Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Review of Vector Calculus
Lecture – 1.2
Gradient, Divergence, and Curl operators

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So, next what we will do is, we will dive in deeper and look at how to generalize the idea of the gradient and some other commonly used operators.

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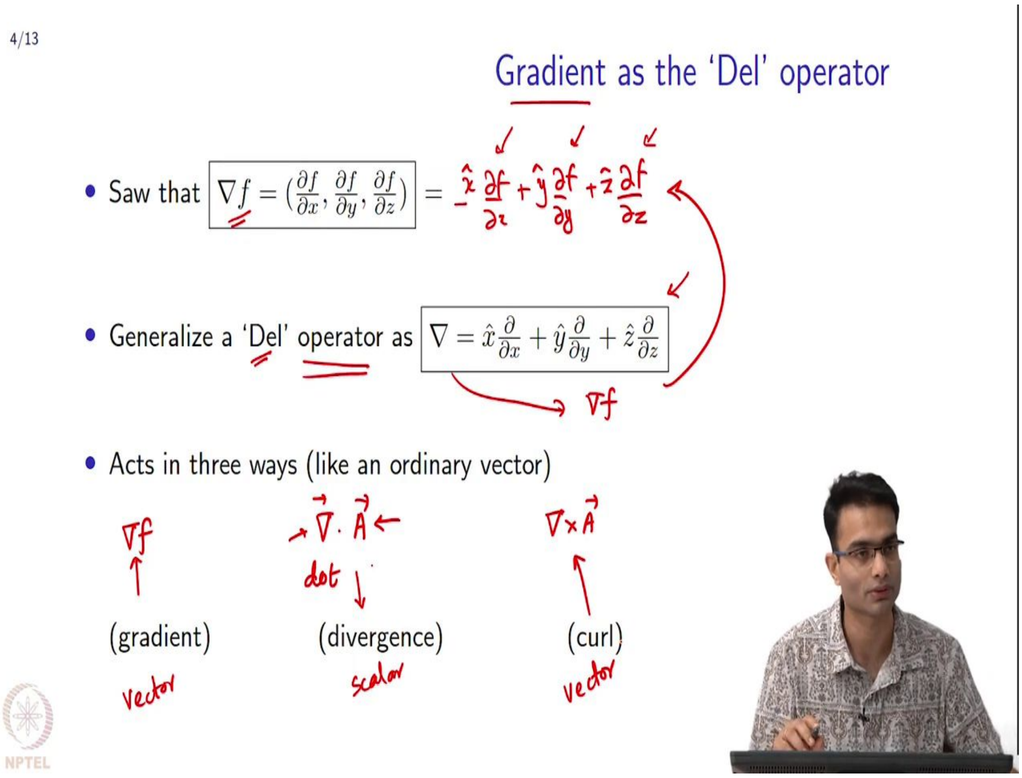

So, now this gradient that we have defined we do not have to always make it look like this right. So, we have written ∇f in terms of the three components and I have already written this, I will write it again this is how I will write it. So, note I just did a small change I wrote the unit vectors on the left hand side for convenience.

Now, you can see that there is f is appearing again and again over here in this expression. So, one act of abstraction is let us get rid of f and define an operator right. So, I can call this gradient thing, I can call it the ∇ operator which is like this. So, now, this operator is in need of some function to act on always. So, if I write this over here it is incomplete in the sense that it does not tell me what it is doing on anything, it needs a force to act or a field to act on and that field in this case is a scalar field.

So, when I plug in over here grad this expression if I take over here and put f I get back this expression. Now this operator over here is very much like a vector. So, it obeys the usual laws of vector calculus. So, for example, of course, we know that the gradient is that so, I can take the dot product between two vectors. So, first is let say gradient. So, scalar function this is called the gradient. Divergence so, this is take the dot product between two vectors so, this is like the dot product right. So, this is a vector I can take the dot product, but when one of these guys is the del operator this act is called the divergence of the vector. So, this was dot.

The other thing is cross product right. So, we know that I can take two vectors and define its cross product. So, similarly I can take these two vectors and take a cross product. So, the cross product we know is in the language of vector calculus it is called the curl of the vector "a". So, we started with the gradient, we defined a del operator, the del operator is something that stands by itself you can see it has three components. So, it is a vector given a vector I can take its dot products it is called the divergence, I can take the cross product it is called the curl or I can make it act on a scalar function.

So, remember so, this quantity over here is a vector right, this quantity over here will be a scalar right and this quantity over here is going to be a vector. So, these are the sort of nuts and bolts in which the whole language of Maxwell's equations will be written so, this is to give us some confidence on these operators.

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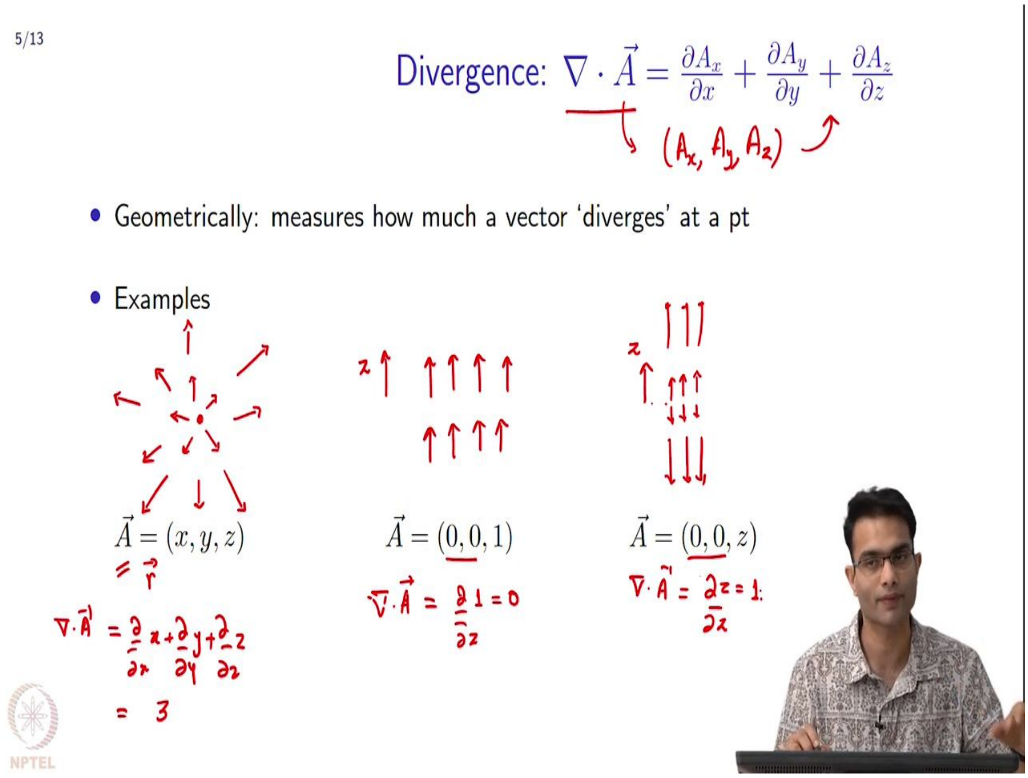

So, let us look at these operators now one by one ok. So, the first operator is the divergence. So, divergence very simply I have taken a vector A and it has three components so I am going to write it as A_x, A_y, A_z when I take the dot product, this is what I get as we said before this is a scalar ok. Now there are some very beautiful geometric meanings of what these quantities are. So, the divergence it sort of measures how much a vector diverges from a given point ok. So, this is the intuitive geometric pictures.

So, let us take a few examples. So, let say that I have some point over here, let us call this the origin and I am trying to plot a vector field like this. So, what is this vector field? This is nothing but the position vector in spherical coordinates x y z those are the component. So, if

I plot this on the origin it will always be pointing in which direction? Radially outwards right, and as I go away and away from the origin the magnitude will increase.

So, they will be represented like this, start with as I go further outside I can draw longer length, the length of the vector denotes the magnitude and it is always radially outwards. So, everywhere in at any point in space it is pointing outwards over here. Now you can see that if I calculate at this point over here, the vector field is sort of diverging away from one point. So, we intuitively expect that the divergence of this vector will be non-zero.

You can see if I take $\nabla \cdot A$ over here what am I going to get; I am going to get $\partial/\partial x + \partial/\partial y + \partial z$ and as going to be equal to 3 right so, it is a non-zero quantity. The next thing the next example that I have is a vector field that is constant in space. So, it is constant along the z axis so, if this is let say the z axis, then it has constant length everywhere, no matter which point in space I plot it this vector field is like this right.

So, it is not really diverging out from anywhere, it is sort of all flowing like a river flowing in a constant speed everywhere it is going a same way. So, when I take the divergence of this vector field over here, what do you expect, only first two turns are 0 I am going to be left with this and which is 0. The next example that I have is like the river, but it is catching speed as it goes along the z direction.

So, if this is the z axis over here then and let say this is z equal to 0. So, which way is it going to go? So, at first small z its value is going to be small let say and then the magnitude picks up over here, and when I go to negative z, it is going to change direction again the magnitude is small and so on. So, again this vector field has an intuitive idea that it is diverging.

So, when I calculate the divergence over here again these two guys are going to contribute nothing and I am left with it has unit divergence. So, this gives us a very nice geometric intuition of what is going on with a vector field. We are trying to get ways of quantifying measuring what a vector field looks like what it does. So, divergence is the first tool over here.

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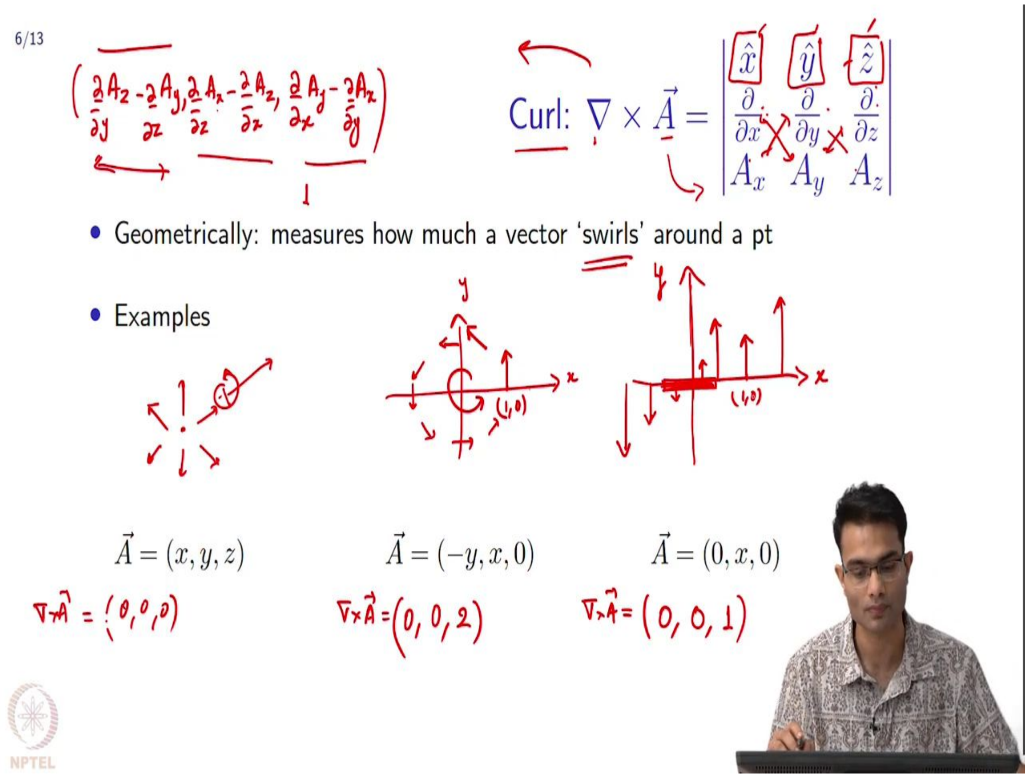

The next quantity of interest it is the curl right. So, curl is the generalization of the cross product and just like how we take the cross product between two vectors we write down x, y, z when we write down the first guys components which are here, here and here and the components of a going to the bottom row.

So, we can write we all know the rules of cross product. So, if I were to write this in the component form so, let us write in bracket form. The first component will be $\partial A_z/\partial y - \partial A_y/\partial z$. The next component will be along the $\hat{y}$ direction so, that is going to be $\partial A_x/\partial z - \partial A_z/\partial x$ and the third quantity will be along $\hat{z}$ so, that quantity is going to be $\partial A_y/\partial x - \partial A_x/\partial y$ ok. So, this is a cyclic way of doing it when I am looking at this guy then I take this difference when I take this guy I, this is the first guy and this and then minus this. Similarly, when I take this guy it is going to be this difference, those of you who rusty on this just you can do a few practice examples yourself it is not very hard.

Now unlike the divergence operator the curl operator measures how much a vector swirls or how much it twists about a point. So, again keeping with the river analogy, if I have a whirlpool of water that is a place where it is swirling. So, the curl is something that measures how much a vector field is swirling. So, if I take a few examples over here. So, the first example is the same vector field that I had in the previous example right. So, from the origin going radially outwards; so, does this look like a vector field that is swirling in anyway? Intuitively no does not seem like it is swirling anyway. So, let us confirm our intuition, let us calculate the curl of a over here. So, the first component we can see is this guy over here.

So, what is A z is z so, $\partial A_z/\partial y$ is z which is 0, A y is y this is 0, similarly A x is x, $\partial A_x/\partial z$ is 0 so, I am going to get a big fat 0. So, it is the 0 vector and that make sense because this vector field over here, it is not swirling, it is just; it is diverging out, but it is not swirling, it is not rotating. One other way of visualizing it is if I put a small paddle type will it start rotating right, if I put a small paddle here it will just get pushed outwards, it will not rotate about its own axis like this.

Now here is another example this is in the x y plane so, let us draw the x y plane over here. So, what is this vector field look like? So, let us take for example, the x axis every point along the x axis over here, the quantity is how should I draw this field over here? So, would it be correct to draw something like this. So, for example, let us take one point over here ok so, let us call this 1 0 right. So, this is 1 0 what are the components of this vector over here. So, it is going to be a will be (0,1,0) right.

So, that is a vector of unit length pointing in which direction? y hat right. So, this is going to be like this. Now let us takes one point over here. So, let us call this 1, 1 right. So, x equal to 1, y is equal to 1. So, this is now which way? So, the x component is - 1, 1, 0 right. So, that is a vector which is pointing in which way. So, the x component is negative, the y component is positive right. So, it is going to be pointing like this 45 degrees right.

So, you can keep drawing this you are going to get a something like this; this is a textbook example of a swirl right. So, again we can let us calculate the divergence sorry the curl of this vector field over here. So, let us take the first component over here which is here. So, A_z is 0 right so this term is 0, A_y is x now dx/dz is 0. So, the first component is 0, what about now let us go to the second component over here A_x is.

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Minus; A_x is -y, but the derivative with respect to z is 0, A_z is 0 so, this term is also 0. So, look is our intuition in trouble let us find out. So, let us take the third term over here A_y is x right, now dx/dx that is 1 minus and the other term is A_x is -y right.

Student: Minus 1 y.

Minus 1right. So, minus minus become.

Student: Plus.

Right; so, this becomes 2 . So, this is non-zero right and this is the familiar your right hand curl thumb rule sort of you can twist it along this and the curl is showing you where the vector is non-zero. So, this is the twisting field we can take another example over here, again in the x y plane for simplicity. Now how does this vector field look like. So, let us take some point over here 1, 0 where is this pointing, upwards over here again right over here. What about the entire x axis sorry the entire y axis? x=0 over here. So, if the function is 0 all along over here and as I go to higher and higher values this is going to get larger and larger. When I come to very small over here, what about this point over here, which way will it point if it going to point downwards right because x is negative, but it is always going to be pointing in the plus y direction right. So, the direction is fixed. So, this is third example.

So, let us again calculate. So, what is your intuitive idea, does this swirl or does it not swirl? It does not swirl. So, let see what the answer is. So, first component A_z is as we are going to come over here, A_z is 0, A_y is x so, this term is 0. Like I take the second term A_x is 0 so, this is 0, A_z is 0. Again I have a 0, third term is interesting term. So, A_y is x. So, $dA_y/$dx= 1 and A_x is 0 right. So, this is actually 1 right.

So, you can see that there is a swirl happening over here right. So, these are pointing in this direction these are pointing in this direction, supposing I put a paddle over here like this, you can see that these forces will make this paddle rotate over here. So, our intuition is correct that this vector field has a swirl about it. So, the curl is non-zero.

So, you know these are traditionally terms like the curl and the divergence are terms which in high school we were very afraid of, but you can see that they have very simple geometrical meanings, we get comfortable with it we can write our equation of electromagnetics in it very easily.