Computational Electromagnetics
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Introduction to greens Function
Lecture – 7.7
2-D example: Evaluating Constants - Part 2

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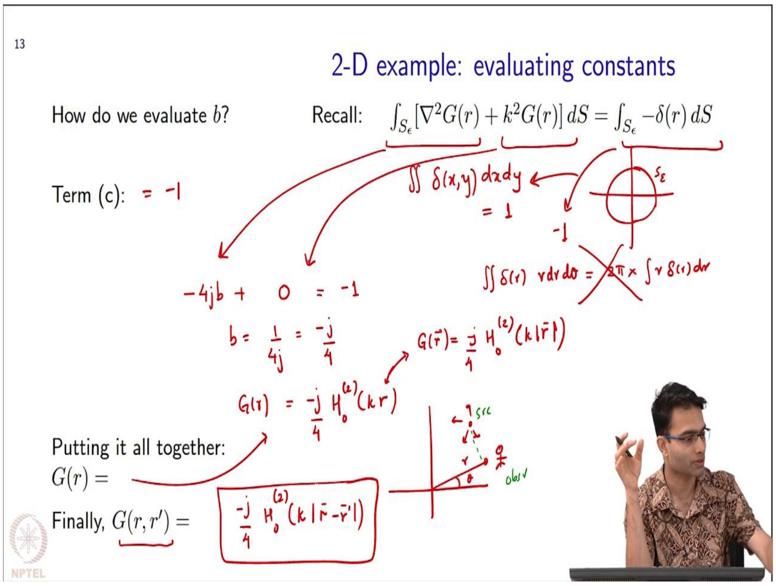

So, the final term that remains is this term on the right hand side ok. So, integrating the delta function and our integral is including the origin over here that is my S_ε right. So, what do we expect?

Student: -1.

-1 right; so this term should just give me a -1. So, here there is one small thing that you have to be careful of, if you the way I have written this looks like a 1-D delta function right. So, if you did you could make a mistake here, if you did you know the following, for example if you wrote this dS as $rdrd\theta$ and you wrote this as $\delta(r)$; you might write this as $2\pi\int r\delta(r)dr = 2\pi$ right. So, do not do it like that right. So, the correct way it I mean the so this is wrong. So, the correct way to do it would be just think of this as a two-dimensional delta function right. So, as $\iint \delta(x,y)dxdy = 1$ right.

So, 2-D delta functions so its integral is just going to be equal to 1. So, that term c is equal to -1. So, finally, what all did we have left over here, the first term gave us.

Student: -4.

- 4jb right, so that gave us -4jb; the second term gave us.

Student: 0.

0 right and finally, over here I have - 1 right. So, $b = -j/4$. And if I put it altogether finally, I can write my Green's function as equal to $(-j/4)H_0^{(2)}(kr)$ right.

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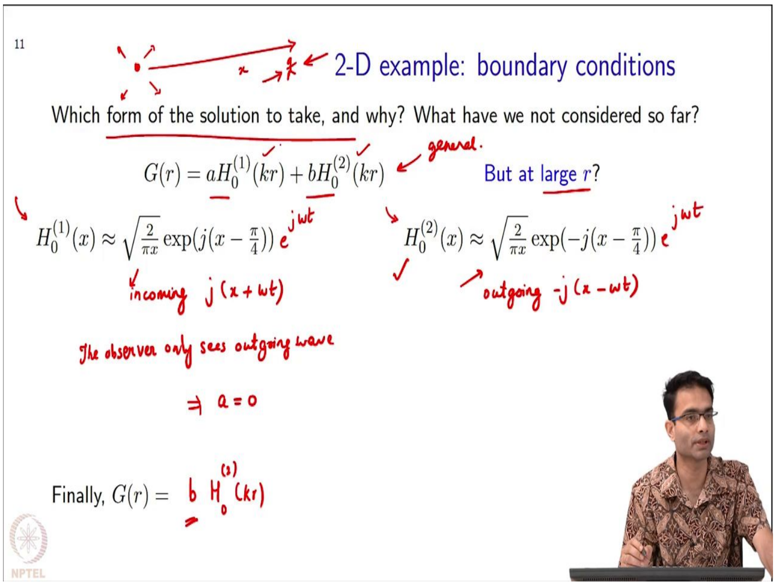

So what I started with here? I had started with two terms and I am left with only the second term right so, $H_0^{(2)}(kr)$ ok. So, another thing that is easy to get confused by because we were being a little not very careful with our notation, I have simply written r over here right, this r lets identify where it came from in the differential equation ok.

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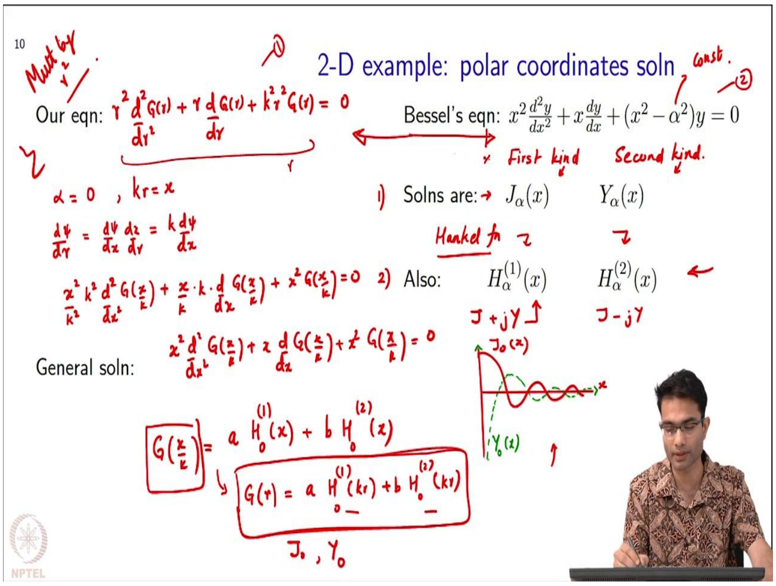

So, this r came from identifying this wave equation over here with the Bessel differential equation correct. So, over here what is our r? It is a scalar because it matches with the x over here right, there is a one to one mapping and that is the r that I maintained everywhere right. So, finally, by the time I come to the expression for Green's function over here $G(r)$, what is the meaning of this r? It is a scalar and what is; I'm in polar coordinates; so this value is always positive, it is because it is distance radial distance right. So, if I want to write this in more precise way where $G(\vec{r})$, r can be anywhere then how should I modify this?

Student: $\vec{r}$ (Refer Time: 03:41).

Or just the $|\vec{r}|$ right; so, $H_0^{(2)}(k|\vec{r}|)$ ok so, it is a minor thing to if you retrace the derivation you will see why this $|\vec{r}|$ should be kept over here. Again depending on which text you look at you might find instead of $H_0^{(2)}$ you might find $H_0^{(1)}$ and the reason for that would be +j by yeah, but why would you choose; why would anyone write the Green's function as $H_0^{(1)}$?

Student: It is invert (Refer Time: 04:19).

Invert is physically not allowed; invert waves are not physically allowed, but still you will find some places where, in some text books you will find the Green's function written as some constant times $H_0^{(1)}(kr)$. What is and it is correct and why would that be correct and why would this also be correct. So, remember we had when we wrote our solution to the Bessel differential equation I got two terms $H_0^{(1)}$, $H_0^{(2)}$. How do I choose $H_0^{(2)}$?

Student: Outward (Refer Time: 04:45).

Before I came to outward wave what would I have to remember, time convention. I chose $e^{j\omega t}$ therefore, $H_0^{(2)}$ was a outgoing wave, but if my time convention is $e^{-j\omega t}$.

Student: $H_0^{(1)}$.

Then it will be $H_0^{(1)}$ because $H_0^{(1)}$ then is the outgoing wave the wave has to be outgoing. So, you have to see in your time convention which one to choose again. So, this is a common difference between physics text and electrical engineering text. Physics people tend to use $e^{-j\omega t}$ electrical engineers tend to choose $e^{j\omega t}$.

So, you should just be aware because otherwise you will be off everywhere by minus sign. And in are in the case of computational electromagnetic, there is a mix of both kind of authors so you it is a genuine possibility of a mix up ok. So, we are going to stick to the convention of $e^{j\omega t}$ everywhere that is why we get $H_0^{(2)}(kr)$ right.

Now, what we had done to simplify all our math was that we put the delta function at the origin right; and that is how we got this simple looking differential equation. Now let us make it general. So, let us consider the case where I have my delta function at any location r', how will I get back my solution now? I have to remember we had agreed on what we will do, once I have got the solution all that I have to do is.

Student: Shift the origin.

Shift the origin. So, what will be the final form that I should write so $(-j/4)H_0^{(2)}(k|r-r'|)$ because this is implicitly making r', the origin and finding out the distance of r from r' right. So, just arguing this physically allows us a lot of simplification. Instead if you are chosen your location of a delta function to not be the origin and insisted on solving everything; you would have had to keep the theta dependent terms right.

Because if for example, in this coordinate axis if my delta function is over here and my observer is over here at θ and r then; obviously, the way this source is emitting there is a theta dependence for this guy over here. So, I cannot; I would not have been able to remove the θ dependent terms ok, but finally, all that matters is; all that matters is this distance, the distance between the source and the observer alright.

So, this is the form of the 2D Green's function that we will use. Now I have mentioned this previously this is a special function, Hankel function consisting of Bessel functions right. The main idea is that these functions are not you cannot write them in closed form ok, but that does not mean we cannot numerically see what it looks like? So, let us see what it what this function actually looks like.

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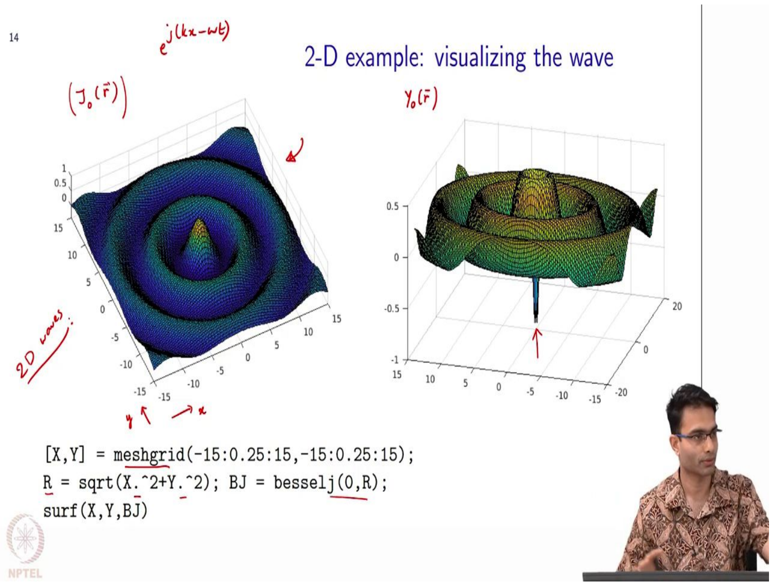

So, that is what we have over here. So, what I am showing you over here this is your $J_0(r)$; r since I have plotted it in the plane. And this is your $Y_0(r)$ ok. So, for example, this is your x-axis and this is your y-axis alright.

So, what I am doing? I am just squaring x and y and at each value of x and y I am plotting this function ok. It is a real value; J_0 is the real valued function it has a value finite value everywhere, so you can see the maxima of this function is happening where.

Student: Origin.

Origin right that is the maxima its yellow colored yellow shaded over here and it begins to decay ok, as you go away from it just like a wave. For Y_0 what is the only difference? Notice this.

Student: (Refer Time: 08:52).

Right, It is going its plunging to $-\infty$.

Student: At 0.

At 0 at the origin right; and again it is like a wave, but it is decaying as I go away from it ok. In MATLAB its very very simple to generate this so, you use your command called meshgrid to generate a grid of X, Y points and you evaluate the radial distance this is element wise operation and just find out your Bessel J. So, they have Bessel J, Bessel Y and so on that is how you generate this different things and the surface plot is what you will used to plot this 3-D function ok. Now just out of curiosity have you seen something like this in real life?

Student: (Refer Time: 09:39).

Student: Ripples.

Ripples; ripples were in the point or even closer to home in when you are have a bath, you put water in your bucket you see ripples like this right on the surface of the water right. Actually if you look at the differential equation for the water surface it turns out to be the same Bessel differential equation so the solution is Bessel function. And if you had to choose between J_0 and Y_0 for water on the surface what would you choose?

Student: J_0.

J_0 right, because physically Y_0 does not make sense since is going to $-\infty$ right. So, from today whenever you have a bath you will remember Bessel functions right, because they are everywhere in nature wherever there is a 2-D kind of a wave problem. So, 2-D wave is a Bessel function. So, for in sort of high school when I was any one said plane wave, what would you do you said $e^{j(kx-\omega t)}$. But, if I plot the magnitude of this the amplitude of this is unchanged it is the wave that goes off to infinity it never decays.

So even though it is a simplest kind of wave, it is the most unphysical kind of wave this on the other hand these are 2-D waves, which physically you can see that they as you go far away from the center they begin to decay of which is what we expect ok and subsequently we will come to 3-D waves also. Any questions about this, ok so you got a physical idea of what is happening. The waves that satisfy our differential equation are just a linear combination of J naught and Y naught, and why did we choose that linear combination? Because it gave us a certain physical meaning of outgoing waves so that is what we had.