Electromagnetic principles
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Module 2.4
Electromagnetic principles - Dipoles

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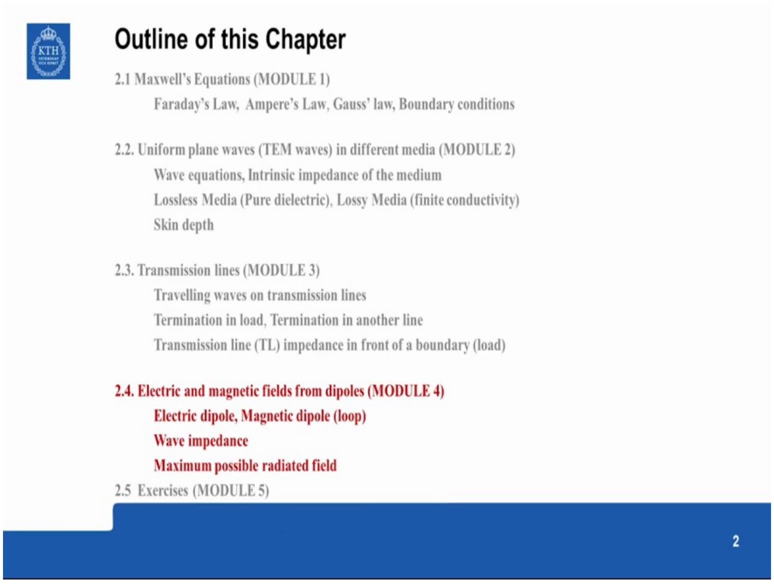

Review of electromagnetic principles continuation. In this chapter we will inspect electric and magnetic fields from small dipoles; these are the electric dipoles and the magnetic dipoles. Magnetic dipole is basically a small loop; we will introduce the concept of wave impedance and also find the expression for maximum possible radiated field.

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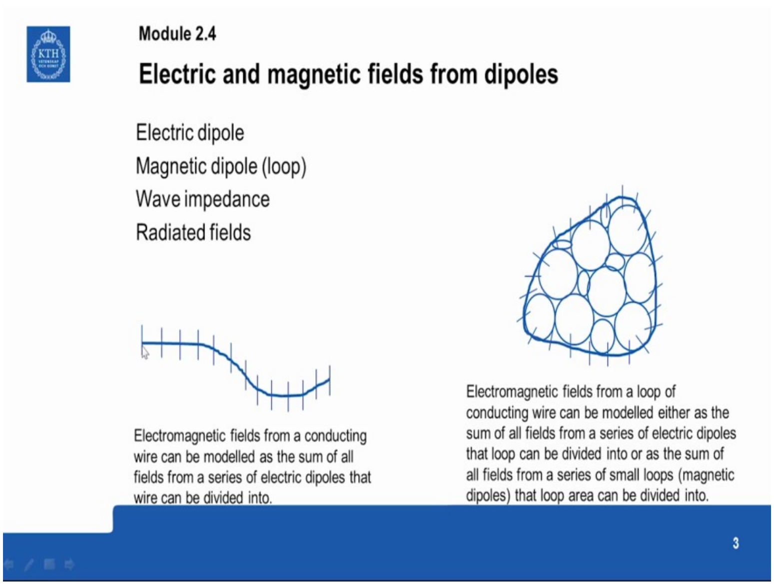

Now take this picture on the left over here, if we want to find the radiation field from a piece of wire, this piece of wire can be component leads or it can be a track in the printed circuit board or it can be just a connecting wire between two circuits. So if we want to find the radiation field from such a wire then you can divide that into very small dipoles, these dipoles are shown here. Then if you know the expression for electric and magnetic fields from a small dipole like this then you can add up all these dipole fields to find the total field, so that is the principle involved. So the basic expression that we want to know is that from the small electric dipole, these are called electric dipoles.

Now you can imagine another scenario in which you have a wire that is closed like a loop like that. So here we have 2 possibilities; one possibility is that you can divide this wire as we did before into very small dipoles, find the electrical and magnetic fields from each of these dipoles at a point where we are interested in finding the fields then sum it up. Another way is that, this area of the loop that you can divide into very small loops so that it covers more or less the complete area, then these are called the magnetic loops or magnetic dipoles. Then from there you can find the total magnetic field, so there are 2 possibilities in finding the fields.

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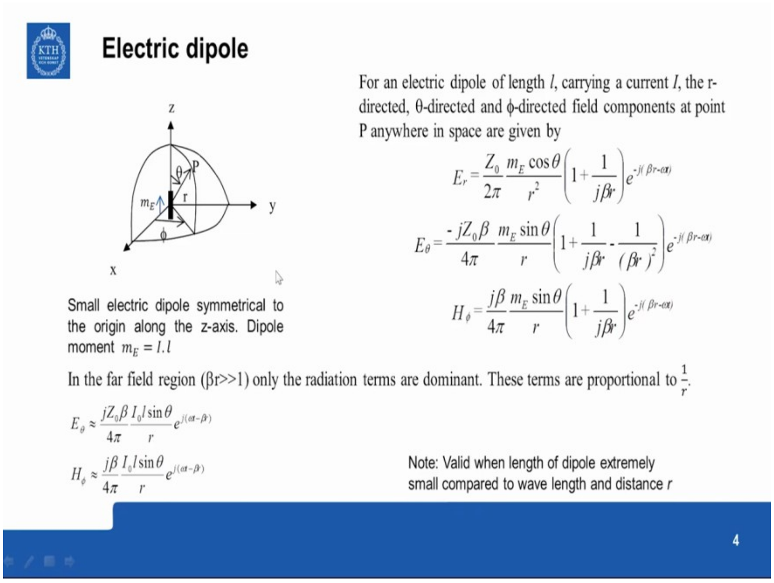

Now first consider the electric dipole, so this is represented by a small piece like this which has a dipole moment, this dipole is a small part of a wire of length ***l*** and carry a current I then the electric dipole moment is defined as

($M_e = I \times l$) M subscript e = I times L, so the current is directed in the Z directions so this is Cartesian coordinate X, Y, Z what is shown, but we will find expressions in spherical coordinate because that is more convenient for us, so this spherical coordinate is defined like this. So if this is the any point is space p then from the origin to this point the distance is called R that is one of the coordinates.

Then from the Z direction an angle (θ)Theta so this is the second of the coordinate then the rotation φ(Phi) from the X axis, this orthogonal system R Theta and Phi, so field is expressed in terms of R Theta and Phi, the orthogonal spherical coordinate system. Now if that is the case, we can find expression for the R component of E field, the Theta component of E field and the φ(Phi) component of H field. So H will have only one component that is Phi because if you have current in this way then the fields are around this in the φ(Phi) direction so we have only φ(Phi) components for the H field from the symmetry. Then for the E field we have only R component and theta component and you do not have any φ component for the E field.

$$E_r = \frac{Z_0}{2\pi}\frac{m_E \cos\theta}{r^2}\left(1+\frac{1}{j\beta r}\right)e^{-j(\beta r-\omega t)}$$

$$E_\theta = \frac{-jZ_0\beta}{4\pi}\frac{m_E \sin\theta}{r}\left(1+\frac{1}{j\beta r}-\frac{1}{(\beta r)^2}\right)e^{-j(\beta r-\omega t)}$$

$$H_\phi = \frac{j\beta}{4\pi}\frac{m_E \sin\theta}{r}\left(1+\frac{1}{j\beta r}\right)e^{-j(\beta r-\omega t)}$$

Now if you look at the expression for the electric field and the magnetic field, Z_0 is the free space impedance 377, we will not go into the details of this equation because we are interested in only one term of this equation usually. Now what you can notice is that this is varying with respect to the distance, now it can vary as R square or R cube or if it is θ component it can vary as inversely proportional to R or inversely proportional to R^2 or R^3, so here you can see that it can vary as 1 over R or 1 over R square. So as you are moving far away from this dipole, by the way it is assumed that the length L of the dipole is so small compared to the distance that you are interested in the field as well as wavelength involved, then only this expressions are true, this is a very small dipole compared to the wavelength as well as the distance R.

Now very far away from the dipole, 1/ R^3 times and 1/R^2 times fall off very fast, and basically the terms that are significant in value are 1/R terms. So where are the 1/R terms? So in this you do not have because this is either 1/R^2 or 1/ R^3 so here you have one term that is varying as 1/R and here also you have 1 term varying as 1/ R, so these phase are more dominant far away from the dipole. So let us look at the expressions for those fields so E(θ) and H(φ) and they are orthogonal to each other, these 2 terms and far away from the dipole this is almost like transverse electromagnetic waves, so the pointing vector or the energy flow is in the R direction faraway.

$$E_\theta \approx \frac{jZ_0\beta}{4\pi}\frac{I_0 l \sin\theta}{r}e^{j(\omega t-\beta r)}$$

$$H_\phi \approx \frac{j\beta}{4\pi}\frac{I_0 l \sin\theta}{r}e^{j(\omega t-\beta r)}$$

And θ component and φ components are lying in the plane perpendicular to the direction of propagation on the fields. So this is the expression for the θ component and this is the expression for the φ component, they are proportional to the current and also it varies as a function of Sinθ so it is angular dependent. So when Sinθ is 90 degree, the field will be maximum, and when Sin Theta is equal to 0, in this direction the field is 0 so the radiation pattern is more like this as I am drawing here, so this will be the radiation loss faraway.

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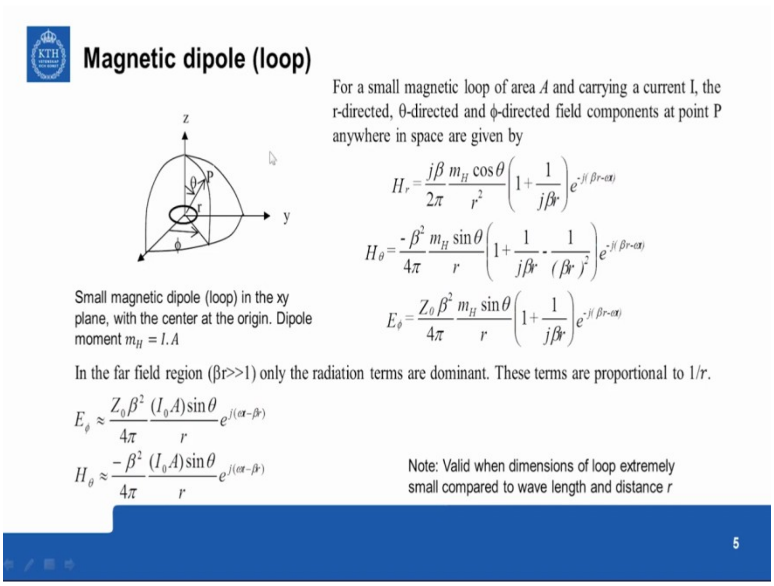

Now consider the case of magnetic loop, so here again from the symmetry there is a loop here in the XY plane, again spherical coordinate is defined and this has a dipole moment I times A, where A is the area of the loop. So the shape of the loop is not very important even though for convenience it shown as round, it can be any shape as long as this loop is very small compared to the wavelength and the distance where we are interested in finding the field, so the shape does not matter really so this is the dipole moment.

Now here also, from the symmetrical configuration you can see that okay any small voltage can drive the current easily around it so it will create magnetic field as well, it can create an electric field drop around the loop, so you can see that electric field will be in φ direction because it can easily create an electric drop. Then magnetic field will be in both R direction as well as Theta direction so you have 3 components only, other components are 0. So here also these components can vary as $1/R^3$ or $1/ R^2$ or 1/R and far away when Beta are far less than 1, Beta is 2 Pi/ Lambda.

$$H_r = \frac{j\beta}{2\pi}\frac{m_H \cos\theta}{r^2}\left(1+\frac{1}{j\beta r}\right)e^{-j(\beta r-\omega t)}$$

$$H_\theta = \frac{-\beta^2}{4\pi}\frac{m_H \sin\theta}{r}\left(1+\frac{1}{j\beta r}-\frac{1}{(\beta r)^2}\right)e^{-j(\beta r-\omega t)}$$

$$E_\phi = \frac{Z_0\beta^2}{4\pi}\frac{m_H \sin\theta}{r}\left(1+\frac{1}{j\beta r}\right)e^{-j(\beta r-\omega t)}$$

So at far distances we are interested only in terms that is varying as 1 over R because those terms will be the dominant one and others will be approaching 0 so those terms are written over here and you can see that Phi and H are orthogonal to each other and they are also orthogonal to the direction of propagation, so far away you can assume that this produces something like a TEM wave. Now E field and H field both are proportional to the dipole moment I_0 times area of the loop and also Sin Theta, so here also the radiation component is I mean radiation is maximum when theta equal to 90 degree in this direction of the plane of the loop and minimum or 0 perpendicular to the loop in the Z direction far away from the field.

$$E_\varphi \approx \frac{Z_0\beta^2}{4\pi}\frac{(I_0 A)\sin\theta}{r}e^{j(\omega t-\beta r)}$$

$$H_\theta \approx \frac{-\beta^2}{4\pi}\frac{(I_0 A)\sin\theta}{r}e^{j(\omega t-\beta r)}$$

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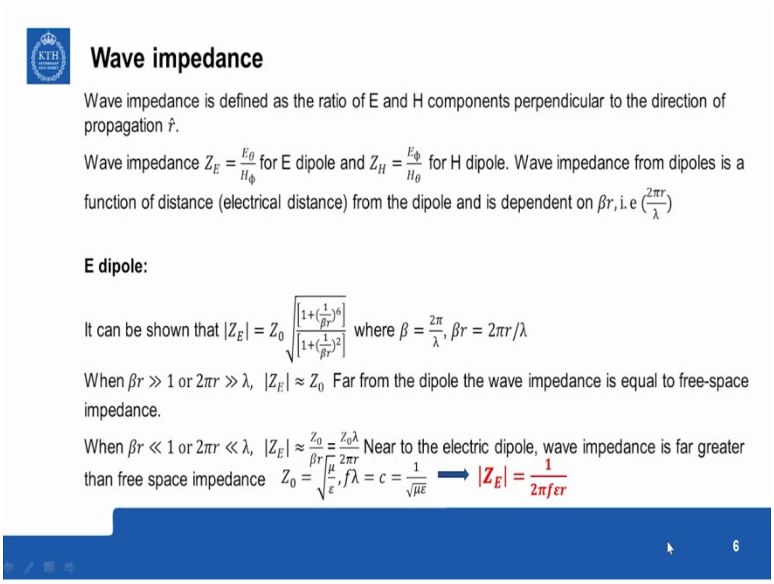

Wave impedance is defined as the ratio of electric field to magnetic fields but those components that are perpendicular to the direction of propagation. So if direction of propagation is R away from the origin, then the electric and magnetic fields in a plane orthogonal to that direction is taken to find the wave impedance. So for electric dipole, it becomes ratio of Theta component of the E field to the Phi component of the H field, and for the magnetic dipole it becomes the Phi component of the E field and θ component of the H field. So this wave impedance is a function of distance or electrical distance from the dipole and it is also dependent upon Beta r, now let us find the wave impedance of the E dipole.

$$Z_E = \frac{E_\theta}{H_\phi}$$

$$Z_H = \frac{E_\phi}{H_\theta}$$

$$\beta r = \frac{2\pi r}{\lambda}$$

So if you take the expression for the θ component of E and φ component of H that you have seen in the previous paragraph these expressions.

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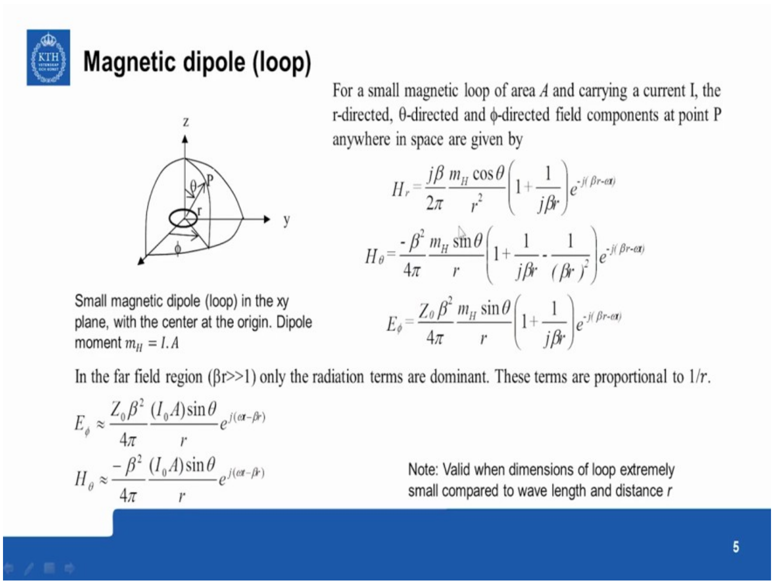

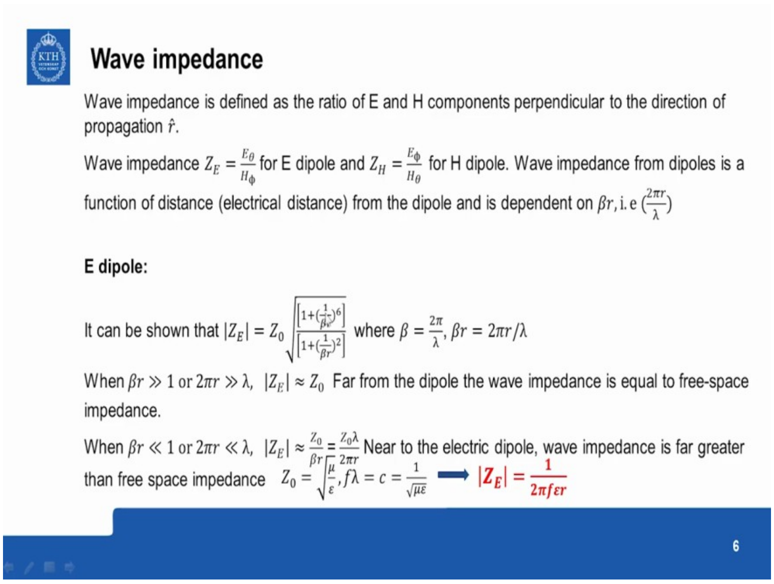

Then you can see that it can be simplified by this expression, β = 2π/λ and βr = 2πr/λ. Now if we have βr far greater than 1 or far from the dipole, the far field condition then this becomes very small and basically what is in the square root is just 1, so you can see that wave impedance for electric dipole far from the dipole is nothing but free space impedance Z_0 that is 377 Ohms, but situation is different when βr is far less than 1 or near to the dipole. When wave impedance, you can see that now βr is small so this becomes very big i.e one by βr, so simplifying you get it as Z_0 by βr and wave impedance is far greater than free space impedance, Z_0 that is $\sqrt{\frac{\mu}{\varepsilon}}$ or $c = \frac{1}{\sqrt{\mu\varepsilon}}$.

Now the expressions for wave impedance, simplifying you get as $\frac{1}{2\pi f \varepsilon r}$, where Epsilon is the electric permittivity. So you can see that very close to the dipole wave impedance becomes quite big and far from the dipole it should be this, but of course this expression is valid only very near to the dipole Beta r far greater than 1, beyond that it is not valid.

$$|Z_E| = Z_0 \sqrt{\frac{\left[1+\left(\frac{1}{\beta r}\right)^6\right]}{\left[1+\left(\frac{1}{\beta r}\right)^2\right]}}$$

When $\beta r \gg 1 \vee 2\pi r \gg \lambda, |Z_E| \approx Z_0$ Far from the dipole the wave impedance is equal to free-space impedance.

When $\beta r \ll 1 \vee 2\pi r \ll \lambda, |Z_E| \approx \frac{Z_0}{\beta r}$ = $\frac{Z_0 \lambda}{2\pi r}$ Near to the electric dipole, wave impedance is far greater than free space impedance

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}, f\lambda = c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad |Z_E| = \frac{1}{2\pi f \varepsilon r}$$

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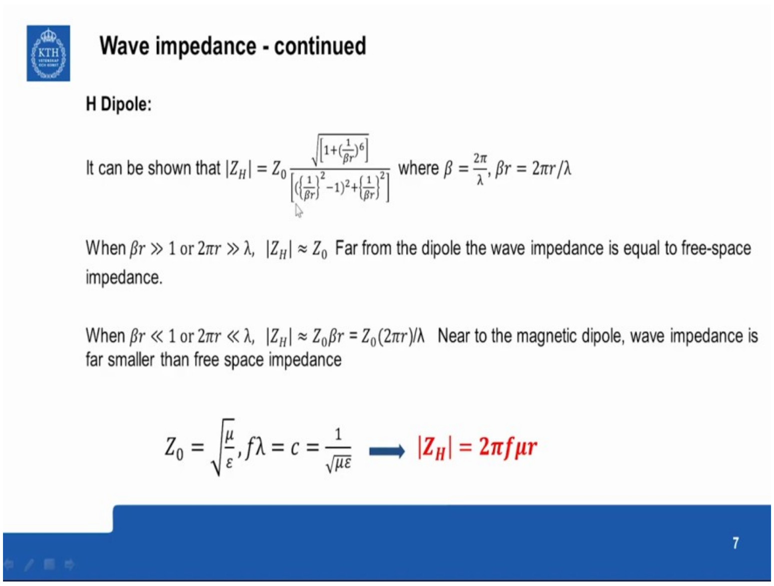

Now we can find wave impedance for H dipole, so from the expressions for the E field and H field and the ratio of that, it can be shown that as wave impedance equals Z_0 multiplied by this expression, where Beta equal to 2 pi by Lambda, $\beta r = 2\pi r/\lambda$ (Beta r = 2 Pi r b y Lambda). Now here we can take 2 conditions; one is when Beta r is far greater than 1 so under that condition you will see that wave impedance is nothing but the free space impedance. And when Beta r is far less than 1, you will see that it is given by free space impedance multiplied by 2 Pi by r by Lambda, so near to the magnetic dipole, wave impedance is smaller than free space impedance and it is $2\pi f \mu r$, Mu is magnetic permeability.

$$\left(\frac{1}{\beta r}\right)^2 - 1¿^2 + \left(\frac{1}{\beta r}\right)^2$$

$$¿$$

$$|Z_H| = Z_0 \frac{¿\sqrt{\left[1+\left(\frac{1}{\beta r}\right)^6\right]}}{¿}$$

When $\beta r \gg 1 \vee 2\pi r \gg \lambda, |Z_H| \approx Z_0$ Far from the dipole the wave impedance is equal to free-space impedance.

When $\beta r \ll 1 \vee 2\pi r \ll \lambda, |Z_H| \approx Z_0 \beta r$ = $\frac{2\pi r}{Z_0 ¿}$)/λ Near to the magnetic dipole, wave impedance is far smaller than free space impedance

$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}, f\lambda = c = \frac{1}{\sqrt{\mu\varepsilon}}$ $|Z_H| = 2\pi f \mu r$

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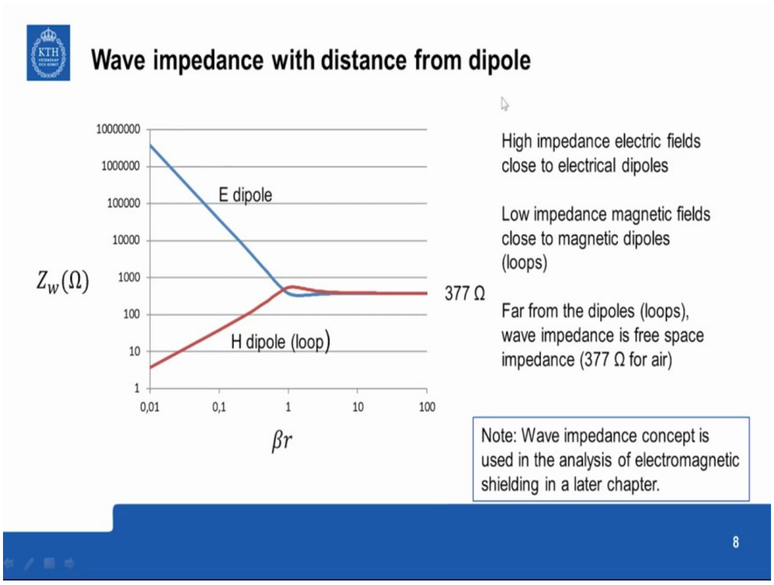

So we have two different kinds of expressions near to the dipole for electric dipole and magnetic dipole, whereas far from the dipole both gives free space impedance, so let us plot this out. So in this, wave impedance is plotted with distance from the dipole so to normalise for the frequency or wavelength, we take β is 2 π r/ λ as the X axis, so βr = 1 will be 2πr = Lambda, when λ equal to 2 π r, then it becomes 1 then this is the wave impedance and this blue line is for electric dipole and red line is for magnetic dipole.

You can see that for the magnetic dipole wave impedance starts from a very low value, the impedance then increases as distance is increased, it reaches 377. For the E dipole it starts from a high impedance when you are very close to, then it comes down and reaches 377. So often when we have a problem of magnetic fields we talk of low impedance magnetic fields and we often talk of high impedance electric fields when we are close to the source, so the reasons for those expressions are coming from this graph. High impedance electric field is close to electric dipole, low impedance magnetic field close to magnetic dipoles, far from the dipole wave impedance is free space impedance for air.

Wave impedance concept we will be using quite extensively in the analysis of electronic shielding in later chapters, so that we will talk about what is ECL to shield against high impedance electric fields, whereas it is very difficult to shield against low impedance magnetic field or we will say that shielding is more difficult when it is low frequency magnetic field.

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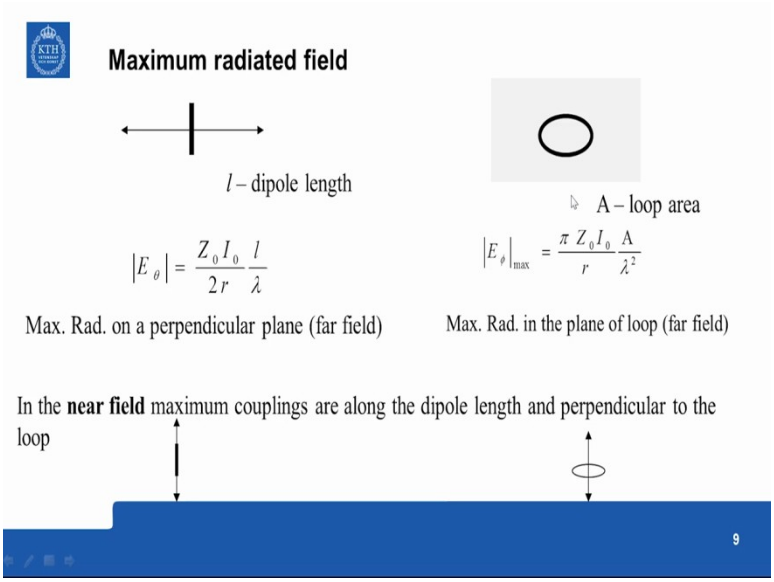

Now, where will be the maximum radiated field? So this is a dipole, first we look at the far field, maximum electric field we have seen before when θ = 90 degree, so in this direction expression is given by this, so maximum radiated field is proportional to the current as well as electrical length of the dipole L by Lambda, and inversely proportional to the distance from the dipole so this is perpendicular, maximum radiation field is on a perpendicular plane to the dipole. Now here, maximum radiation is in the plane of loop in the far field and A is area of the loop, so maximum field in the φ direction is proportional to the current inversely proportional to the distance and ratio of the area of the loop divided by Lambda square wavelength square.

$$|E_\theta| = \frac{Z_0 I_0}{2r}\frac{l}{\lambda}$$

$$|E_\phi|_{max} = \frac{\pi Z_0 I_0}{r}\frac{A}{\lambda^2}$$

Now in the near field maximum couplings are along the dipole length and perpendicular to the loop. So this is opposite to that in the far field so in the near field maximum coupling is along the dipole for the electric field and perpendicular to the loop for the magnetic loop.

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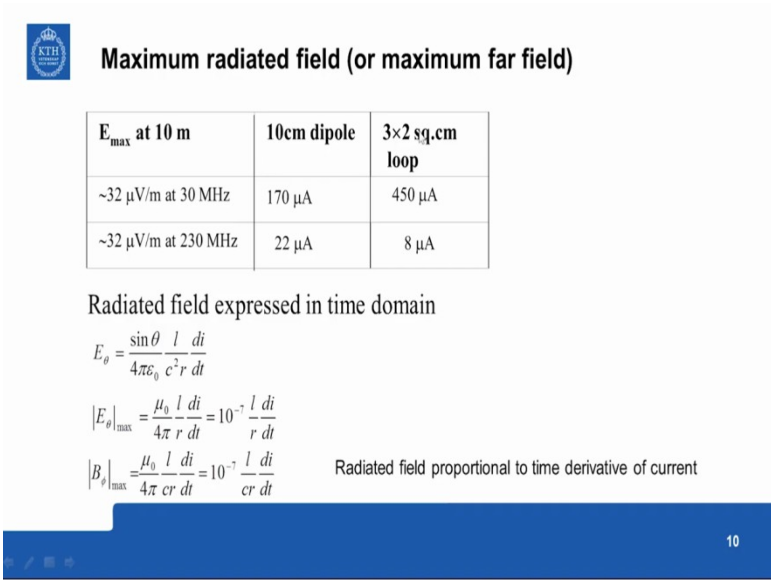

Now this shows example calculation for the maximum radiated field at a distance of 10 meter and 10cm dipole versus 3 by 2 square centimetre loop, again shape of the loop can be anything. In the standards it has been specified that electric field should not exceed 32 micro volts per meter at 30 megahertz, so you will already exceed this limit if you have a small dipole carrying a current of 170 microampere, and if it is for this small loop 450 microampere will be the limit, above that you already exceed this value permissible value by the government regulations.

Similarly, at 230 megahertz to reach this value you need only 22 microamperes for dipole and for a loop you need only 8 microamperes because for the loop 8 divided by Lambda square that is why the changes are much faster here. Now sometimes we can find the radiated field in time domain other than the frequency domain, then you can see it is proportional to rate of change of current dI by dt and the length of the dipole and inversely proportional to the distance and Mu$_0$/4 Pi these are all constant, C is also constant you can write it in this way 10 to the power – 7 l by r d I by dt volts per meter. And B Phi, flux density, is 10 raise to - 7 divided by C r d I by dt Weber per meter square, so you can see that ratio of E and B it is speed of light, radiation field is proportional to time derivative of the current.

$$E_\theta = \frac{\sin\theta}{4\pi\varepsilon_0}\frac{l}{c^2 r}\frac{di}{dt}$$

$$|E_\theta| \frac{\mu_0}{4\pi}\frac{l}{r}\frac{di}{dt}^{-7}\frac{l}{r}\frac{di}{dt}_{max}$$

$$\left|B_{\varphi}\right| \frac{\mu_0}{4\pi}\frac{l}{cr}\frac{di}{dt}^{-7}\frac{l}{cr}\frac{di}{dt}_{max}$$