**Computational Electromagnetics**
**Prof. Uday Khankhoje**
**Department of Electrical Engineering**
**Indian Institute of Technology, Madras**

**Applications of Computational Electromagnetics**
**Lecture – 14.08**
**Antennas – Hertz Dipole – Part 2**

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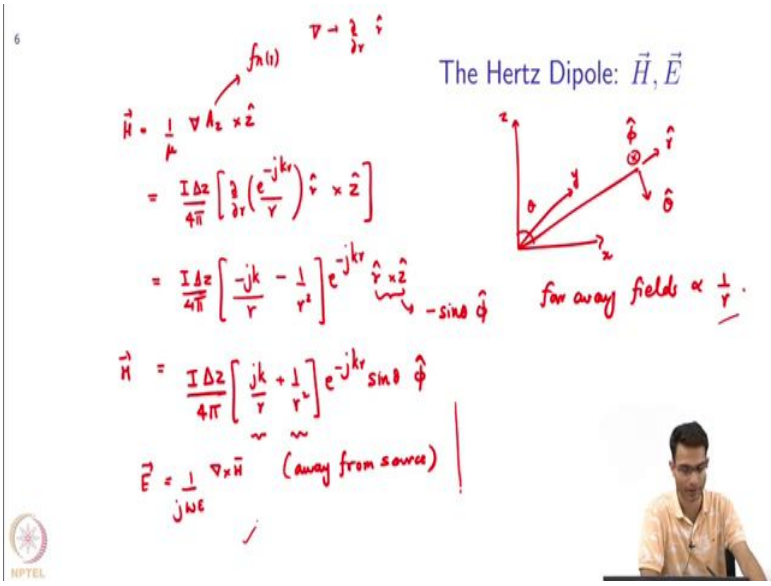


Right. So, we will continue our discussion of this Hertz Dipole. The only sort of a thing to remember about this hertz dipole it was a very very small current element and because it is very small, you can assume current is approximately constant thing right.

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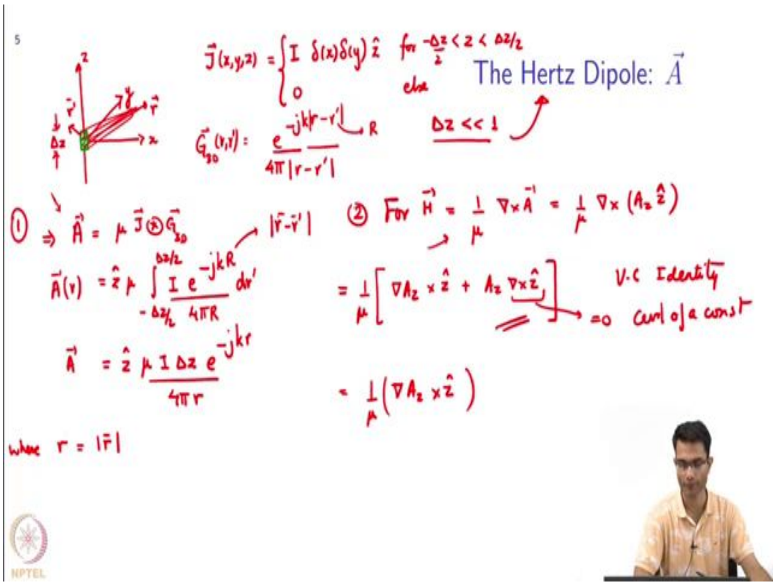


So, what are the steps that we did? Given the given the current we could calculate the $\vec{A}$ vector using this relation over here, convolution of unknown current with Green's function I got $\vec{A}$ I got $\vec{H}$ I calculated little bit cleverly making use of both Cartesian and polar coordinates right you are something like $\hat{r} \times \hat{z}$.

So, I have mixed up these coordinate systems and I have got a very nice expression over here. And we noticed that far away the fields go as $1/r$ ok. Then we said that the electric field is simple to obtain once I get the magnetic field just by taking the curl right. So, if I now go to take the curl of this expression, you can sort of see what coordinate system will I use? Spherical right because it has r theta phi everything is there inside it.

So, spherical is a best thing do you think it will be a very beautiful expression right it's going to have everything in it right. So, I am not going to write down that curl operator and derive it, but this is the whole expression over here.

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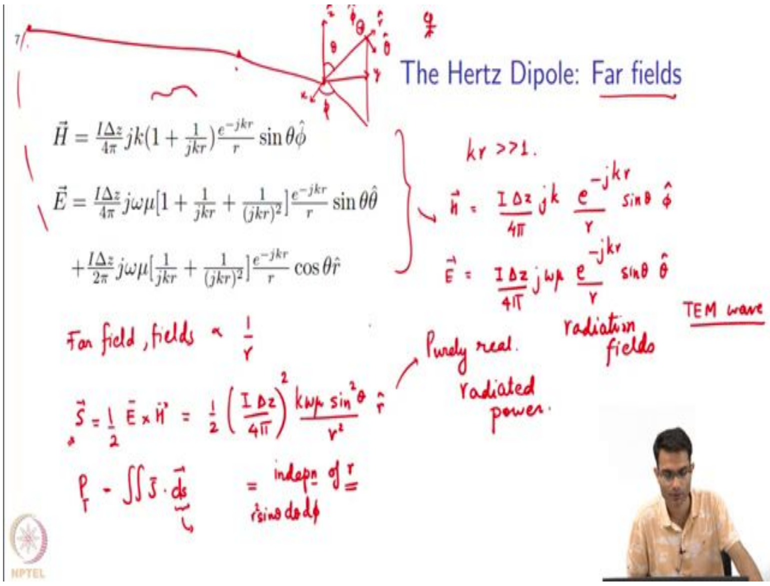


So, this is the magnetic field which we already saw first expression and the second is obtained a sort of when I take the curl of this I get two components. So, there is a this was in the phi hat direction then I have a $\hat{\theta}$ and an $\hat{r}$ ok. So, just to remind you that this is my $\hat{r}$ direction right. Which way is? This is my $\hat{\theta}$ and this is my $\hat{\phi}$. So, $\hat{\theta}$ is pointing downwards over here and $\hat{\phi}$ is which way?

Student: (Refer Time: 02:15).

Into the board right. So, this is my phi hat that is what this coordinate system looks like that.

Student: (Refer Time: 02:26) we want to ask that (Refer Time: 02:27).

$\hat{r} \cdot \hat{z}$ was written.

Student: Dot.

Sorry cross $\hat{r} \times \hat{z}$ was written as.

Student: Sin theta.

$\sin\theta\, \hat{\phi}$.

Student: (Refer Time: 02:42).

Right direction between the I mean the perpendicular direction is $\phi$ and the angle between them is $\theta$. So, $\sin\theta$ right the unit vector some $|r|\ |z|$ is one fine. So, this is the especially you get you notice that the expression for the electric field is fairly scary looking, but this is the exact expression that is there are apart from the fact that the length of the dipole is small there is no other approximation here right.

So, looking at this the first thing to do is let us take a simple simplified version of this, when we say that we are going to look at what are called far fields ok. In this problem very naively I am going to say far field is when $kr >> 1$ ok. There are more refined definition of this we will come to them later. Under this approximation, what happens to the magnetic field? I have two terms right in this bracket.

Student: (Refer Time: 03:44).

I will just keep the first term because $kr >> 1$ right. So, in net case the magnetic field magnitude becomes

$$\vec{H} = I\Delta z/(4\pi)\, jk\, e^{-jkr}/r\ \sin\theta\ \hat{\phi}$$

What happens to the electric field? From how many terms are there? 5 terms right. So, I get to keep I should keep that most leading term will be just one of them which is the one term right. So, this will become

$$\vec{E} = I\Delta z/(4\pi)\, j\omega\mu\, e^{-jkr}/r\ \sin\theta\ \hat{\theta}$$

So, that is why we say that in the far field, the fields are going $1/r$ right in amplitude at least has a function of r is going is $1/r$ ok. So, we have already made one comment about how this is different from your Coulomb's law right ok. Anything else that you can notice from here? Besides, there is a lot of interesting physics happening over here. So, what is its saying? And when I score stands far away from the source what do the fields look like?

So, which way will the Poynting vector be? So, Poynting vector

$$\vec{S} = 1/2\,\vec{E} \times \vec{H}$$

So, which way is my Poynting vector? So, my Poynting vector is along $\hat{r}$ and my electric field is along $\hat{\theta}$ my magnetic field is along $\hat{\phi}$. What kind of a configuration is this what does it remind you off?

Student: TEM.

TEM its a Transverse Electromagnetic wave because the electric field and magnetic field are perpendicular to each other and they are perpendicular to the direction of power transfer. Isn't that also the case in a plane wave?

Student: Yes.

Yes right. So, this is your TEM wave. No component of electric or magnetic field in the direction of propagation. So, this is the simplest kind of wave that we have studied right. In fact, it looks identical to the 1D plane waves you have studied, the only difference is that this a $1/r$ in the denominator and so, we call this as a 3D plane wave right.

So, what do you expect will happen to this Poynting vector? You already noted that it's going to be in a direction in the $\hat{r}$ direction I mean in the $\hat{r}$ direction what will be the magnitude? So, this is going to be a half of this whole thing is going to be there $I\Delta z/(4\pi)$, $-j \times j$ is going to give me a $+1$. So, I am going to have a $k\omega\mu$ right. So, $k\omega\mu$ what else?

Student: (Refer Time: 07:25).

Pardon me.

Student: (Refer Time: 07:26).

$\vec{H}^*$ that is why $j$ becomes $-j$.

Student: (Refer Time: 07:31).

Whole square correct and $\sin^2\theta/r^2$ in what direction? $\hat{r}$ direction right I think I have got all these terms right now. So, I have got a power that is going as $1/r^2$ ok. So, you should keep this in mind this have derived for hertz dipole, but this is true for any antenna structure in general far away from the antenna. Even if you take the base station I mean in the mobile base station and you go far away from you will find the fields coming from it are of this form.

So, if I integrate if I want to find out how much is the total power leaving this right. So, what would I do? I would integrate supposing I wanted the total power I would integrate this Poynting vector over what? A sphere of radius some $r$ right. So, if I did for example, ds over here. So, you can do this calculation right this is not a very difficult calculation find the surface element on a sphere and integrate this is going to be what is your $ds$ over here?

Student: (Refer Time: 08:49).

I mean a little bit more generally.

Student: $r\, dr$.

$r\, dr\, r \sin\theta\, d\theta\, d\phi$. So, when you integrate this for constant I mean over you will get $4\pi r^2$ which is a surface area right.

Student: (Refer Time: 09:14).

Sorry there is no dr. So, it's going to be $r^2 \sin\theta\, d\theta\, d\phi$. So, when I integrate this over the surface what will happen to the $r$ square term?

Student: Cancels out.

Cancels out. So, the answer I mean we can do this integration, but it's not very interesting right the answer is independent of $r$. So, what does that tell you?

Student: (Refer Time: 09:44).

No matter where what I mean this is also mixed physical sense no matter where I integrate from, the power leaving this sphere should not change right. So, that is what I get this is just confirming our intuition. Because a sphere encompasses the entire $4\pi$ studied in, so, the power if I calculate because it encloses the dipole. So, I whether I integrate here or here the power that is leaving the surface going to be the same. Power density on this wave will be different that is captured by S.

And you can see as you get closer at smaller r power density increases because its $1/r^2$ right. The other thing to note over here is this S expressional over here its purely real. Even though there was a good chance that something could have something could become imaginary over here because they are all is $j$ and so, all in the field expression this is purely real right. So, this is its what circuit element has purely real power dissipated in a?

Student: Resistor.

Resistor right. So, this is like if you want to make a circuit equivalent of its like power that is going through a resistor once its goes its goes. no need to recover. So, this is also called . So, these are called radiation fields and this is called radiated power.

Student: (Refer Time: 11:08).

No, I think these are very simple sort of confusion over here. This is the power Poynting vector. This is the power density right. So, this is total power I am calling it total power integrated over sphere if I go very, very far away the power is still there it's going as $1/r^2$ right even though it becomes very very small, I am also integrating over a large area if you want being over that way right. So, it's going to be there.

Student: One point (Refer Time: 00:40).

Pardon me.

Student: (Refer Time: 11:42).

Yeah.

Student: (Refer Time: 11:45).

So, if I this is my antenna over here there is some power over here, if I go further over here right. So, the power is this I mean let us say a Poynting vector the Poynting vector is going as $1/r^2$ right. So, it's going to drop. So, its low over here it's even low over here the power is dropping that is ok, but now if I integrate over this whole thing over here I get the same number that is what you expect right you go far away from this source the field intensity drops. So, this is your near field consideration now not surprisingly that the other thing that we should look at is.

Student: (Refer Time: 12:24).

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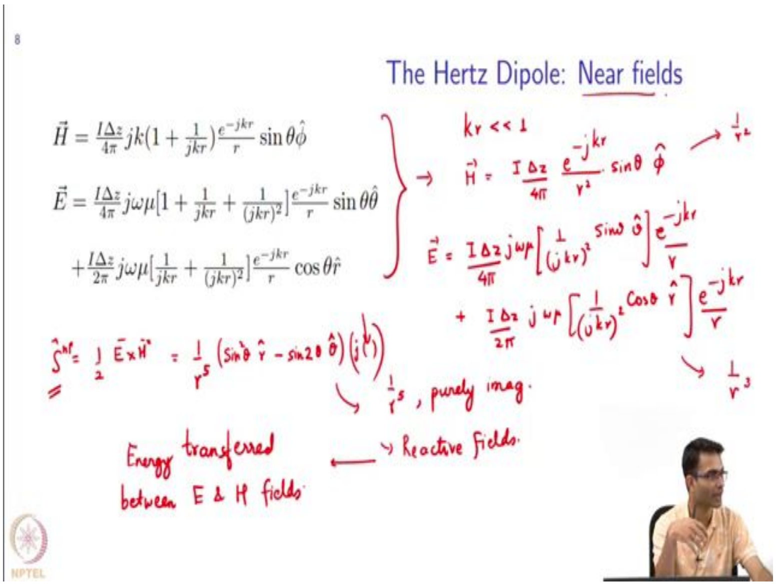


Also this is far field the other thing we should look at is what happens in the near field right. So, if the near field what we can say is that, $kr << 1$. So, I am looking at regions very close to the antenna right. So, what happens to these expressions? So, $\vec{H}$ for an example which term will I keep?

Student: Second term.

The second term right. So, $1/kr >> 1$ right. So, I will get $I\Delta z/4\pi$, j and j cancels off. So, I get a $k/r$ sorry there is no k it will become the $j$ and $k$ cancels off. So, I get a $e^{-jkr}/r^2 \sin\theta\,\hat{\phi}$. So, magnetic field anyway is in the same direction. Electric field let us see what happens electric field which term should I keep?

Student: Only the.

Only the $1/(jkr)^2$ right. So, I am going to get $I\Delta z/4\pi j\omega\mu$ when I have $1/(jkr)^2 \sin\theta\,\hat{\theta}$ and then I have a $e^{-jkr}/r$.

Student: (Refer Time: 14:05) ok.

And one more term is there this is $\mu$. So, I have a $j\omega\mu$, then I have a $1/(jkr)^2$ and I have a $\cos\theta$ along $\hat{r}$ direction ok. So, in contrast to the far field case the magnetic fields are going as $1/r^2$ the electric fields are going as $1/r^3$ right $1/r^3$ reminds us of what from electrostatics?

Student: Dipole.

The electric field due to dipole right. So, this is acting like it seems like a dipole right. So, now, if I calculate the Poynting vector in the near field same thing I will do $1/2\,\vec{E}\times\vec{H}^*$. What do you expect?

Student: (Refer Time: 15:04).

For a lot of terms right, but I mean what are the essential features over here I should get a $1/r^5$. What directions will I expect? So, there is a $\hat{\theta}\times\hat{\phi}$ is going to give me a? $\hat{r}$ right. So, I am going to have $\sin^2\theta\,\hat{r}$ and then I have a $\hat{r}\times\hat{\phi}$ which is going to give me something along $\hat{\theta}$, $\sin 2\theta$ it was $\cos\theta\times\sin\theta$ along $\hat{\theta}$ and a bunch of constants which are not so important. Well what is important about these constants are let me just write it over here you said this as a j into something ok. You can see that because $\vec{E}\times\vec{H}^*$ both the terms of $E$ they have a $j$, $H$ has no $j$ right. So, the one j term is going to remain over here right. So, what are the key features is $1/r^5$ then it is purely imaginary right.

So, this is different from your previous case. So, these are called your reactive fields, now if I ask you if I integrate over some sphere over here what should you get, will it be $r$ dependent or not? So, it should be 0 why should be 0 that will not be 0.

Student: (Refer Time: 17:12).

So, one answer is that it will be the integral will be r dependent it should not be $r$ dependent what is the catch?

Student: (Refer Time: 17:18).

No I am not integrating over the source the source is there and some small distance away from a time integrating.

Student: (Refer Time: 17:26) we have this.

No. So, actually the thing is that when we wrote down these electric and magnetic fields we made the approximation $kr << 1$. Now if I want to find out the total power leaving the sphere I should include all the terms right, if I do that I will find out that the power is independent of $r$ and that makes the physical sense the I want power leaving at right should we will not change. But here I am taking only out of these 5 terms I am only keeping 2 terms.

So, there will be some sort of mismatch over here that is ok, but we are not I mean the main point is we are not going to integrate this term over the whole sphere this is just telling us what is the dominant part of the Poynting vector what is the main contribution to Poynting vector in the near field. So, then this fact that it is purely imaginarily and reactive reminds us of which circuit element?

Student: Inductor.

Inductor or capacitor right. So, what happens in the near field of any antenna is that the energy keeps shuffling between the electric field to magnetic field like in LC circuit, energy goes from an inductor to capacitor and back and forth same thing happens over here. So, this I mean back and forth keeps happening and there are you heard of these wireless power transfer applications right those work best when you do it in the near field because you are able to access this energy by the time you come to the far field its becomes purely real. So, it's only a resistor drop, but here this energy can be exploited. So, for example, these you know toothbrush chargers and all, you would block it over there. So, it is the stuff is happening in the near field we are able to transform more power.

Student: (Refer Time: 19:13).

There is one more.

Student: (Refer Time: 19:15).

Negative signs may be there I mean this is once you have consensus over here.

Student: (Refer Time: 19:19).

$\hat{\theta}$.

Student: (Refer Time: 19:23) r square (Refer Time: 19:24).

Its $\hat{r} \times \hat{\phi}$. So, $\hat{r} \times \hat{\phi}$ that is $-\hat{\theta}$ right.

Student: (Refer Time: 19:41).

Where do you see a $j^2$?

Student: (Refer Time: 19:50).

Well that the expression is correct because in the other case what term do you get $\hat{\phi} \times$?

Student: (Refer Time: 20:01).

No again what is $\hat{\theta} \times \hat{\phi}$? $\hat{r}$. But also minus term there know.

Student: Full minus sign.

That is the full. So, all those consensus are inside this bracket we do not care about it relative to these two there is a minus sign ok. So, reactive fields and energy is transferred between the E and H fields.

Student: (Refer Time: 20:37) power is another side.

Yeah.

Student: (Refer Time: 20:40).

The total power what we conserve.

Student: (Refer Time: 20:44).

The Poynting actual Poynting vector will be conserved regardless of what you choose to integrate ok. So, in a course on the antenna theory we would spend a lot more time deriving this and understanding all the features, we want to sort of get to the CEM Computational ElectroMagnetics part of it. So, this is just by means of giving some introduction to it.