Digital Signal Processing
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Lecture 27:
troduction to the DTFT, Properties of the Z-transform (1)
Properties of the Z-transform: linearity

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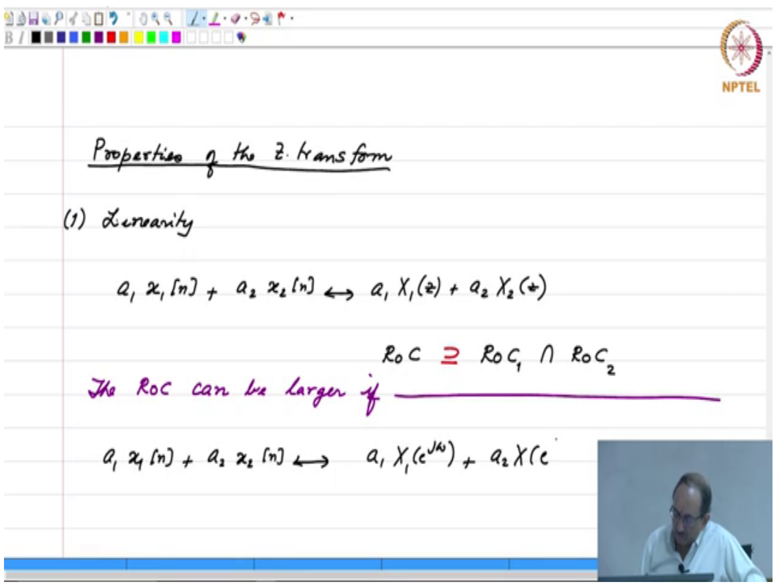

So, now we will start looking at properties of the Z-transform. So, now that we have the definition of the DTFT with us, for each property we will set the property for the Z-transform and also the corresponding property for the DTFT. So, the first property of course is linearity, $a_1x_1[n] + a_2x_2[n] \longleftrightarrow a_1X_1(z) + a_2X_2(z)$. So, this property immediately follows from the linearity of the sum. And, the RoC will be at least as large as the intersection of the individual RoCs. If X_1 had RoC_1, X_2 had RoC_2, this will be at least as large as the intersection.

And, the RoC can be larger if we will come to this in a minute. And, the corresponding property for the DTFT is this, again pretty straightforward here. Notice that, whenever we state a property for the Z-transform, you will also not only give the expression, but also make a statement about the RoC. On the other hand, when we are looking at the DTFT, no other qualification is added and that is because?

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Yes. So, as far as the DTFT is concerned, the DTFT exists on the unit circle; therefore, you do not have to make any further qualifications in terms of properties, because we assume that if the DTFT exists, the RoC is on the unit circle, all right.

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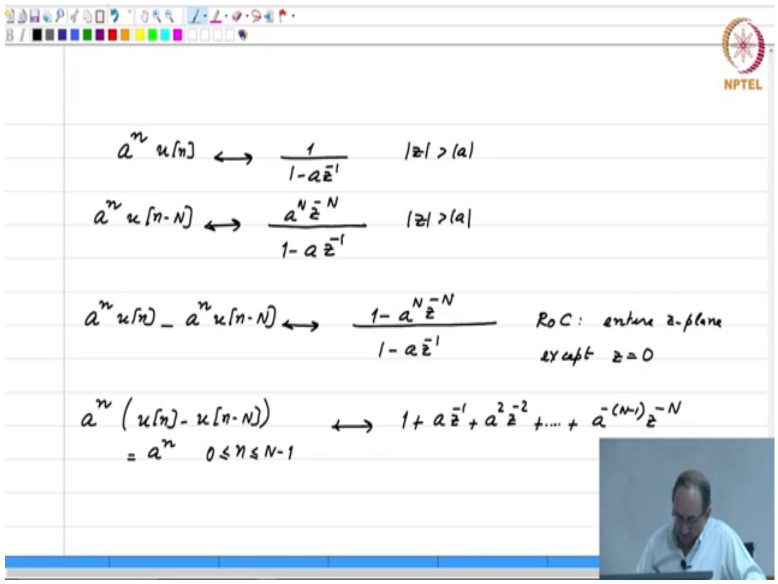

Now, let us look at this. So, this is the first sequence $a^n u[n]$ and the second sequence is $a^n u[n-N]$. And, the next property that we are going to see is the delay property and in just a couple of steps we will be able to show what the Z-transform of this second sequence is. Right now, you can either work it from first principles or take this on faith. So, this is $|z| > |a|$, all right.

And, now if you look at this sequence, that is $a^n u[n] - a^n u[n-N]$, then this of course is $\dfrac{1-a^N z^{-N}}{1-az^{-1}}$; and the RoC is the entire z-plane except $z = 0$. So, here is an example where the RoC is larger than the individual RoCs. And what is happening here is, so this is nothing but $a^n\big(u[n] - u[n-N]\big)$ and this is nothing but a^n for $0 \leq n \leq N-1$.

And, this Z-transform by definition is $1 + az^{-1} + a^2z^{-2} + \ldots + a^{-(N-1)}z^{-N}$. So, this is a finite duration sequence that we have seen that for finite duration sequences, the RoC is the entire z-plane except possibly for 0 and or ∞. In this particular case, only 0 is not part of the RoC. So, what is happening here is that if you look at this expression, so these two are clearly identical, because they are the Z-transform of the same sequence.

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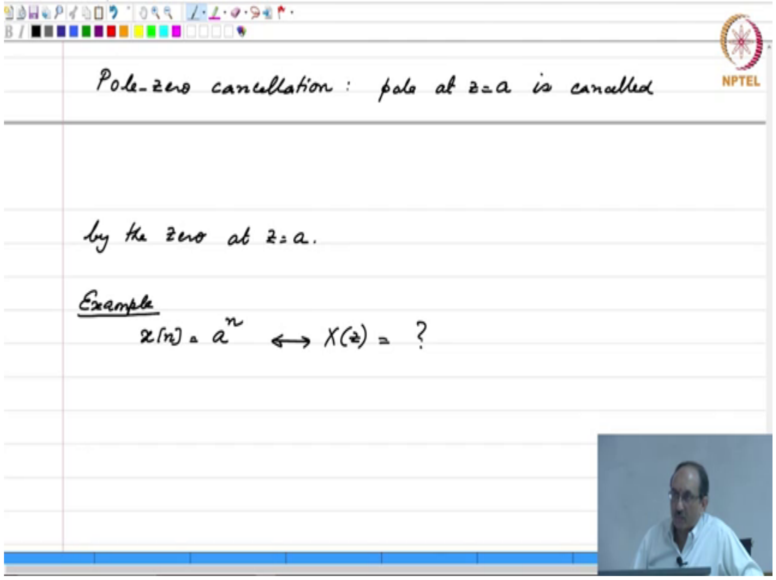

And, the region of convergence is larger because of pole-zero cancellation, ok. So, the pole at $z = a$ is cancelled by the zero at $z = a$, so this makes the RoC larger. So, this is exactly the same as what was happening in the continuous-time Laplace, continuous-time function and its associated Laplace transform.

If you had $a_1x_1(t) + a_2x_2(t)$, the corresponding Laplace transform was $a_1X_1(s) + a_2X_2(s)$ and the RoC was once again, at least as large as the intersection of the individual RoCs. There as in the case here, you can have an RoC that is larger if there is pole-zero cancellation. So, this is exactly a mirror of what was happening there.

And, since in this course, we will be interested only in functions with rational transfer function, if there is a pole-zero cancellation, the RoC will be larger, that is all. Suppose, you had $x[n] = a^n$, so the corresponding Z-transform is what? Use the linearity property and evaluate the Z-transform.

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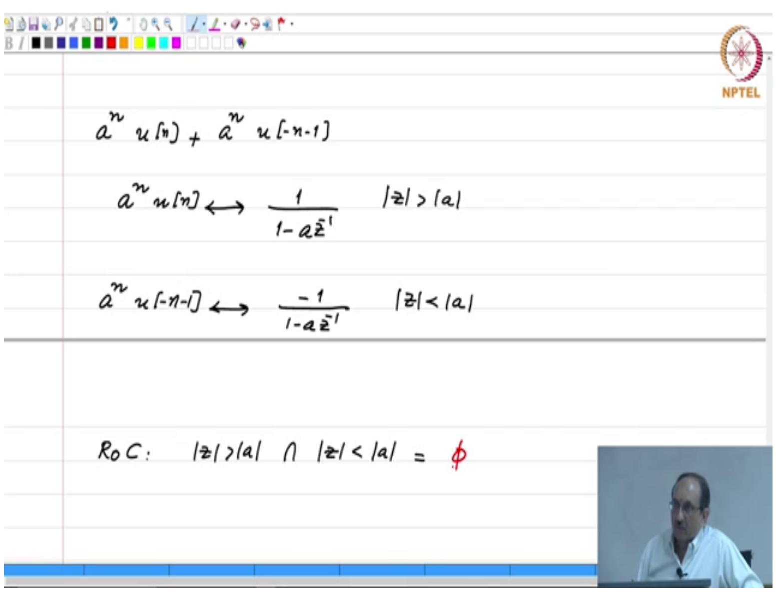

So, this can be written as $a^n u[n] + a^n u[-n-1]$. So, why do not you work out the transforms in the individual case and then the final transform. And, $a^n u[-n-1]$ has Z-transform $\frac{-1}{1-az^{-1}}$. And, if you are not careful superficially, if you add these two in the Z-transform domain, the two terms cancel and you will get 0 to be the answer which is clearly not right.

What is happening here is, this is $|z| > |a|$ whereas, here this is $|z| < |a|$ and the final RoC is $|z| > |a| \cap |z| < |a|$ and this of course, is null.

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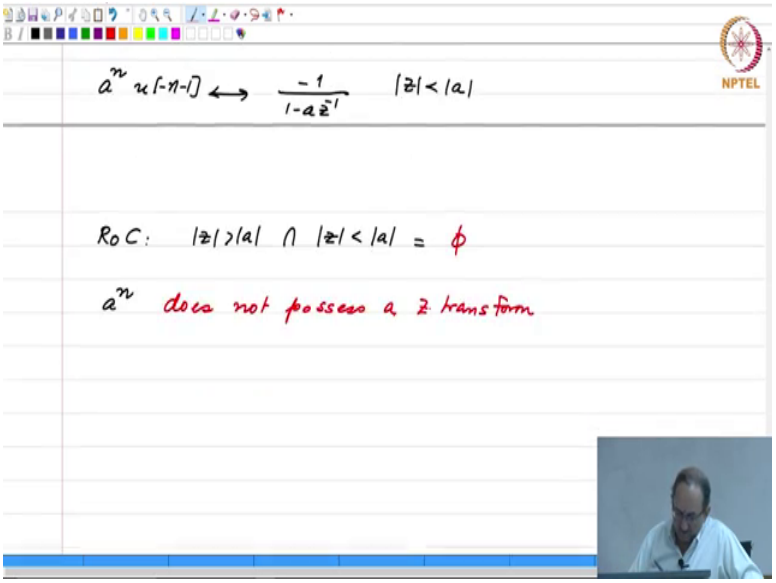

Therefore, a^n does not possess a Z-transform. So, this is the counterpart of e^{at} not possessing bilateral Laplace, $e^{at}u(t)$ has Laplace, which is $\frac{1}{s-a}$, RoC is $Real\{s\} > Real\{a\}$; $e^{at}u(t)$ has Laplace, but e^{at} over all t does not possess bilateral Laplace. So, this is exactly the counterpart of that.