Digital Signal Processing

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Lecture 17:
LTI Systems (3) -Systems governed by LCCDE

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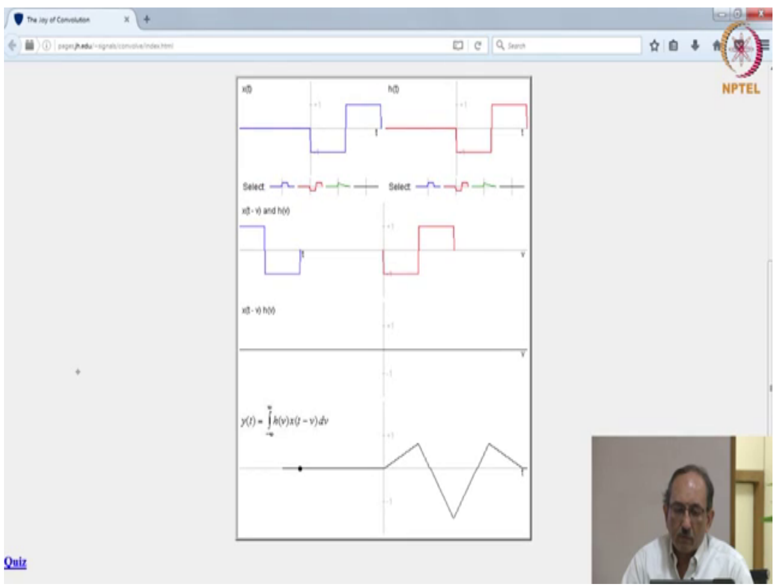

Now, what we will do is we will continue with where we stopped at. We had looked at systems and their properties and we had looked at LTI systems and as I had mentioned towards the end of last class, we are going to concentrate on a particular subclass of LTI systems, namely those systems that are governed by linear constant coefficient difference equations, which is the counterpart to linear constant coefficient differential equations.

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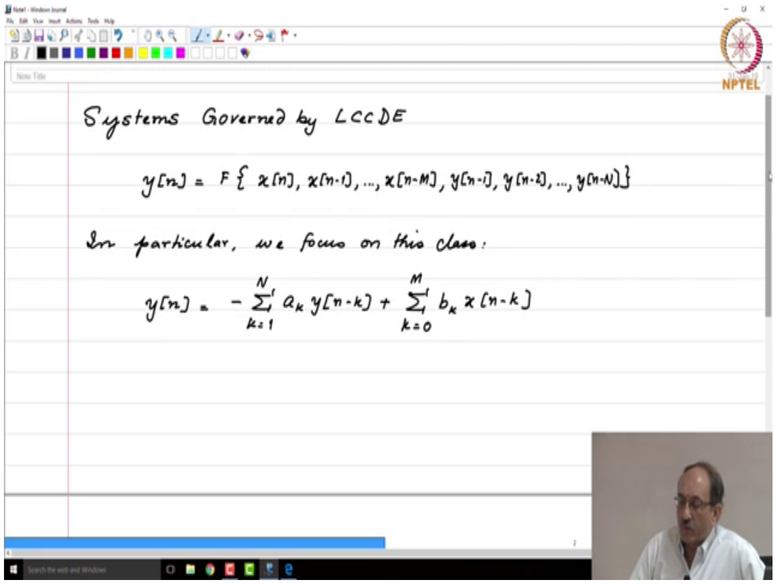

Now, let us look at Systems governed by LCCDE. So, what we have been looking at so far is the general case, general class of LTI systems. So, we now we will focus on a specific subclass. So, we will look at systems whose output $y[n]$ is a function of $x[n]$, $x[n-1], \ldots, x[n-M], y[n-1], y[n-2], \ldots, y[n-N]$ i.e., $\mathcal{F}\{x[n],\, x[n-1], \ldots, x[n-M], y[n-1], y[n-2], \ldots, y[n-N]\}$.

So, we are going to look at systems whose output depends only on these samples not on anything else. Even this is too general. In particular, we do not want $\mathcal{F}$ to be an arbitrary function, we want $\mathcal{F}$ to be the class of linear constant coefficient difference equations. Therefore, in particular, we focus on this class. That is we want $y[n]$ to be of this form,

$$y[n] = -\sum_{k=1}^{K} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k].$$

So, this is the specific form of $\mathcal{F}$ that we are interested in. So, this is clearly a linear constant coefficient difference equation and this is the counterpart to systems being described by linear constant coefficient differential equations. There also, you saw general systems, then you focused on LTI systems and within LTI systems you focused on systems that are governed by linear constant coefficient differential equations. So, this is the counterpart to that. Now, by the way, why did you focus on systems governed by linear constant coefficient differential equations in that course? Why that particular class?

Student: (Refer Time 04:05).

You can solve them. You mean you cannot solve systems other than that form.

Student: We can, but (Refer Time 04:21).

Forget about ease of solving. Suppose, ease of solving is not an issue

Student: (Refer Time 04:28) initial condition (Refer Time 04:30) like canceling out each other.

How can initial conditions cancel out each other?

Student: (Refer Time 04:8).

I mean there are cases where initial conditions do not. I do not know what exactly you mean by canceling out, but assuming for the moment.

Student: (Refer Time 04:50).

Ah Student: (Refer Time 04:51).

Oh ok. So, you are concentrating on incrementally linear versus this, but in general right, why linear?

Student: Checking it linear (Refer Time 05:08).

No why you said that you want. That is the question right. Yeah, you are about to say something?

Student: What is the (Refer Time 05:16).

Ah.

Student: (Refer Time 05:20).

Ok, but suppose there are systems that are non-linear.

Student: Sir (Refer Time 05:24) operator is ligand we can find (Refer Time 05:28).

So, that is again you are coming to your ability to solve them correct. When you say homogeneous, I mean particular integral complementary functions all tied to your ability to solve. So, if you take 1 step back and said I can solve even another class of equations, right.

Student: (Refer Time 05:58).

Yeah, but then this is subclass, there are LTI systems there are not in fact, that is a good question. Can you give an example of an LTI system in continuous-time that is not in the form of a linear constant coefficient differential equation? So, this you must know from what you have learnt already.

Student: (Refer Time 06:25).

Say that again. No, that is a signal right. Here, we are talking about a system. Can you give me an example of an LTI system that is not governed by a linear constant coefficient differential equation? Because, LTI systems that is a larger class LCCDE is a subclass. So, I am asking you to give me a member of something that is LTI, but still not LCCDE in continuous-time. Again, so, these are the kinds of question I want you to have asked or continue to ask even when you see some concept here, right.

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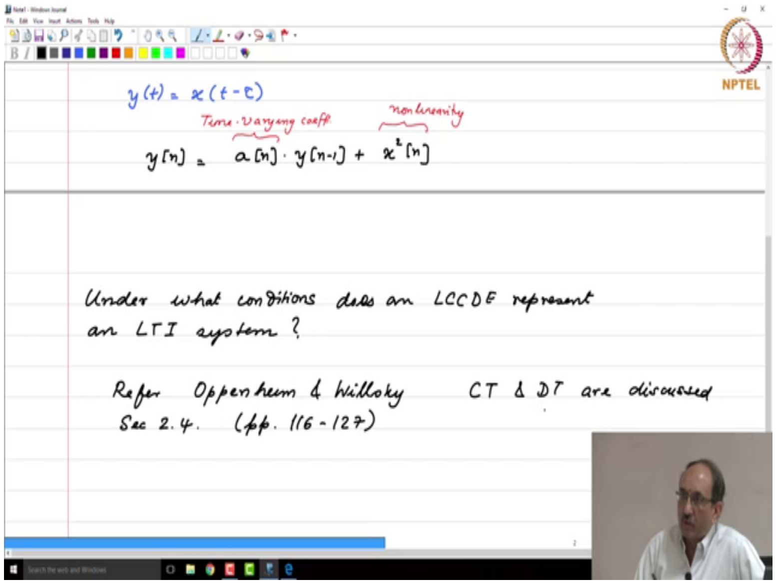

So, the answer is very simple. So, $y(t) = x(t - \tau)$. So, this is a pure delay and this is not a system that belongs to that class. The answer is very simple, I am sure you must have encountered delay in CT, alright. So, but still this begs the question as to why are you so much enamoured by systems governed by linear constant coefficient differential equations in continuous-time and same question will arise in the discrete-time as well.

So, in discrete-time, among linear time invariant systems, we will focus only on this particular subclass alright. Now, just to put this in perspective, if you have $y[n] = a[n]y[n-1] + x^2[n]$. So, here, what you have is a time varying coefficient. This also looks like a difference equation, but here you have a difference equation with time varying coefficients and here you have non-linearity whereas, we are going to focus only on the class of linear constant coefficient difference equations. So, some of these open questions we will revisit later. Yeah, go ahead.

Student: (Refer Time 09:32).

So, in the difference equation the question is why is the negative sign there. Is that the question? ok. We will later, what we will do is we will convert this into the transform domain and the transform that we are talking here is the z-transform. So, this is analogous to what was happening in the CT case. There you had a linear constant coefficient differential equation and there you looked at the system in the transform domain by taking the Laplace transform. So, same thing we will do here and when you look at this in the transform domain, we will talk about why this minus sign. This minus sign is not very fundamental, you can always rewrite this as c_k where $c_k = -a_k$, that is all.

So, the question to ask is under what conditions an LCCDE or rather does an LCCDE represent an LTI system? So, this is an important question to ask. So, I want this to be left as an exercise to you. So, please refer Oppenheim and Willsky Section 2.4 pages 116 to 127.

So, this talks about both continuous-time case as well as the discrete-time case, that is CT and DT are discussed. The CT case is the linear constant coefficient differential equation, in the DT case is the linear constant coefficient difference equation. And then Oppenheim and Willsky, they work out the continuous- time case example in a very detailed manner and in one of the exercises, the DT case is developed paralleling exactly what was happening in the CT case and we will just make note of the conclusion that falls out of this.

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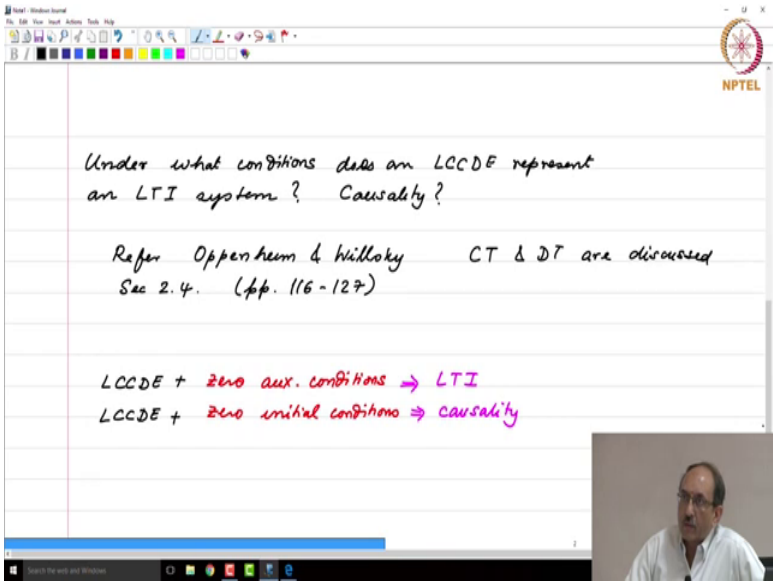

So, LCCDE + zero auxiliary conditions, this implies LTI. So, if the input output relationship is described by a linear constant coefficient either differential equation or difference equation, that represents an LTI system provided you have zero auxiliary conditions. And LCCDE + zero initial conditions, this represents causality. So, the other question that you want to ask is not only do you want linearity and time invariance, you also want causality. Yeah, go ahead.

Student: (Refer Time 13:31).

Yes, no. So, LCCD by itself does not represent an LTI system, that is the point that is made here. Merely the input output relationship being represented by an LCCDE does not guarantee an LTI system. LCCDE + zero auxiliary conditions is when you have LTI, + zero initial conditions is when you have causality. Does that answer the question?

Let me make some brief remarks, I want you to refer to this section and then probably if there some questions, I can always clarify. When you have a linear constant coefficient differential equation, to solve that, you would solve it, first you would solve the homogeneous equation and then you will solve the particular integral.

If you have an nth order differential equation, the homogeneous equation will have n unknown constants. To pin down the solution uniquely, you need auxiliary conditions given to you which help you to evaluate these unknown constants and help you to pin down the solution uniquely, alright. If these conditions are not given, you cannot find the solution to this equation. To help you to get the final solution is why you need these auxiliary conditions, alright.

Student: Sir.

Yes.

Student: We can also use an initial condition as auxiliary condition.

So, auxiliary conditions are not necessarily at what we call as $t = 0$ or $n = 0$. They can be specified at any point in time. Initial conditions are auxiliary conditions specified at the origin. So, that is why that is tied to causality.

Student: Sir.

Yeah.

Student: (Refer Time 16:06).

There is some result that states, that connects LTI and causality. I will look that up and get back to you.

Student: (Refer Time 16:30).

Yeah, that is why there is a connection between LTI and causality. There is a specific theorem that I will write off the top of my head I do not recall. I can look it up and get back to you. Linear operators in Engineering and Science Naylor and Sell, Springer-Verlag, the theorem is there in that book. I need to locate that and discuss what the implications are, alright.

So, this is something that you need to read upon to make sure you understand why LCCDE alone by that input output class alone does not represent an LTI system, you need some more conditions for it to represent an LTI system. And the intuition behind this is for it to be a LTI system, 0 input has to produce 0 output. So, its tied to that concept here.