Digital Signal Processing

Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 75:
The Discrete Fourier Transform (1)
- DFT definition
- Inverse DFT definition
- Connection between DTFS and DFT

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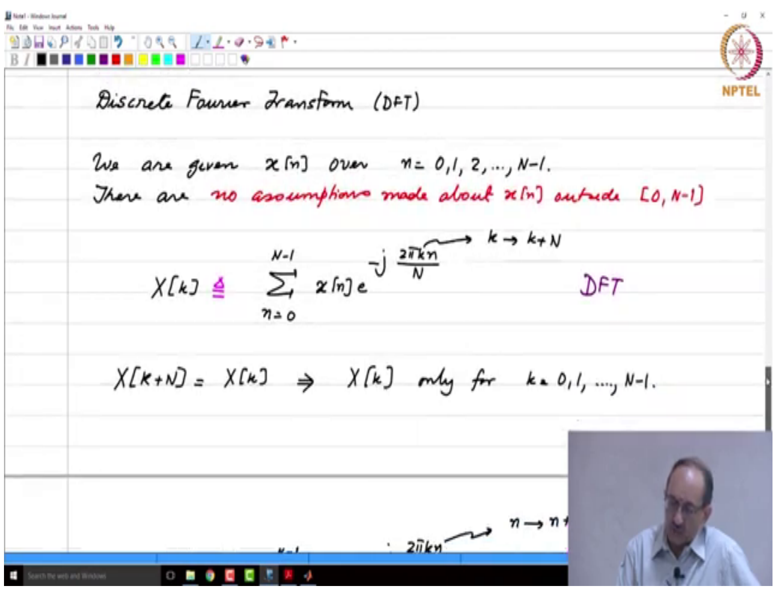

Now, we are going to look at the Discrete Fourier Transform and as I had said, this is DFT. So, this has the potential to be the fifth Fourier series representation. Let us now define this, then see how it looks like and then we will see whether it is the fifth Fourier representation that we are going to encounter. So, we are given $x[n]$ over $n = 0, 1, 2, \ldots, N-1$. So, N data points are given to you. We are further told, there are no assumptions made about $x[n]$ outside 0 to $N-1$.

Now, what do we mean by this? Suppose, I give you N data points and then I tell you there are no assumptions about $x[n]$ outside these values. If you want to make an assumption about $x[n]$ outside 0 to $N-1$, one natural assumption that will come to your mind would be?

Student: 0.

0, all right. So, it is in that context what we are saying is, no assumption is made about $x[n]$ outside 0 to $N-1$. And then somebody comes along and says, ok, you want the DFT, here is the DFT, almost god given. This is the DFT take it, do not argue with me. So, $X[k]$ is defined like this, $\sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$, this is the definition. And, now if you look at this, immediately you can realize $X[k+N]$, wherever k is there if you replace little k by $k+N$, what happens? Right.

So, here you need to replace k by $k+N$, does anything change? No. Therefore, $X[k+N] = X[k]$. So, this immediately tells you, you need to be worried about $X[k]$ only for what values of k?

Student: (Refer Time: 03:53).

0 to $N-1$, because $X[k]$ is periodic with period N. Therefore, you need to worry about $X[k]$ only for k values in the range 0 to $N-1$. So, this is one immediate fall out of this. So, now this person who was so kind to give you the definition of the DFT, was also kind enough to give you the inverse DFT.

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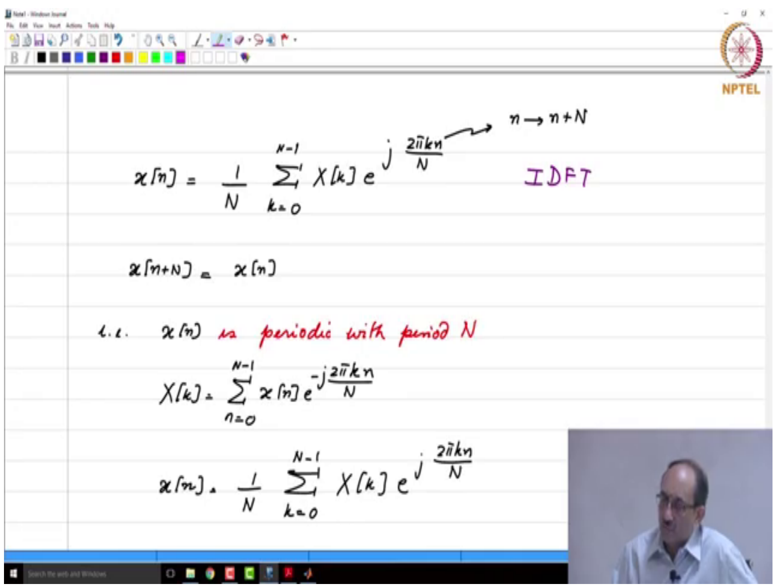

So, this person then says, ok I gave you the DFT definition, I will not leave you in the lurch, I will also give you the inverse DFT definition. And hence this definition turns out to be $x[n] = \frac{1}{N}\sum_{n=0}^{N-1} X[k]e^{j2\pi kn/N}$. So, this was the DFT and this is the definition of the inverse DFT.

Then, this person to whom this definition was given immediately observed this, what about $x[n+N]$. That is, here now you are going to replace n by $n+N$, you will get back $x[n]$. Therefore, when we started off this DFT by saying, no assumptions about $x[n]$ is made about $x[0]$ to $x[N-1]$. In the DFT framework, there is no escaping the fact that $x[n]$ is really periodic. So, that is $x[n]$ is periodic with period N.

Therefore, just to summarize, we have $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$ and $x[n] = \frac{1}{N}\sum_{n=0}^{N-1} X[k]e^{j2\pi kn/N}$. So, these are the DFT and IDFT relationship. By the way, $X[k]$ is called as the k^{th} DFT coefficient and sometimes this is also called as the k^{th} bin; k^{th} coefficient or k^{th} bin. So, k is also called as the bin index, standard term used in the literature.

So, one thing that this clear is, given a sequence which has N values so, N point sequence gives you a transform that is also N points. N point sequence gives you N point transform. Now, we can immediately see that, this DFT which was not the name to any of the four Fourier series representation that we had seen so far and when I had started that before giving what the definition was, I told you it could be the potentially the fifth representation, turns out to be nothing but the discrete time Fourier series.

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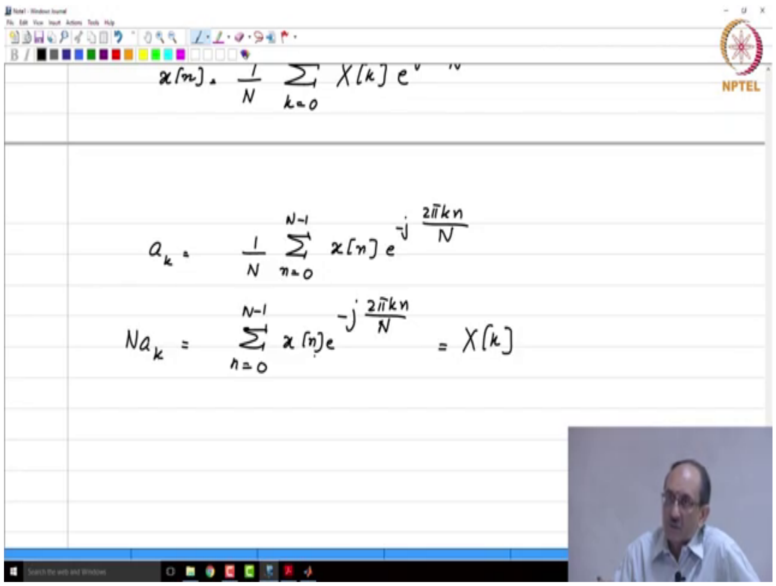

Recall, that a_k, the Fourier series coefficient was given by this, $a_k = \dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$. This was the Fourier series definition. Na_k is nothing but $\displaystyle\sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$, this was nothing but $X[k]$. Therefore, the DFT is not the fifth Fourier representation, but it is nothing but the discrete time Fourier series with a slight change in definition.

Again, everything is consistent, because in the DFT framework, even though you start off by saying no assumption is made about the data outside the observation window 0 to $N-1$, the DFT framework immediately produces periodicity. The sequence in the DFT framework is periodic, because $x[n]$ is the same as $x[n+N]$. The Fourier series coefficients are also periodic, because $X[k]$ and $X[k+N]$ is the same; they are the same. Therefore, the sequence is periodic, the transform is periodic.

In fact, the famous Cooley – Tukey paper 1965, which talks about the fast Fourier transform or FFT which we are going to discuss after we are done with the DFT and it is properties, that papers title was An algorithm for machine computes, they did not say FFT or DFT. An algorithm for machine computation of complex Fourier series was the title of that very famous paper, that also brings up this point. So, all the Fourier series the representations that we have seen so far could not be represented on a machine except this, which is the DFT; DFT is the same as DTFS with a slight modification.

So, continuous time Fourier series, the signal is, independent variable is continuous, no good for machine representation. The Fourier series coefficients are infinite number, even though they are discrete; no good. Continuous time Fourier transform, both the signal and the transform are in general infinite duration, not only that independent variable is continuous. DTFT is nothing but CTFS in disguise; again, no good for machine representation whereas, look at this.

The sequence is periodic with period N therefore, you have to deal only with N independent samples; finite number, good for machine representation. Transform, the DFT coefficients are only N in number. Therefore, N-point signal produce N-point transform, that also can be represented on a machine. The only thing that you will have to worry about is quantization, because you do not have infinite precision, but then throwing as many bits as needed you can control quantization. And for all practical purposes, you can at least in the first course, you need not worry about quantization effects.

Therefore, of all the Fourier representations we have seen so far, the only one that is amenable for machine implementation machine competition is the DFT or the DTFS; DTFS and the DFT are synonymous. So, no other representation can be implemented on a computer. Question?

Student: But there are time sequence

Yes.

Student: Why not?

So, again this is a very good question you are asking. You should have asked this question when Fourier series was taught to you, continuous time Fourier series. So, there, you are assuming a signal to be periodic; $x(t + T)$ is the same as $x(t)$. There also, it is of infinite duration. The fact that it was independent variables continuous is one part, but the infinite duration part, you still have to deal with it.

Student: (Refer Time: 13:53).

So, the no signal is of infinite duration, right. So, that objection also, the one that you raised in this context, you should have raised in that context as well.

Student: (Refer Time: 14:05).

Yes now, but your objection was to the fact that this is of infinite duration and you said then there is no signal that is, that has infinite duration, correct? So, your objection to this was from that angle, not from the storage point of view. But, again you should have asked this question when Fourier series was taught to you. No signal has infinite duration. The universe itself is supposedly only 15 billion years old, earth is 4 billion years old or something like that.

So, no signal could have begun before the universe was created as physicists tell us. So, yet we deal with signals that start at $-\infty$. So, what gives, so what is the use of those representations there?

Student: (Refer Time: 14:58).

It helps in mathematic. So, it help surely does. So, how is it used, how does it help you here? Yet you are using this for a practical application, correct?

Student: Sir (Refer Time: 15:14).

Student: Infinite is related.

All right. Infinite is relate as far as engineers are concerned, you should be saying this to the math people. So, the point that you are making is?

Student: (Refer Time: 15:29).

Very good. So, basically these things help you to kind of have a conceptual framework that in which you can bring the ideal signals and not worry about the approximation that you are making, just like an impulse is a very good approximation for a short duration pulse. So, all transients would have died down when you turn on the function generator and apply to us circuit.

Quickly, the transients would die down and then you will observe these steady state response and that circuit is best analyzed using steady state theory thinking of infinite duration sinusoids that were switched infinitely long ago. So, these are models that are used in practice to approximate actual workings.