Digital Signal Processing

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Lecture 66:
Linear Phase (4), Sampling (1)
Frequency requency response, linear phase IIR filters, sampling

Let us get started. We are looking at linear phase and group delay. And towards the end of last class, we talked about the possibility of IIR filters having exact linear phase. And let me recap that example.

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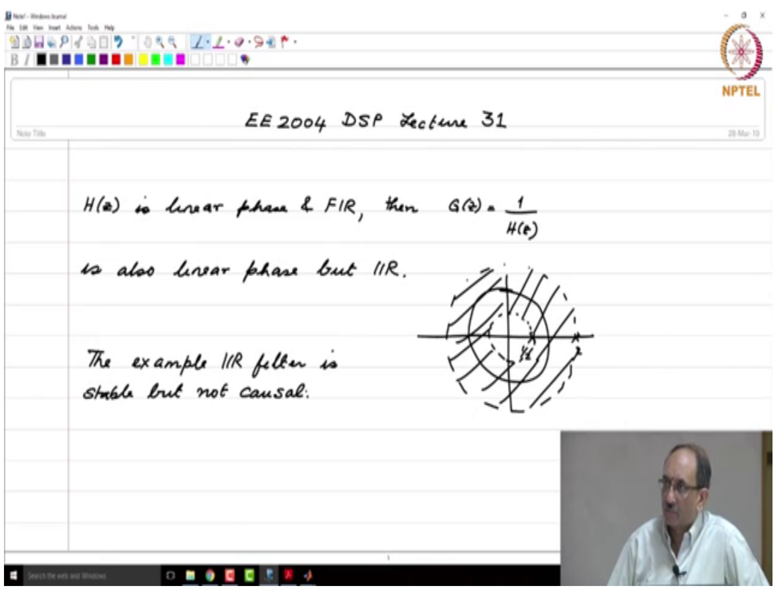

So, if $H(z)$ is linear phase and FIR, then $G(z) = 1/H(z)$ is also linear phase but it is IIR. The ideal low pass filter with a certain delay attached to it, is also IIR and linear phase. And here is another class of IIR filters that are linear phase and this has been derived from an FIR filter inverted. And we also made the general remark that, in general, FIR filters are much larger order to achieve the same specifications.

And hence IIR filters are more desirable because they have lower order. So, this begs the question if you can have IIR filters with linear phase, then why even bother about FIR linear phase filters? So, that was the question that we were looking at. And the answer is, remember, but the answer is motivated by the observation that for linear phase, if you had root inside, you will have to have a root at the reflected position.

And in the example of the FIR starting off with the FIR filter and then coming up with $G(z)$ which is $1/H(z)$, all the zeros of the FIR filter have now become poles. Therefore, you now have poles both inside and outside the unit circle at their reflected positions. So, what is the problem then? It is, it can be made stable if you have the region of convergence being the annular region between the smallest radius pole outside the unit circle and the largest radius pole inside the unit circle.

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So, in this case the filter is no longer causal. Therefore, if you had a filter with poles like this, say this is (1/2) and this is 2 and this is exactly linear phase. And then and also stable, if the region of convergence is this annular region. It is stable but not causal.

So, the example IIR filter is stable but not causal. Clements and Pease, Mark Clements and student Pease 1987, I think I have got the year right, 1987 or 1988 published a paper in which they came up with an IIR filter that is causal and has exact linear phase.

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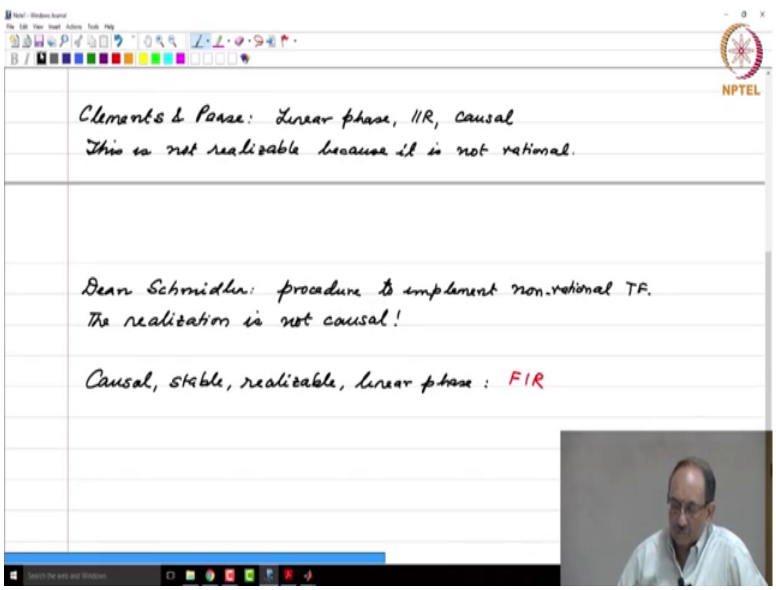

So, linear phase, IIR, causal. So, your objection that the example that is considered is not causal is now met with this paper. Causal, you want causality? Ok, I will give you causality, you want linear phase, you have linear phase, IIR? Yes, this is IIR. So, then really everybody should be using these filters and clearly we are not.

So, the question is what gives? So, this may seem like a slightly unfair questions because you have no clue what is contained in this paper. Nevertheless you can make a guess, perhaps but if I tell you what the problem is, you will say, yeah of course, yeah that seems very much plausible. The point is this is not realizable because it is not rational. But what is the use of filter that is not realizable, then came Dean Schmidlin in 1995 or 1997.

So, in that paper, Dean Schmidlin outlined the procedure to implement non rational transfer functions. And in fact, one of the examples that is in that paper is precisely the Clements and Pease filter, how to implement it? So, again this begs the question, all right. You said that was not realizable because it is not rational, but then here comes Dean Schmidlin and he has given a procedure to implement this non rational transfer function.

And as a particular example has given this Clements and peace filter as one of the things that can be implemented. Now you would think there is really no excuse for not using this and the answer is of course, no one is using this. So, what gives? The answer is the realization is not causal. Therefore, if you want causal, stabled, realizabled, linear phase filters, realize will be typically mean rational transfer function.

If you want causal, stabled, realizabled, exact linear phase, you are stuck with no choice, but FIR filters. Therefore, causal, stable, realizable, linear phase; this is the only game in term. So, as I always say this, I do not think I have said this as part of this lecture, there is no free lunch. And the price you pay for this is very high order relatively speaking. So, that is the way it is.