Computational Electromagnetics
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Review of Maxwell's Equations
Lecture - 3.3
Uniqueness Theorem

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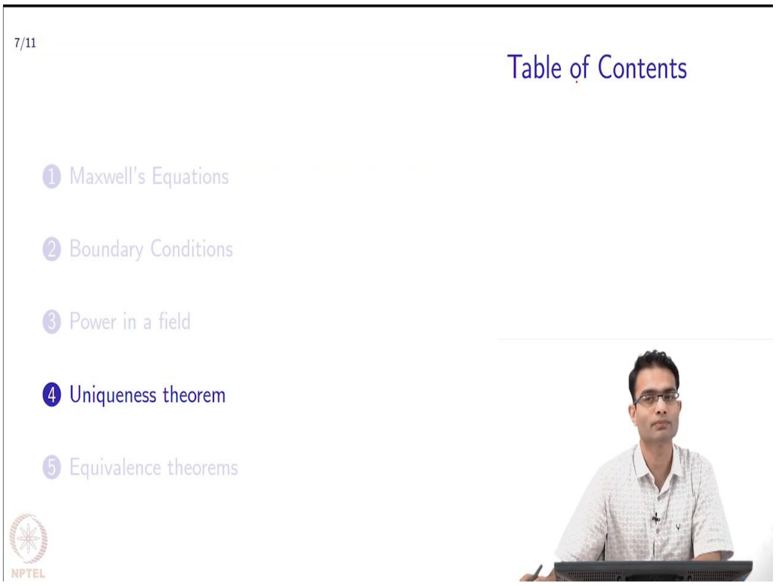

So, we will continue our discussion, now with the discussion of the Uniqueness Theorem in Electromagnetics. So, before we go in to the details of the uniqueness theorem, there is a very practical reason why we should be interested in something which is uniqueness and that is this. Supposing I take a practical situation let us say there is an antenna, in this you know mounted on this room that is radiating feels everywhere; and you was someone who was taken this course on CM is asked to calculate what are the fields. Now you do it and if the answer is not going to be unique, you know what is the point of doing the calculation right.

So, thankfully for us there is a very powerful theorem in electromagnetics which guarantees us, that under certain conditions the answer that we get after our laborious calculations will be unique. So that makes it whole worthwhile ok.

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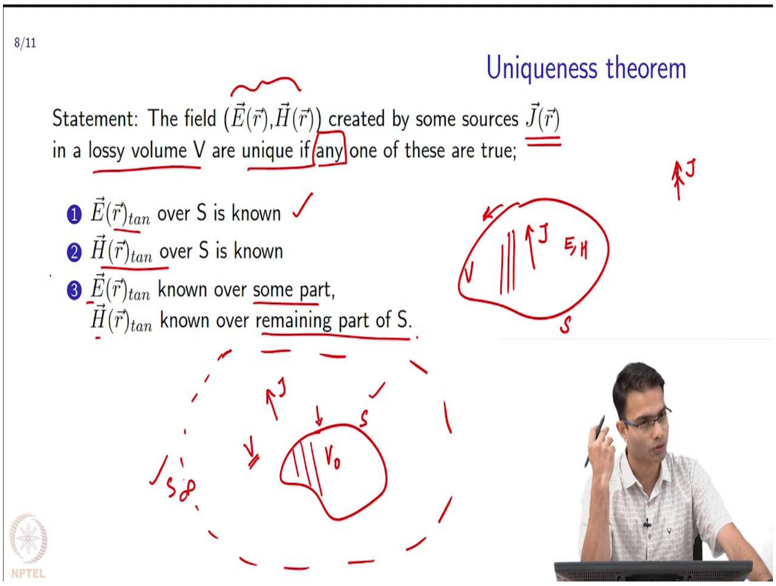

So, the statement of the theorem is as follows ok. So, it says that the field and by field I am talking about E and H created by some sources J ok. So, let us take some volume over here and let us put some current source over here J and this is going to produce a field E, H.

So, what is the theorem saying that, if I take a source J, inside a lossy volume V then the fields are unique under any of these three conditions ;it is not all its any of these three conditions. So, what are these three conditions? So, this volume has some surface S, this is saying that E tangential over this surface. Supposing I know E tangential over all of S ok; if that is specified then whatever you get as the result of a calculation the fields they are unique, there is no other solution that is correct ok.

The next condition is analogous because we have seen that there is a sort of symmetry between E and H fields. So, it says if you do not know $\vec{E}(\vec{r})_{tan}$ and if you know $\vec{H}(\vec{r})_{tan}$ over all of S, then the field is unique and then there is of course, the third option is a sort of a linear combination that you give me $\vec{E}(\vec{r})_{tan}$ over some part and $\vec{H}(\vec{r})_{tan}$ over the remaining part.

So, under these conditions any of these conditions the field that you will get is unique. So, this also tells you that if you want to approach a CEM problem, you need to specify the tangential E fields on the boundary. Once you do that you are guaranteed that whatever procedure you use for solving these equations, we will give you a unique solution.

Now, you might ask supposing I put an antenna like this over here, and it is in free space absolutely nothing anywhere. So, in this case will the fields be unique there is no boundary anywhere it is in middle of interstellar space; will the fields that you get from this antenna supposing I have specified the current over here will they be unique? Answer is yes because.

Student: Boundary has a.

The boundary has is at infinity and because this is physical field it cannot go on forever. So, trivially the $\vec{E}(\vec{r})_{tan}$ and $\vec{H}(\vec{r})_{tan}$ is 0 right; so, in this case I have specified $\vec{E}(\vec{r})_{tan}$ and $\vec{H}(\vec{r})_{tan}$ implicitly without even thinking about it at infinity to be 0. So, in this case the fields are going to be unique. So, elaborating a little bit more on this example, this was an example of a closed volume. You may come across another different kind of volume, which is like this; let us say this is let us say volume V_0 and there is a current source over here J and this is volume V ok. And there is no boundary over here this boundaries infinity. And I am talking about region V.

So, now, what do I have to specify for the fields to be unique on which surface. So, this is S infinity and this is S. So, this theorem holds to here also, whatever surfaces are bounding the volume. So, on for this volume V what are the surfaces? S infinity is one surface and S is the other surface, S infinity being infinitely far away fields are physically they will go to 0.

So, I know the tangential fields at infinity I need to tell you what the fields are the tangential fields are on this boundary and then, once these are specified I know that I can proceed with my calculation because my answer will come out to be unique. So, very simple example of this is if for example, this volume V_0 is a solid perfect metal.

So, perfect metal will have $\vec{E}(\vec{r})_{tan}$ field is 0 right. So, once I have such a situation I know there is a fields in this volume we will be unique ok, as just a simple example alright; so is this uniqueness theorem clear. Now you might ask why this lossy volume V right, why should the volume V lossy, what if the; what if there was no loss in this medium.

So, the answer to that is not clear just from the statement of the theorem, we will have a small exercise in the homework problem where you can work it out. When you take Maxwell's equations and solve them over some volume, you will find that you need to specify some tiny amount of loss for this theorem to hold to, practically this loss can be really really small. So, that you can effectively treat it like a lossless volume ok; even though we say free space is lossless right even air we say its lossless, but actually there might be some really tiny amount of loss in it which allows this theorem to be applicable question.

Student: So, can we say when the field is not unique and set a example.

So, if these conditions are not satisfied then we cannot make any statement about the uniqueness of the theorem of the fields that is all there is ok. So, you may be able to construct examples where you have not specified $\vec{E}(\vec{r})_{tan}$ or $\vec{H}(\vec{r})_{tan}$ over the entire boundary, in that case you may get many solutions which satisfy the partial information which you given that is possible yeah.

Student: Does the send the theorem talk about a charge density.

Charge density so we are in this particular case we are dealing with a case where we are talking only about currents ok. So, we are talking about electrodynamics not electrostatics cases, but a the theorem could be extended also in that case.

Student: You know static charge present in the.

Yeah in this we will not deal with static. In fact, Griffiths has a very nice discussion about it a static charge is not going to sit there it will fly off somewhere over the other, unless it is in the place where there is no fields whatsoever sitting there forever.

So, we practically speaking when you think of antennas radar cross section of antennas you know the issue of charge is sitting out there is not very relevant ok. All of those charges once they start moving they become currents and we already dealing with currents. So, this was about the uniqueness theorem.