Digital Signal Processing
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Lecture 16:
LTI Systems (3)- Java applet demo of convolution

I want to spend a few minutes pointing out some aspects of convolution, just in case, you may not have observed some of the aspects associated with it. So, I am going to use the Java applet of John Hopkins website. Before I use that, let me draw a couple of waveforms and point out the kind of things I want to show there.

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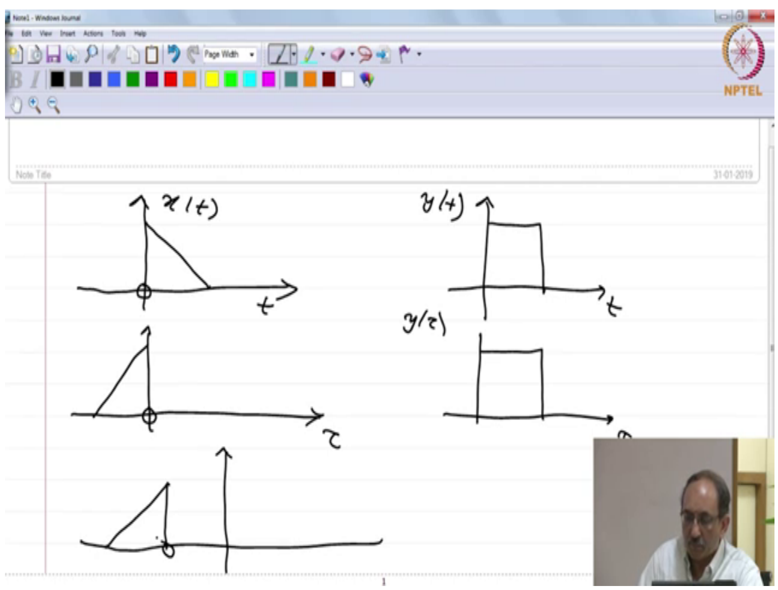

Of course, I have two signals that I want to convolve, all of this is going to be in continuous-time, but exactly the same idea applies to discrete-time as well. So, if I have two signals that I want to convolve. So, this is $x(t)$ and this is $y(t)$. Now, I keep one signal fixed and I am going to change the independent variable to τ. So, this becomes $y(\tau)$. What I am going to the other signal is I am going to do two things first, I am going to flip and then shift so, again the independent variable becomes τ, I flip it.

The point you have to keep in mind is the point of reference will be always the origin of the signal that you are going to flip and shift. So, after flipping because you have not shifted, the origin stays as it is. Then when you shift, you can either shift it to the left.

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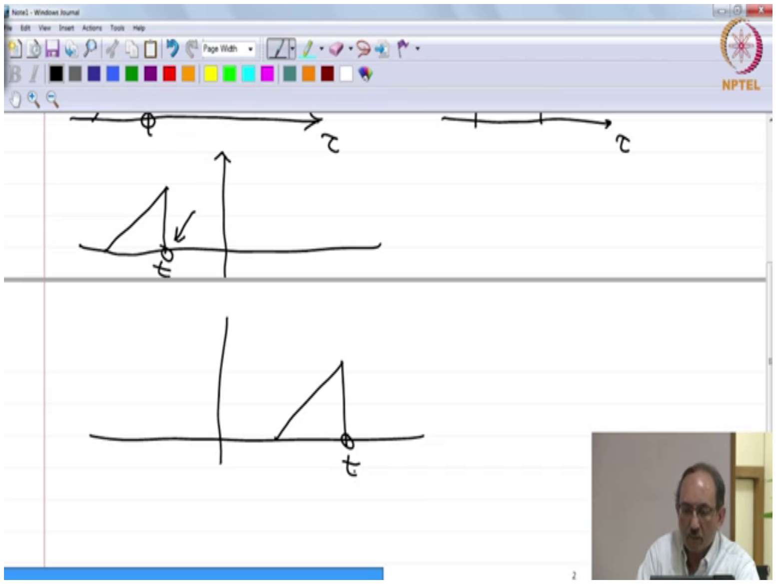

Or, shift this to the right and for each of these cases, you need to pay attention to this point. This point will be t. Similarly, this point will be t. So, the output will always be associated with this location, alright. So, this is the main point. So, when you talk about $y(t)$, you are talking about t that is given by this variable here, where the independent axis is τ, but t is this parameter and t is the parameter associated with the shift.

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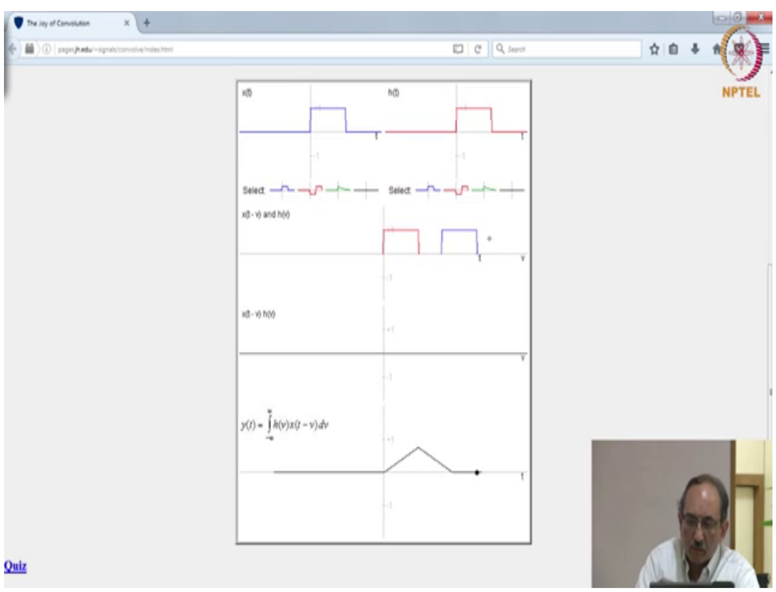

So, with this let us go back and look at this applet. So, here it gives you the choice of certain predefined waveforms. So, I can choose this to be the first waveform, I can choose the second waveform also to be the same. So, these are the two different waveforms that you want to convolve. And then in the second panel, one waveform is kept as it is, the second waveform is flipped and shifted and if you notice here, this is the origin of the waveform that we have flipped and we have shifted.

So, if you note that this is the origin, if you flip it, the pulse flips around because it is a square pulse, you do not see any difference. However, if you notice where the origin is attached, so, this was the original origin. After shifting, remember the pulse was to the right of the origin before flipping whereas, after flipping the pulse is to the left of the reference point and this origin is marked as t and now I can move this to the left or to the right, all right.

And this third panel is going to show the product. Right now, there is no overlap therefore, the product is 0. The last panel is going to show the area under the product and if you look at this dot here that is aligned to this t.

Therefore, when you move to the left or to the right, this dot in the bottom most panel which is going to give you the result of the convolution will always be tied to this reference point. So, now, you keep moving, no overlap and at this point you just start to overlap these two pulses. The third panel shows you the product and the last panel shows the area under the product. Therefore, when you slide it across, the third panel shows the product and as you keep sliding in, product has more area and at this point you have reached the maximum overlap.

So, the area is maximum and then when you slide out, the overlap decreases and hence the area also decreases, convolution result decreases, the decrease is linear and you get this triangular output ok. So, this is pretty straight forward, but I want you to focus on the fact that the output is tied to the origin of the signal that is being flipped and shifted. That is the key point.

So, this t and this dot are tied together that tells you what the convolution the result is, right. So, this is pretty straight forward and now let us clear the first curve. Now, I will again convolve two square pulses except now, the first square pulse is going to be this.

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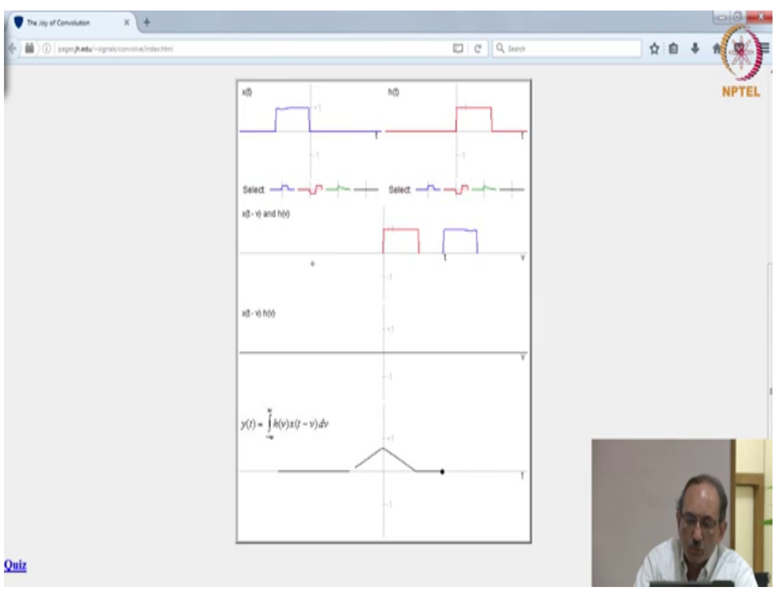

So, it is roughly the same width as the first one and now, let us look at where the t point is. Now, you can expect the pulse which is now to the left of the origin after flipping, it will be to the right. Therefore, the reference point which was at the leading edge before, now will be at the trailing edge so, that you can see here. So, right here is the flipped and shifted pulse. Notice now where this t is attached to, it is attached to the origin of the first pulse, right. And now you see the reference point is here and now when I start to move in, at this point of shift the overlap starts to happen and when I slide it across, I get something like this.

Now, you see the difference in these two cases and where this variable of shift t which is what determines the output is. So, this clearly illustrates the difference between these two cases. And just to explore this point more, if I now have a pulse that is shifted here. So, now, this is the origin when I flip and shift, this point will continue to be the reference point.

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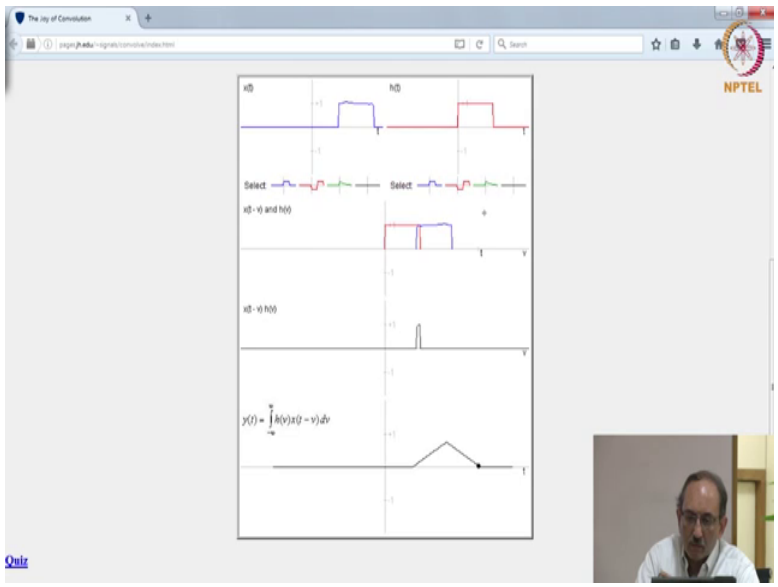

So, now look what happens here right, the origin point that was here is now here after flipping and shifting and this is indeed the point of reference. As you now move this across, t, the overlap will happen only when t occurs later. So, when you do this, you get the output.

Therefore, when you have a pulse that is delayed, the convolution also is delayed and the feel for that is seen from this plot because the output is tied to the origin of the original pulse. So, now, you see this. And the final point I want to illustrate is remember, I had mentioned that convolution is a smoothing operation because it involves integration or summation, integration in the CT case and summation in the DT case.

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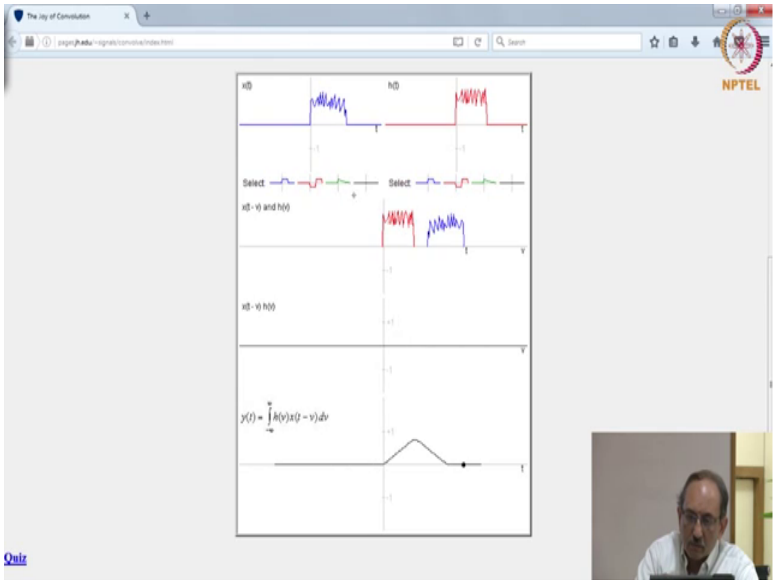

Now, let us take two square pulses that are now noisy. So, this is one pulse so, you can think of this as a square pulse plus some random noise and now, here is another pulse which is similar. So, now, what I am going to do is now, I am going to convolve them. If if these were two clean pulses, you would expect a triangular shape.

Now, let us see what happens here all right, the final result is still not too different from a triangle because all these variations have been smoothed out. So, this kind of nicely illustrates the fact that when you integrate, all these fluctuations get smoothened out. So, if you have not seen before, you may not have guessed just based on equations. And so, you can play with this applet and get a feel for how this works.

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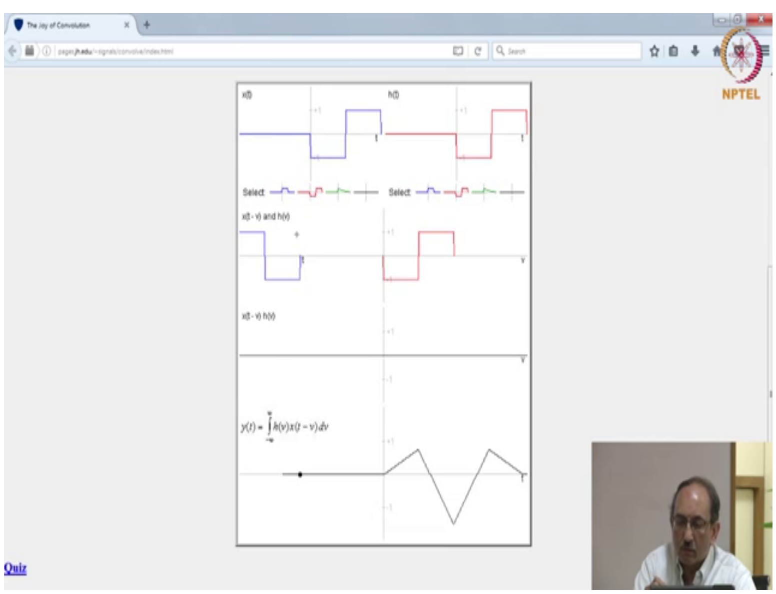

So, this is `pages.jh.edu/~signals`, I will put this link out on Moodle, but one thing that is possibly might be a problem is because of security problems more and more browsers are not allowing Java applets to run and this is an old version of Java and old version of the browser.

So, I think Firefox put out a notice saying they will no longer support Java. And internet explorer seems to be the only last choice, but when I tried it on internet explorer on this machine, it did not work. Anyway, if you can get it to work and play around with it, good for you. But in general, this site is a good site to take a look at, lots of fun experiments are there. So, this is as far as convolution is concerned.