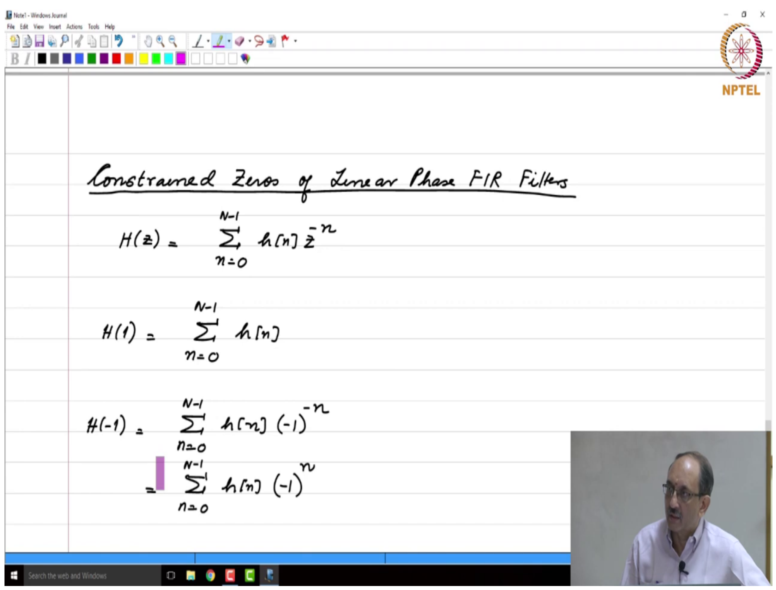
Digital Signal Processing
Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 64:
Linear Phase (3)
-Constrained zeros of linear phase FIR filters

(Refer Slide Time: 00:20)



Another important aspect of Linear Phase FIR filters. We are going to look at constrained zeros of linear phase FIR filters. So, $H(z) = \sum_{n=0}^{N-1} h[n]z^{-n}$. So, this is the Z-transform of an FIR filter, general case. Now we are going to examine two specific points in the z-plane and then we are going to see whether no matter what $h[n]$ is, our zeros present at these two points.

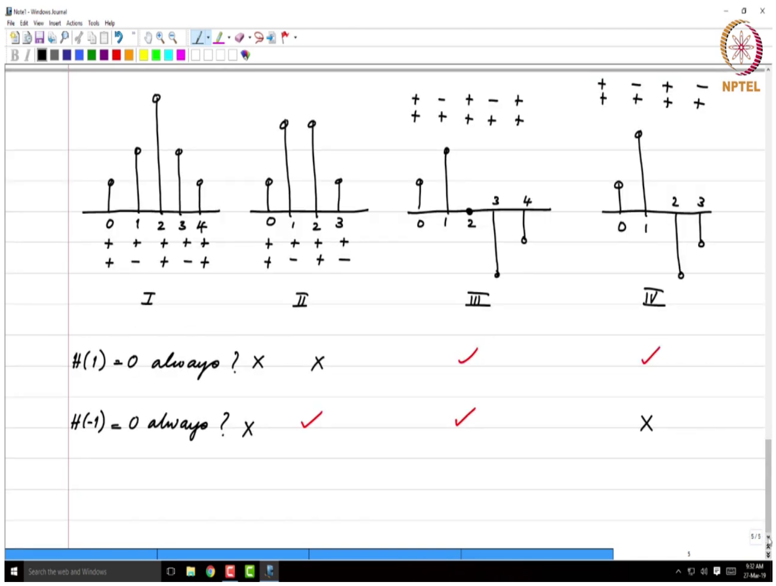
And the points we are going to look at are $z = 1$. So, $H(1) = \sum_{n=0}^{N-1} h[n]$, you put $z = 1$, this is what you get. The other point that we are going to look at is $H(-1)$. So, this is $H(-1) = \sum_{n=0}^{N-1} h[n](-1)^{-n}$, which is the same as $H(-1) = \sum_{n=0}^{N-1} h[n](-1)^{n}$; $(-1)^{-n}$ is the same as $(-1)^n$.

So, we are going to examine the Z-transform of the linear phase FIR filter and look at the Z-transform at $z = 1$ and $z = -1$. To see whether there is a zero at $z = 1$, all you need to do is, you need to sum of these samples; that is what this says. Sum up $h[n]$ hence, examine whether $H(1) = 0$ independent of the values of $h[n]$. Whether this Z-transform of $H(1) = 0$ always, no matter what $h[n]$ is.

The other location is at $z = -1$, rather than summing up $h[n]$ itself, you multiply it by $(-1)^n$ and then sum up the coefficients; the resulting sequence. And then, see whether the resulting number is always

0. If it is, then you can make the statement that there is a constrained zero either at $z = 1$ or $z = -1$ depending upon whether each of these statements is true or not. So, we will get a feel for this by looking at simple examples.

(Refer Slide Time: 03:14)



And then the generalization will very easily follow. So, we are going to look at types I, II, III and IV in that order. And if you recall, type I is even symmetry and odd length. Therefore, so this is clearly a type I filter; the symmetry is even and the length is odd. yYou have 0 to 4, 5 samples therefore, length is odd. Therefore, to look at whether $H(1) = 0$, always you need to sum up all these samples.

Therefore, you add up all these things. And, then you want to ask this question, is $H(1) = 0$ always? And the other question, is $H(-1) = 0$ always? So, these are the two things we want to answer. So, clearly, if you add up all the sequence, will you always get 0? No, therefore, for Type I, this is not the case that $H(1) = 0$ always. Now the other thing is, we have to examine it $H(-1)$.

So, you need to multiply it by $(-1)^n$ first therefore, this is $+, -, +, -, +$. If you now look at these samples, for example, this sample corresponds counterpart is this. After multiplying by $(-1)^n$n, in this particular case, they maintain the same sign. And if you look at this, this gets multiplied by -1, this gets multiplied by -1, right; because this gets multiplied by $+$ in this particular case.

Now, if you add them up, is it the case that this will always be 0? No, all right. Now let us look at Type II; Type II is even symmetry as Type I, but the length is even. Therefore, so, this is a simple example of a Type II filter; even symmetry even length. And now, if you add up all these samples, it is not in general true that this will always be 0 independent of the values of $h[n]$.

Therefore, there is no constrained zero at $z = 1$. Now, let us look at the other thing, now you have $+, -, +, -$. Now if you look at this, this is multiplied by $+1$, its counterpart is getting multiplied by -1. And therefore, if you add them up, these two terms will cancel. Similarly, this and this now have opposite signs and you add those, they will cancel.

Therefore, independent of $h[n]$ for the Type II case, $H(-1)$ will always be 0. Therefore, you are forced to have a zero at $z = -1$. And one immediate implication of this is, if this is zero at $z = -1$, you are guaranteed that in the Z-transform, it will have a factor $(1 + z^{-1})$; that is what this means. Now let us along these lines quick. We will be able to see what is going on here.

Now we are looking at type III. Type III is odd symmetry and odd length, therefore if the first sample is 0, the last sample has to be like this. If this is like this, this will be like this, 2 and 3. Center sample of course, is 0 because of the odd symmetry. Now you are going to sum up all these samples. If you sum up all these samples, will they be 0? Yes, because this after all is odd symmetric.

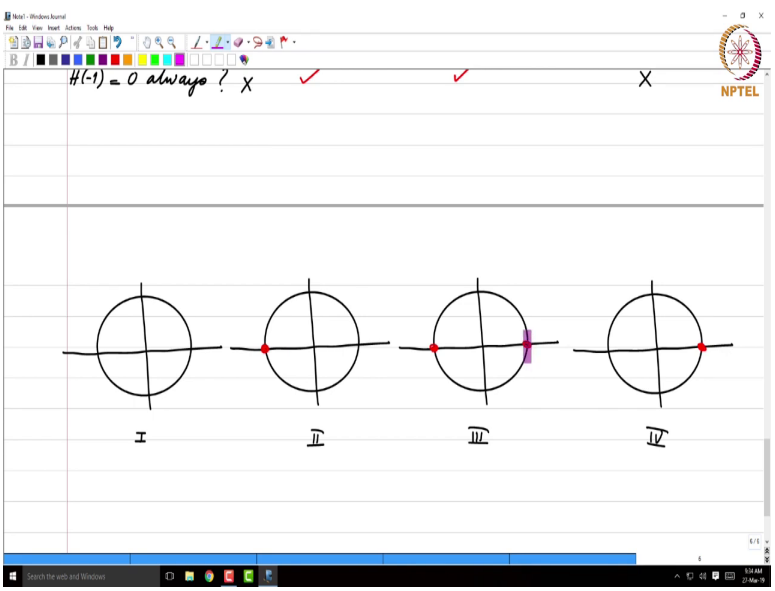
And if you sum up all these samples of an odd symmetric finite duration sequence, you are guaranteed to get 0. And now the other possibility is, you have to multiply by $(-1)^n$. Again, if you look at this, this and this after multiplying by $(-1)^n$ continue to be having opposite sign. So, they will cancel.

So, this gets multiplied by -1, its counterpart also gets multiplied by -1. Again they have opposite signs, they will add up to give you 0. Center sample of course is 0. Therefore, you are able to see that $H(-1)$, is it always 0 for this case? Yes. Therefore, for Type III, you have constrained a zeros both at $z = 1$ and $z = -1$.

And immediately, you can see that $(1 + z^{-1})(1 - z^{-1})$ will be a factor of $H(z)$. And the last case, of course is Type IV. So, if this were the zeroth index sample, this has to be like this; so, this is $0, 1, 2, 3$. Type IV is odd symmetry, even lengths. Now, if you sum up all these samples, will the result always be 0? Yes. And now let us look at the last possibility so, this is $+, -, +, -$.

Now, let us look at this sample, it is no $+$ after multiplying by $(-1)^n$. Its counterpart has now changed sign because it is getting multiplied by minus. Therefore, this and after multiplication by $(-1)^n$, these two will not cancel. Similarly here, so, this is minus and this is plus. When you multiply $(-1)^n$ and then add them up, this changes sign and therefore, these two will not cancel. And hence, you can conclude that $H(-1)$ is not always 0 for type IV.

(Refer Slide Time: 11:08)



So, in the z-plane, so this is I, II, III and IV. So, type I, there is no constraint zero either at $z = 1$ or $z = -1$. Whereas, for type II, there is always a zero at -1. For type III, you have zeros both at $z = 1$ and $z = -1$. And type IV, you have a zero at $z = 1$. So, no matter what the coefficients are, you are guaranteed that these zeros will be present given the nature of symmetry of these sequences.

Now, one implication of this is that, suppose you want to design, say a high pass filter, can you use type II filter is the question? And the answer is you cannot because, for a canonical high pass filter, the gain at π must be 1. Whereas, here the frequency response is forced to go to 0 at $\omega = \pi$ or $z = -1$. And hence this cannot be used as a high pass filter. Similarly types III and IV cannot be used for low pass.

Because for low pass, the canonical low pass filter, the gain at $\omega = 0$ has to be 1, but the frequency response is forced to go to 0 at $\omega = 0$ for type III and type IV. In addition, type three cannot be used for a high pass filter, because there is a zero at $z = -1$ or $\omega = \pi$. Therefore, the presence of these zeros, kind of restrict certain kind of filters from being designed.