inductance that you are familiar in circuit theory. Now imagine some sort of a current somewhere in space. So this current is I subscript G. So this current is inducing some flux in a closed path. Now this is an imaginary path, this path need not be made of conductors or anything but of course, if you want to measure something, then it will have a conductor here. Otherwise you know even if you take an imaginary path in here, Faraday's law is true.

Now in the case of a circuit in the form of a closed path, the mutual inductance M between this and this is defined as M is identical to rate of change of total enclosed flux here to rate of change of the current. That is d phi by DIG. So this is the definition of mutual inductance. Now you can do some algebraic relation. You can do like this because this is the total derivative. So MDIG is equal to D phi. Now you take the time derivative on both sides, M DIG by DT equal to D phi by DT.

$$M \equiv \frac{d\varnothing}{d\,I_G}$$

$$Md\,I_g = d\varnothing$$

$$M\frac{d\,I_G}{dt} = \frac{d\varnothing}{dt} = e.m.f$$

So rate of change of flux from Faraday's law, we have seen that it is just like an EMF, a source of voltage. So this is the Faraday's law. So D phi by DT can be equated to minus D by DT surface integral of B dot DS. So you can see that Faraday's law of magnetic field interaction can be modelled as a series voltage source if you are involving a circuit. So this phenomena of magnetic field interaction can be replaced by a voltage source. So the direction of the voltage source need to be determined by Lenz's law as we have seen before and this voltage source VI is given by M DIG by DT. So we have reduced the magnetic field interaction into a voltage source, series voltage source with the circuit. This you will see in the discussion of crosstalk or magnetic field interaction.

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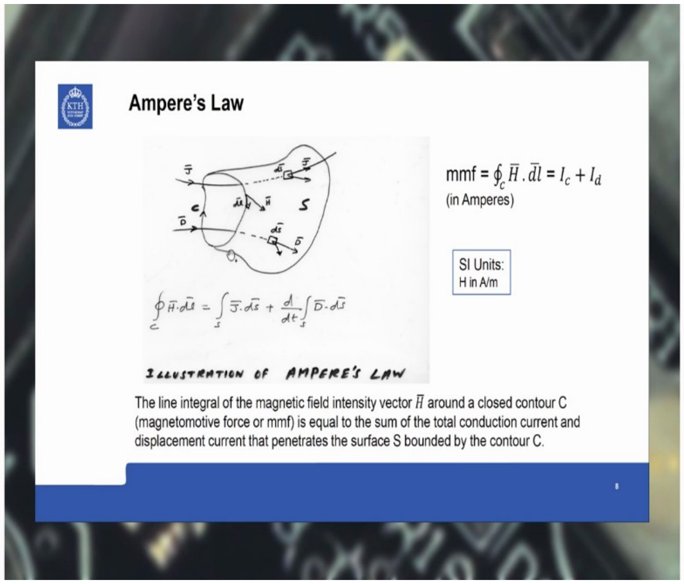

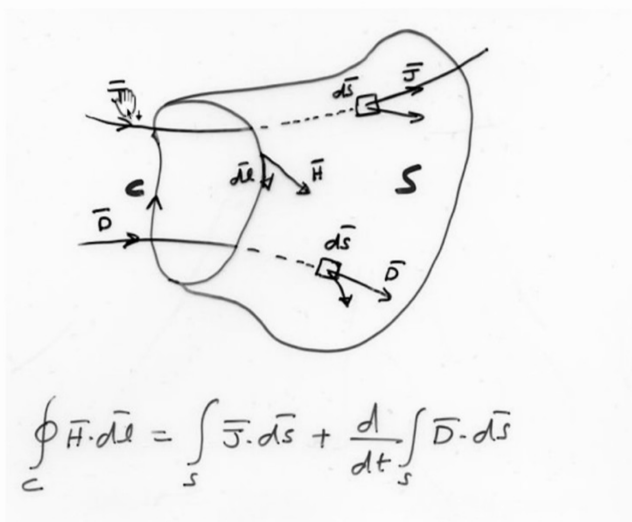

Then we come to Ampere's law. So look at this picture. So here also imagine a plastic bag with one side open. Now there are several vector fields here. One is the electric flux density field D, then we have the free current field J, current density field J. So these are you know some of these are entering the plastic bag and going out of it, it can be in any direction, you can imagine. If you look at this equation, here this is a line integral along this path which is enclosing the surface as, then this total current density and the vertical component of the current density coming out of this.

So if you have current density coming out here and the small elemental area, if you multiply it by that, then you get this small current coming out. So you add up all those small currents and that is given by this. So these are the free currents, total free currents coming out of these surfaces plus now all the electric flux density is coming out, the vertical component of that multiplied by the small area. Actually, this is the displacement current in the Maxwell's equations. So rate of change of integral d dot DS is the displacement current ID.

$$\oint_c H.dl = \int_s J.ds + \frac{d}{dt}\int_s D.ds$$

$$mmf = \oint_c H.dl = I_c + I_d$$

So H is given in amperes per metre. So the line integral of the magnetic field intensity vector, H around a closed contour C or magneto motive force or MMF is equal to the sum of the total conduction current and displacement current that penetrate the surface S bounded by the contour C. So this is Ampere's law.

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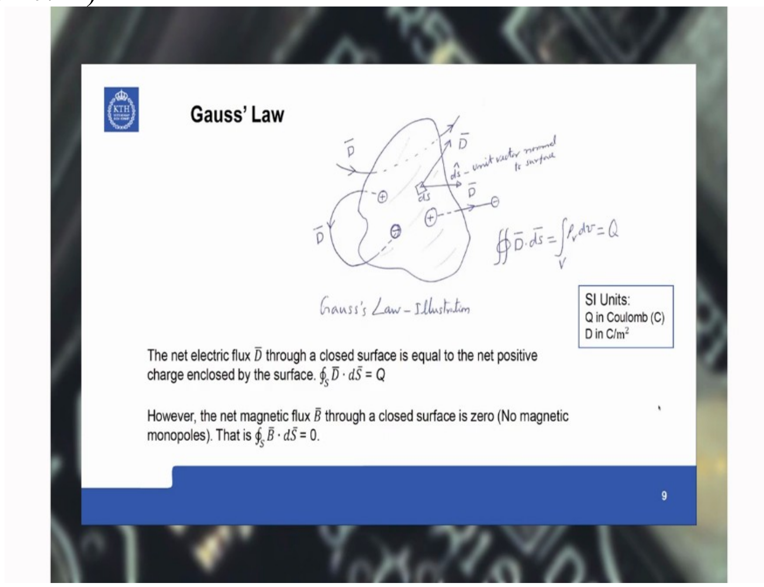

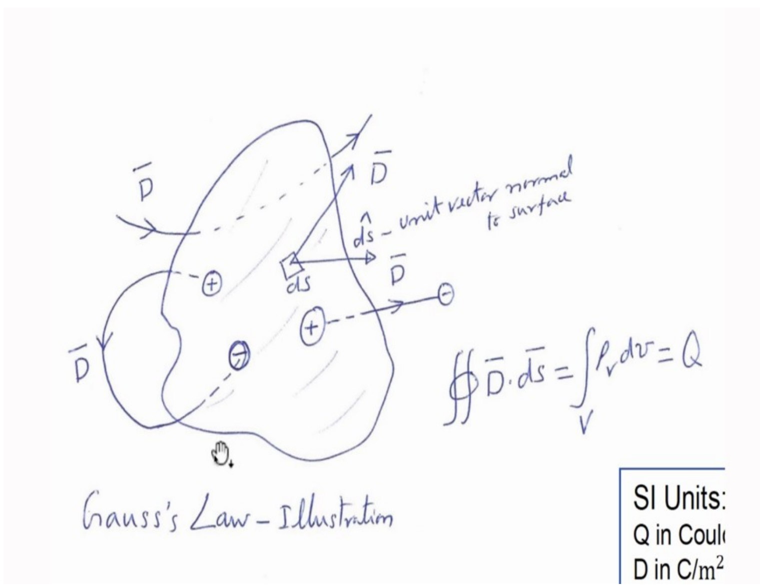

$$\oiint D \cdot ds = \int_V \rho_v \cdot dv = Q$$

Now look at Gauss law. Imagine a closed path. So it can be your plastic bag or balloon where the end is tied up. So you do not have any open end. It can be of any shape. So you have several magnetic flux density fields coming out on the closed path and going in again or just going through it. You can have charges inside, positive charges and you can have negative charges outside. So there will be electric flux lines connecting these 2 charges. Now what Gauss law is stating is that total electric flux density coming out is equal to the charge contained within that closed surface.

That is, you are summing up all the electric flux density lines multiplied by small elemental area, summing it up. And so, it is the total electric flux coming out. That is equal to the charge contained within this volume. It does not matter where those charges are. Sum of all those charges together will be equal to Q. So this is Gauss law. The net electric flux D through a closed surface equal to the net positive charge enclosed by the surface. Now unit of Q is in coulombs and D in coulombs per metre squared.

$$\oint_s D.ds = Q$$

Now instead of electric flux density, if it is magnetic flux density, then we will see that closed surface integral of B dot DS equal to 0, that is because we do not have any magnetic monopoles. We do not have any positive magnetic pole or a negative magnetic pole. So that is why this is always equal to 0.

$$\oint_s B.ds = 0$$

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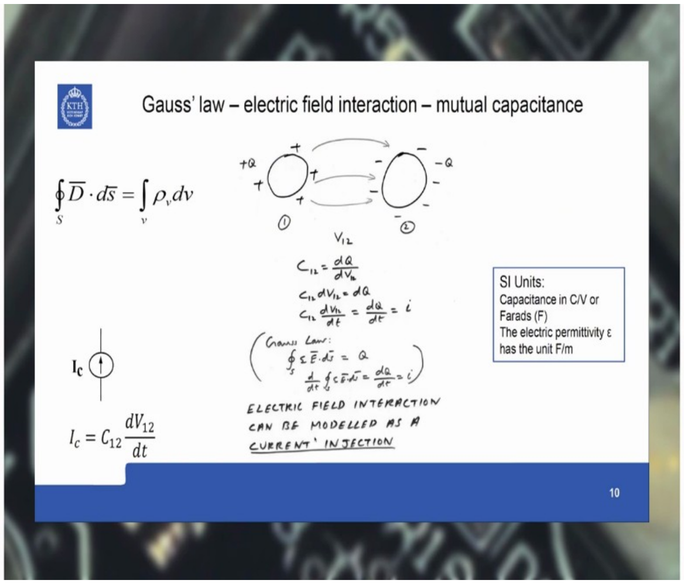

$$C_{12}=\frac{dQ}{dV_{12}}$$

$$C_{12}\,dV_{12}=dQ$$

$$C_{12}\frac{dV_{12}}{dt}=\frac{dQ}{dt}=i$$

Now let us look at the circuit implication of Gauss's law of electric field interaction and how we can tie it up with a mutual capacitance between two metallic bodies. Consider two bodies, one is charged to plus Q positively and the other is charged to minus Q negatively and there is a potential difference between them expressed by V_{12}. So the capacitance between them in circuit theory is defined as $C_{12}=\frac{dQ}{dV_{12}}$ .So this is the definition of capacitance, mutual capacitance.

This is the total differential, you can do this algebraic manipulation. Then you take the time derivative on both sides, then you will see that $\frac{dQ}{dt}$ is nothing but the current i. So it is like a current injection already you can see that. So tie it up with the Gauss law. What Gauss law state is that closed surface integral, so you can take a closed surface along this, of epsilon E which is nothing but D for a linear material, dot DS equal to Q and take the time derivative on both sides, so this is current injection.

So rate of change of field in relation to conductors can be modelled as a current injection. So electric field interaction can be modelled as a current injection. So this electric field is created by these charges here. So look at this current source I_c. I_c is nothing but $C_{12}\frac{dV_{12}}{dt}$, rate of change of voltage. It is this one. Now SI units, capacitance is in coulombs per volt comes from here or we can call it as Farads, the electric permittivity epsilon has the unit Farads per metre.

$$I_c = C_{12}\frac{dV_{12}}{dt}$$