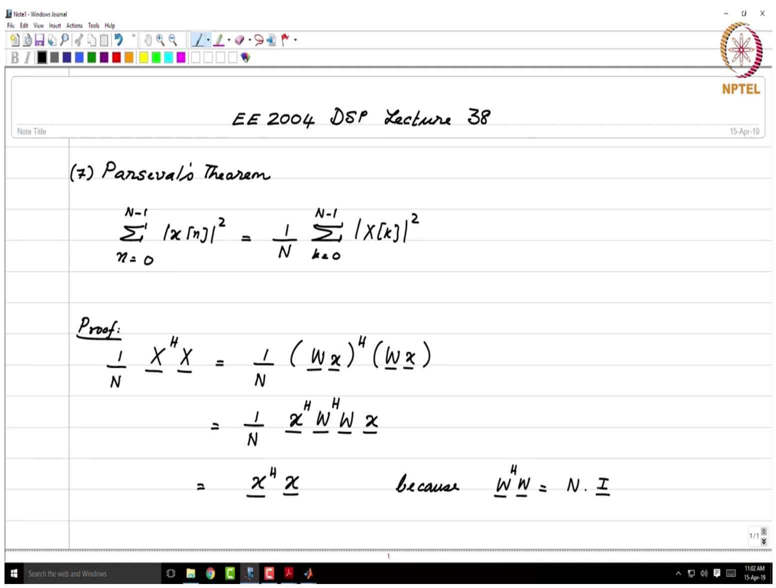
Digital Signal Processing
Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 80:
The Discrete Fourier Transform (4)
- Properties of the DFT (cont'd)

So, we are looking at the Properties of the DFT and the last property is Parseval's.

(Refer Slide Time: 00:24)



And this is quite easy to show. And the property states, the sum of the squares of the absolute value in the time domain is related to the sum of the squares of the DFT values like this and the proof is pretty simple.

If you consider $\frac{1}{N}\underline{X}^H\underline{X}$, this is after all is the inner product on the right hand side. And this is nothing but, $\underline{X}$ after all is the DFT matrix times the data vector and hence $\underline{X}^H\underline{X}$ can be written like this. And from simple linear algebra, $(\underline{W}\ \underline{x})^H$ is $\underline{x}^H\underline{W}^H$ and this turns out to be $\underline{x}^H\underline{x}$ because $\underline{W}^H\underline{W}$ is N times the identity matrix.

Remember, $\underline{W}$ is the DFT matrix and its made up of columns which have the form $e^{j2\pi kn}$ by and these vectors are orthogonal. Actually implication of this is the DFT is a norm preserving transform, because this after all is the norm of the signal or the vector in the time domain and this is the norm of the vector in the frequency domain. And, all it says is at least in this form, the norm in the time domain is a scale version of the norm in the frequency domain.

However, what you can do is, you can redefine your DFT by splitting this $1/N$ which was there in the inverse transform. We can split this factor as $1/\sqrt{N}$ in the forward transform and $1/\sqrt{N}$ in the inverse transform. If you slightly redefine the DFT by splitting this $1/N$ as $1/\sqrt{N}$ in the forward and the inverse transforms, then these norm will exactly be equal without the scale factor. So, the norm in the time domain will exactly be equal to the norm in the frequency domain.

And if you redefine your DFT matrix like that, then $\underline{W}^H\underline{W}$ will just be $\underline{I}$. And such matrices are called unitary; such matrices are called unitary. And the DFT is a unitary transform; it is norm preserving. One of the interpretations that you can make of the DFT coefficients are the vector $\underline{X}$ which is $\underline{W}\ \underline{x}$. You can think of this, in general, if you have a vector and then premultiply it by a matrix $\underline{A}$, what is the interpretation of $\underline{A}\ \underline{x}$ where $\underline{x}$ is a vector? If you have an $N \times 1$ vector $\underline{x}$ and if you pre multiplied by an $N \times N$ matrix $\underline{A}$, what is the interpretation of $\underline{A}\ \underline{x}$?

Student: Projection in all the (Refer Time: 05:42).

Not necessarily projection.

Student: (Refer Time: 05:46).

You can think of it as, I mean the easiest way to visualise this in the 2D case.

Student: (Refer Time: 05:54).

Yeah. So, what, geometrically what is the interpretation of the linear transformation? If you have a 2×1 vector and then premultiply it by 2×2 matrix.

Student: (Refer Time: 06:09).

It will be.

Student: (Refer Time: 06:13).

You will get another 2×1 vector. So, in general, what can you, how can you interpret this geometrically? It will be in general?

Student: (Refer Time: 06:28).

Very good, rotation and scaling. If the matrix were unitary, then there is no.

Student: scaling

Scaling. It is mere rotation. So, such matrices preserve the norm of the vector because you are just rotating, the norm of the vector is unchanged. So, the DFT is an example of a unitary transformation and so true is the continuous time Fourier transform. So, these are norm preserving transforms and the DTFT also. If you recall the Parseval's relationship in the DTFT case, again you can split that $1/2\pi$ scale factor which was there in the inverse transform, you can split it as $1/\sqrt{2\pi}$ in the forward and inverse transform and in which case it will be exactly norm preserving.