Electromagnetic principles
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Module 2.3
Electromagnetic principles - Transmission lines

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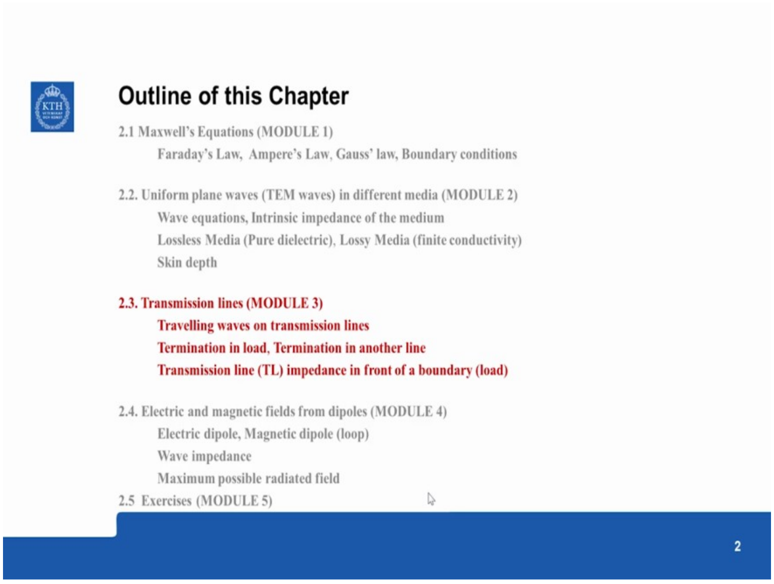

This is module 2.3 of the continuation of electromagnetic principles. In this chapter we will see about transmission lines.

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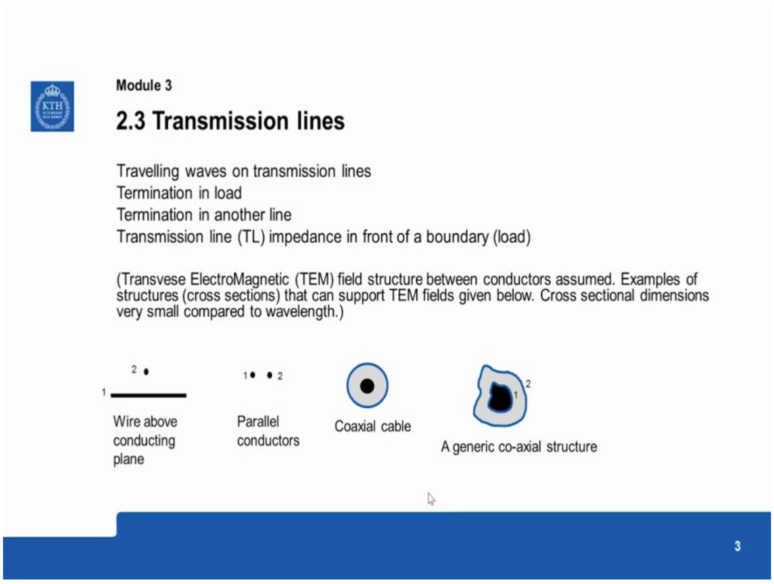

Especially we will see travelling or transmission lines, the solution for that. What will happen when transmission lines are terminated in the load and what will happen when transmission lines are terminated in another line. And also we will look into expressions for transmission line impedance in front of a boundary, that is, in the presence of multiple reflections. Transmission lines support a transverse electromagnetic field structure between these conductors, so this is the basic assumption.

For example, structure cross-sections that can support TEM fields, they are wire above the conducting plane, 2 parallel conductors, a co-axial cable or it could be any generic coaxial structure, its shape may not be very regular as long as the cross-section is the same and the line length is fairly large compared to the wavelength and the cross-section is very small compared to the wavelength, it is possible to have a TEM field structure.

So by the TEM field structure we also assume that we can uniquely define a voltage between conductor 1 and 2, which is an integral of electric field along any path so it will be path independent, whatever path you will take between 1 and 2, it should be the same value or voltage. So there is a decoupling between electric field and magnetic field when there is a TEM field structure. That is, the current in the conductor is uniquely defined by a close integral of magnetic field intensity around this conductor so that will uniquely give the current, so these are some of the basic conditions.

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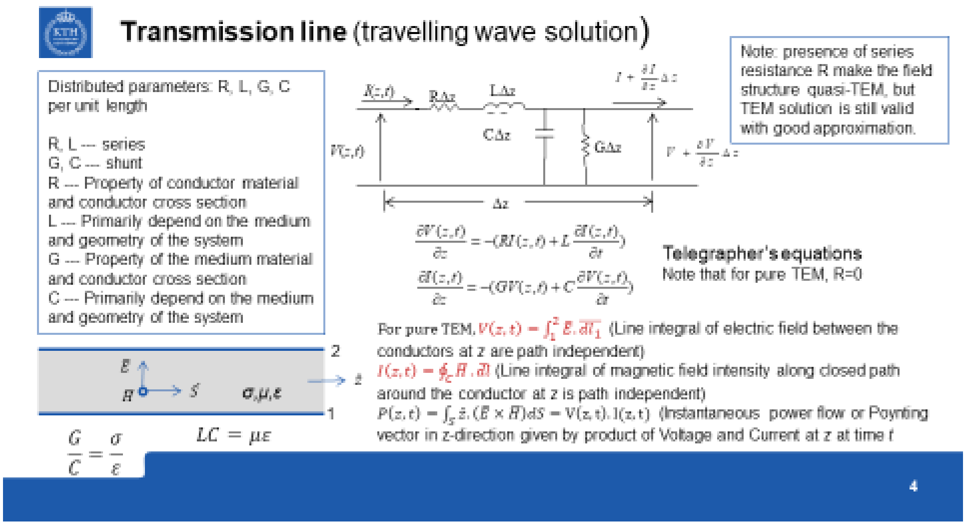

Now, first let us concentrate on this part trying to find the correspondence between the electric and magnetic fields with voltages and currents. So this is a transmission line structure; two parallel conductors so this is the length, the TEM wave is propagating along this Z direction, so E and H fields are perpendicular to the direction of propagation so let us assume the E vector to be like this and H vector is coming out of the paper let us say then this is the direction of the power flow. The medium of transmission line is characterized by these properties: conductivity, magnetic permeability and electric permittivity.

We can have a circuit representation of this transmission line, before that as we said before for pure TEM, the voltage between 1 and 2 is defined as $\int E.dl$, line integral of electric field between the conductors at C are path independent. Similarly, the current across one conductor will be $\oint H.dl$ and the power in the Z direction is given by $\int E \times H.ds$, which is nothing but voltage times the current, so instantaneous power flow or pointing vector in the Z direction is given by the product of voltage and current at z at time T, so these are the connections between voltage and current.

Now we can define, we can take a very small section of the transmission line let us say it is ΔZ in this direction and represented in terms of circuit parameters that we are more familiar with say for example, we can imagine an inductance which is related to the magnetic field interaction and we can define capacitance more related to the electric field interaction, so C and L are values per meter. So it can be absolute values for ΔZ to be multiplied by ΔZ. Then because of this conductivity there is some leakage of current when there is a voltage difference between these two, some current will be flowing through the due to the conductivity of the medium so that is represented by conductance G per unit length multiplied by ΔZ, so this structure can support the TEM, L, C and G.

Now in practice we have always some small resistance however good this conductor is, but there is some series resistance. This series resistance is not part of TEM, this says that there will be voltage drop across this so you can have destruction of TEM wave structure, but if R is small enough still we can assume that TEM solution is approximately valid, G does not cause any problem for TEM structure, so the resistance times ΔZ that is the series resistance component. Now we can derive Telegrapher's equation for the transmission line which are nothing but, first we equate the voltage drop across a close loop like, this that is obtained by this, voltage drop across this, voltage drop across this. That should be equal to the difference in voltage between these 2 ports so that is written over here.

Then we can write the 2nd equation by the current entering and leaving this node over here. So the current entering and leaving and difference is this much and that should be equal to the current entering over here, so you have this negative sign and G(V(Z(t))), voltage across this

+ $c\frac{dV}{dt}$, so we got the current also, so this is the Telegrapher's equation. If you look at these parameters R, L, G and C of that R is property of conductor material and conductor cross-section, whereas L, G and C that depends also on the type of medium that you are using, whether it is air or some other medium.

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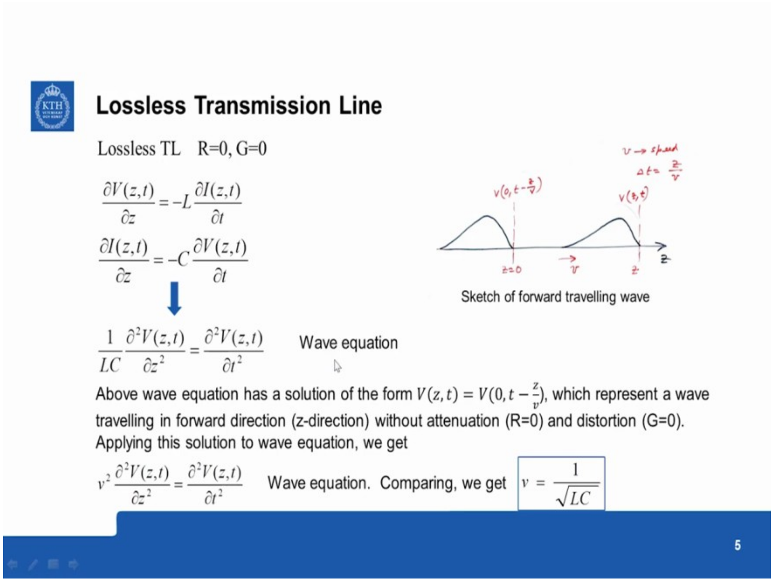

Now consider a lossless transmission line, it does not have any loss that is no attenuation that is R = 0 that is the amplitude are same, all around the transmission lines and load distortion G is equal to 0, no dissipation of the current along the way of the transmission line. So under this condition the previous equations simplify into these two equations, rate of change of voltage with respect to space equals –L $\frac{dI(Z,t)}{dt}$. So it is only spatial derivative of current equal to - $c\frac{dV(Z,t)}{dt}$. So you can manipulate these two equations by writing the 2nd derivative and substituting one into other which you can do at home and see that it is true, then you will get the wave equation, the second degree differential equation.

And here it is $\frac{1}{LC}\frac{\partial^2 V(Z,t)}{\partial Z^2} = \frac{\partial^2 V(Z,t)}{\partial t^2}$. Now we can find the solution for this wave equation using the normal roots in differential equations and one can see that one of the solutions is of the form, voltage at any point in time equal to the voltage that has happened at a time earlier at z=0. For example, voltage at space Z or along the line and that time t equal to the voltage that would have been there at z = 0 at a time Z/V earlier so Z/V is the time required for the wave to travel up to this point and V is the speed of the wave.

So this has the form of forward travelling wave, so V is the speed and Δt is equal to Z / V. So this is one of the solutions, a forward travelling wave without any attenuation or distortion is one of the solutions for this transmission line. Now comparing with the original wave equation we get this V square, V square is the velocity and 2nd derivative of voltage with respect to space and 2nd derivative of voltage with respect to time. Now comparing we get this speed v is nothing but one by square root of LC, so this is the expression for this speed, so this speed is related to the parameter L and C.

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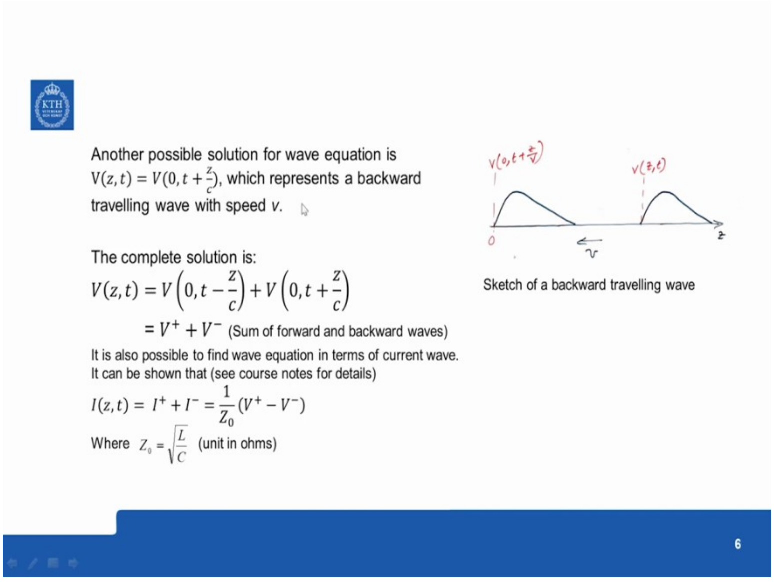

Another possible solution for wave equation is backward travelling wave so you can say that the wave that is present at z would be travelling backwards. So if it is a backward travelling wave it is this kind of reflection in transmission lines that we will see later, so it is T + (Z / V), so the total solution will be the sum of the forward travelling wave and the backward travelling wave. So we denote that by the symbols + and - in the superscript.

It is also possible to find wave equations in terms of the current waves, so we can find the wave equations in terms of the current, then do the same procedure then you will see that there is some of forward and backward current waves and we can define a impedance Z_0 which is defined as $\sqrt{\frac{L}{C}}$ and related to the forward and backward travelling waves. So here this symbol is negative when we are representing in terms of voltage and impedance. This also you can derive it and you can see for yourself that this is true, there is a kind of a homework.

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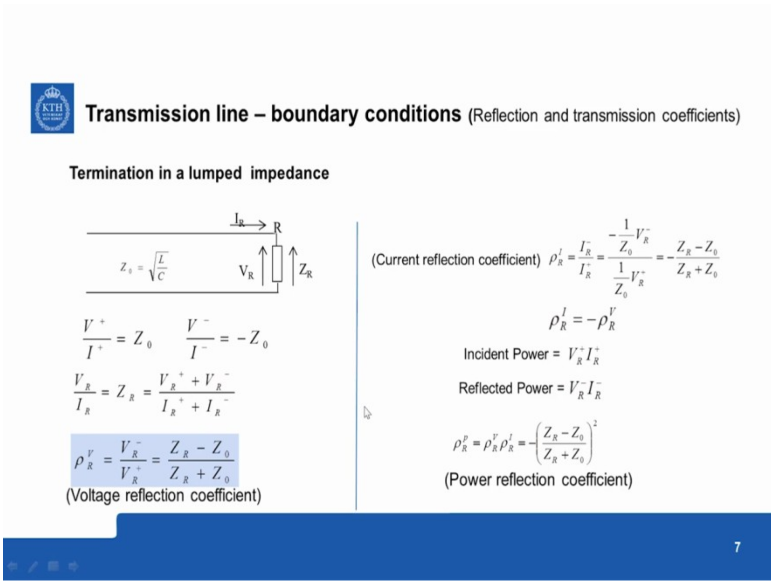

Now let us look at transmission lines boundary condition that is when transmission line is abruptly terminated either by a lumped impedance or it can be another transmission line that we will see in the next page. Suppose, it is terminated in a lumped impedance, and this reflection point here, so we call it as R so we call I_R, V_R and Z_R, Z_R is the impedance so V_R is the net voltage over here, so this is the sum of forward and backward reflected waves over here and the impedance $\sqrt{\frac{L}{C}}$ is obtained as $I^{+¿}\frac{V^{+¿}}{¿ ¿}$ or $I^{-¿}\frac{V^{-¿}}{¿ -¿}$.

Now, if we say V_R / I_R, so boundary condition says that V_R / I_R the net current here and net voltage here should be equal to Z_R, so this is given by summation of forward and backward waves divided by summation of forward and backward currents. You can manipulate this and easily find that this is nothing but from this we can say that if we try to find $V_R{}^{+¿}\frac{V_R{}^{-¿}}{¿ ¿}$ so this is reflection coefficient, reflection coefficient is the wave that is reflected back divided by the wave that is incident. So that is the reflection coefficient for voltage, so that we define with this symbol ρ. So voltage reflection coefficient from this group of expressions you can derive as the termination impedance - the characteristic impedance Z_0 divided by the sum of those impedances, so difference divided by sum, so this is voltage reflection coefficient.

Now we can derive another reflection coefficient also, so intuitively a current reflection coefficient will have a negative sign in front. So you get current reflection coefficient which is negative of voltage reflection coefficient. Now the incident power is product of the forward moving voltage and current waves and reflected power is product of backward moving voltage and current waves. So if you take the ratio of reflected power divided by incident power and simplify it, we will get an expression for power reflection coefficient or we can obtain power reflection coefficient as a product of voltage reflection coefficient and current reflection coefficient, so this is the expression, so here this is the square.

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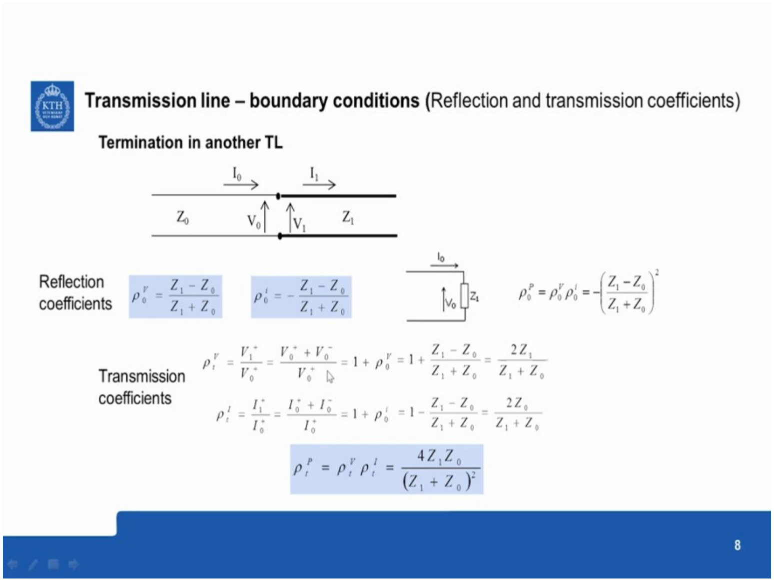

Now consider a case of transmission line boundary conditions when it is terminated in another transmission line. This situation can happen quite often in nature, you are connecting two cables of different characteristics, so let us say that cable one has impedance Z_0, so any current at just left of this junction we call it as I_0, and any voltage just left of this junction we call it as V_0 so the medium 0, then the current that is going into the second conductor, we call it as I_1 and voltage just on the right we call it as V_1 like that, and this transmission line has an impedance Z_1. So note that these 2 currents are the same because some of the energies reflected back from this point and these 2 voltages are different from these 2 voltages.

Now reflection coefficient, for defining reflection coefficient we can if these lines are quite long, we can consider it as termination with other impedance that is equal to the characteristic impedance of this line. Now here it is assumed that this line is quite long and not influenced by what is happening over here, then the reflection coefficient is something like we derived earlier in the previous figure difference in the impedances divided by the sum of the impedances and current reflection coefficient with a negative sign in front. And the power reflection coefficient that is power that is reflecting here, that coefficient can be obtained by this expression so this is very similar to the previous derivation.

Now what is new is that we need to find what is going into this. Of course, whatever is remaining after reflecting is going into this one so that is called transmission coefficient with a subscript T. So voltage transmission coefficient is defined as the forward wave that is going into this medium, so we call it as V_1^+ it is not the same as V_1 you have to understand that, V_1 is a combination of forward and backward waves, so V_1^+ divided by V_0, forward wave that is coming in this direction along the medium 0. So that is equal to so the voltage at this point, the voltage is V_1^+ has to be V_0^+ of the forward wave and reflective wave that will be the net voltage over here divided by forward wave that is going into this medium.

So which will be equal to 1 + you can manipulate this and find out that this is the reflection coefficient into this medium, so that will be nothing but if you write out these equations and you will see that this is 1 and this is this reflection coefficient, V_0^- / V_0^+. So you will get as

$\frac{2Z_1}{Z_1+Z_0}$, so this is what you will be getting. Similarly, for the current reflection coefficient we can write in the similar manner ratio of the forward wave going into this divided by the wave falling into this, so that will be equal to the sum of the forward waves + reflected waves divided by the wave that is going into the 2nd medium, and again that is written as $1 + \rho_0^I$, so

you will see that $\frac{2Z_0}{Z_1+Z_0}$, you will get it like that.

So the expressions are very similar except that for the voltage transmission coefficient, so this is the transmission coefficient for the voltage transmission coefficient you have Z_1 here, and for the current transmission coefficient we have Z_0 over here. Then total power transmission coefficient, this is multiplication of voltage transmission coefficient and multiplication of current transmission coefficient. While talking I might have said that reflection coefficient but these are not reflection coefficient, transmission coefficient, the way to identify is that you look at subscript T, T means transmission coefficients. So, this is the power transmission coefficient into this line.

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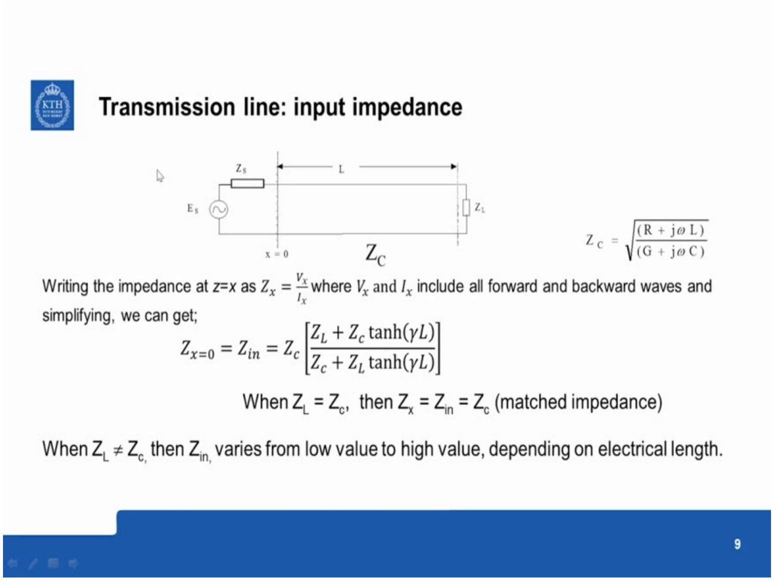

Now transmission line input impedance, often we have situations in which we are connecting a short line or it can be a long line also to an impedance, but they are not infinite transmission lines so you have reflections from here, reflects back so you may have multiple reflections like that. So in that case often it is of interest to see, what is the impedance at a given point X as you are looking into the load side say for example, if you are connecting 2 instruments similarly, we always talk of impedance matching this has got relevance to that. And also when you are connecting equipment to the ground there also this is applicable. So a generic transmission line has characteristic impedance which is given by $\sqrt{\frac{R+J\omega L}{G+J\omega C}}$.

So writing the impedance as Z equal to X as instantaneous voltage at X divided by instantaneous voltages, instantaneous current at X, Vx by Ix. So this Vx and Ix includes all forward and backward waves. If we consider it like that, so you can write it out and it will be a long algebraic expression and simplify and finally you may end up in this type of relationships. Now, of that special case is one that is shown out here when X = 0, so this is the input impedance as seen from the source side.

So you have a load and you have a line and you are connecting this load and the line together to the source, so the source will see an impedance that is given by this expression simplified expression. So that is the input impedance from the source side that is equal to the characteristic impedance times and this is hyperbolic function and this is the propagation constant Gamma, which is frequency dependent. So from this expression you can immediately see they are similar the numerator and denominator except that this L and C are appearing at different places.

So when the characteristic impedance is same as the load impedance, suppose we use a cable that has the same characteristic impedance as the load, then you see that the numerator and denominator are the same and it becomes 1, the input impedance is nothing but the characteristic impedance. So you get the matched impedance, so this is one of the reasons why we always can connect instruments together. Often it may have 50 ohms impedance with many of the standard machine equipment that we use in the lab, so if we use that 50 ohms cable and 50 ohms impedances then we can match.

If the load impedance is not equal to characteristic impedance then input impedance will be varying from a very long value to a high-value because this is propagation constant, so it is periodic, and that depends upon the electrical length of the line. So we will look into some special cases when Z_L is not equal to Z_C these kinds of effects.

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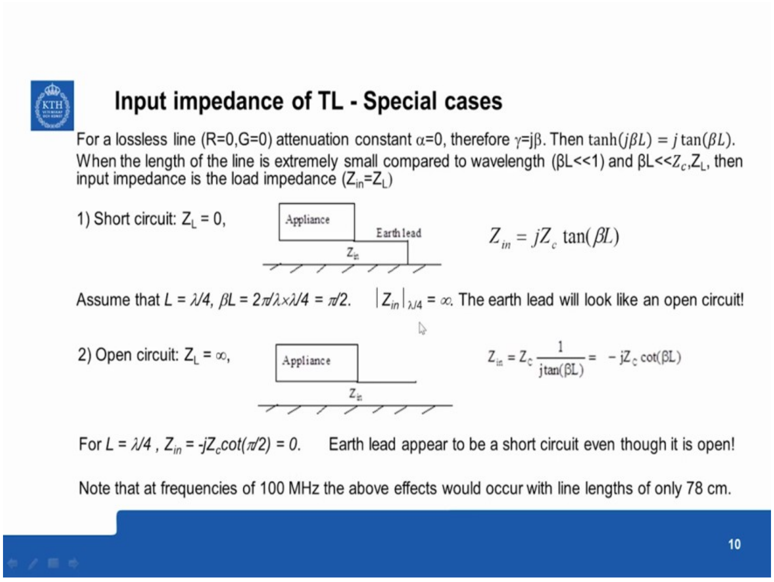

Input impedance of transmission lines; let us look into some special cases. For simplicity we assume a lossless transmission line that is R = 0 and G = 0, it means that attenuation $\alpha = 0$ therefore, propagation constant $\gamma = j\beta$, where β is $2*\pi/$ L. Then from the trigonometric identity we know that hyperbolic transformation tanh ($j\beta L$) equals j tan (βL). When the length of the line is extremely small compared to the wavelength that is $\beta L << 1$, and also βL far less than the characteristic impedance and the load, then we find that input impedance is load impedance always, this we can verify that by looking at the previous expressions because β L when it is too small, this will be equal to 0.

So it means that when the line is extremely short compared to the wavelength, input impedance will be always equal to terminating impedance. Now let us look at some of the cases, suppose transmission line is connected to the ground and short, Z_L=0, then in this particular case from the previous expression.

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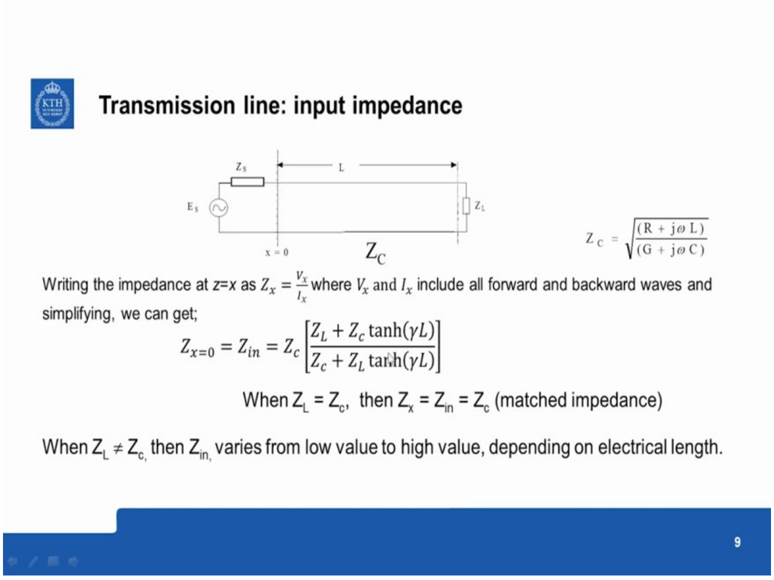

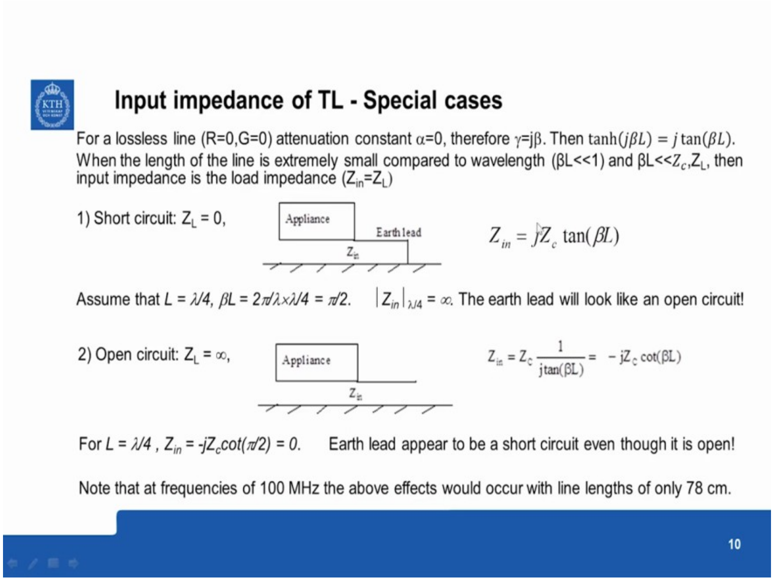

From here you can get that when Z_L = 0, you get an expression that is equal to input impedance is equal to Z_C Tan β L, so this is what you will be getting. Now the length L can be different electrically depending upon the frequency, so assume a case in which this is a quarter wavelength, λ/ 4, then βL=π/2. So substituting this we see that Tan (π/2) is infinity that is Sin(π/2)/ Cos (π/2), so we get the input impedance as equal to infinity even though it is a short-circuit, it should be zero normally we assume, but it is not, it is infinity so the earth will look like an open circuit. So even though you think that you have earth it is not really a short-circuit, it is an open circuit.

Now consider another case in which you are deliberately leaving it open, $Z_L = \infty$. Again from the previous expression for input impedance for this particular case, we can signify the expression to be in this form and assume that your length is λ/ 4 then you will find that input impedance is equal to 0. So even though it is open circuit, from the equipment side it looks like short-circuit, so this is a kind of contradiction. So this kind of funny behaviour can happen at various frequencies and this type of behaviour can happen with lines length of only 78cm at 100 megahertz, at higher frequencies it will be even at lesser length.