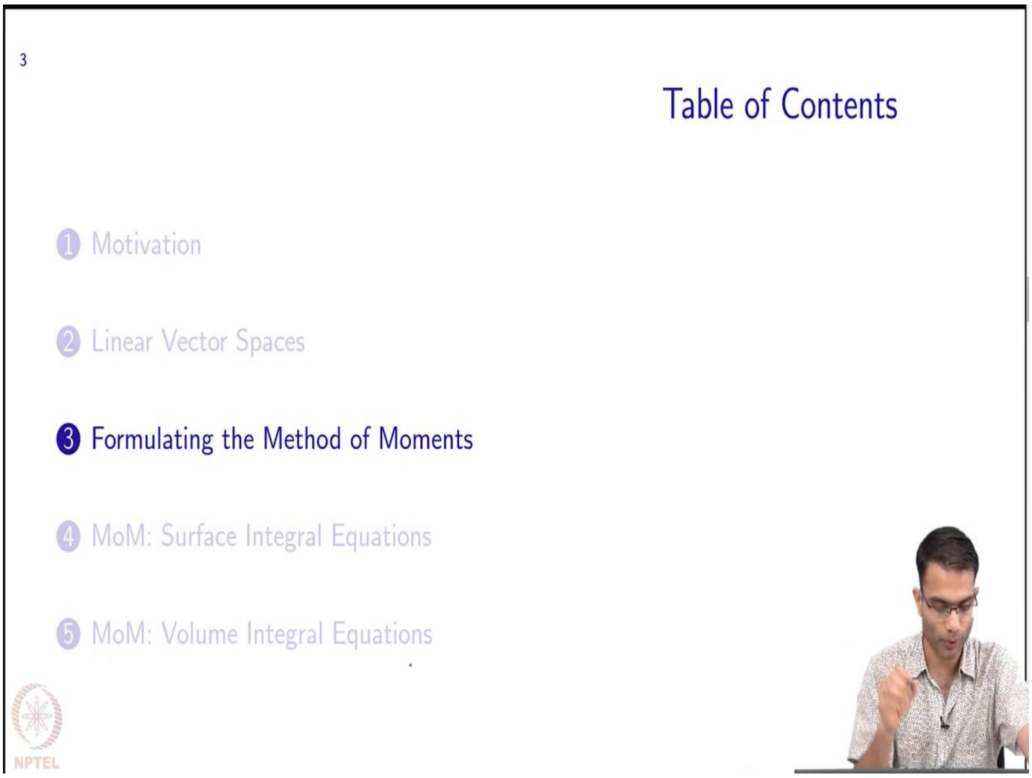
Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

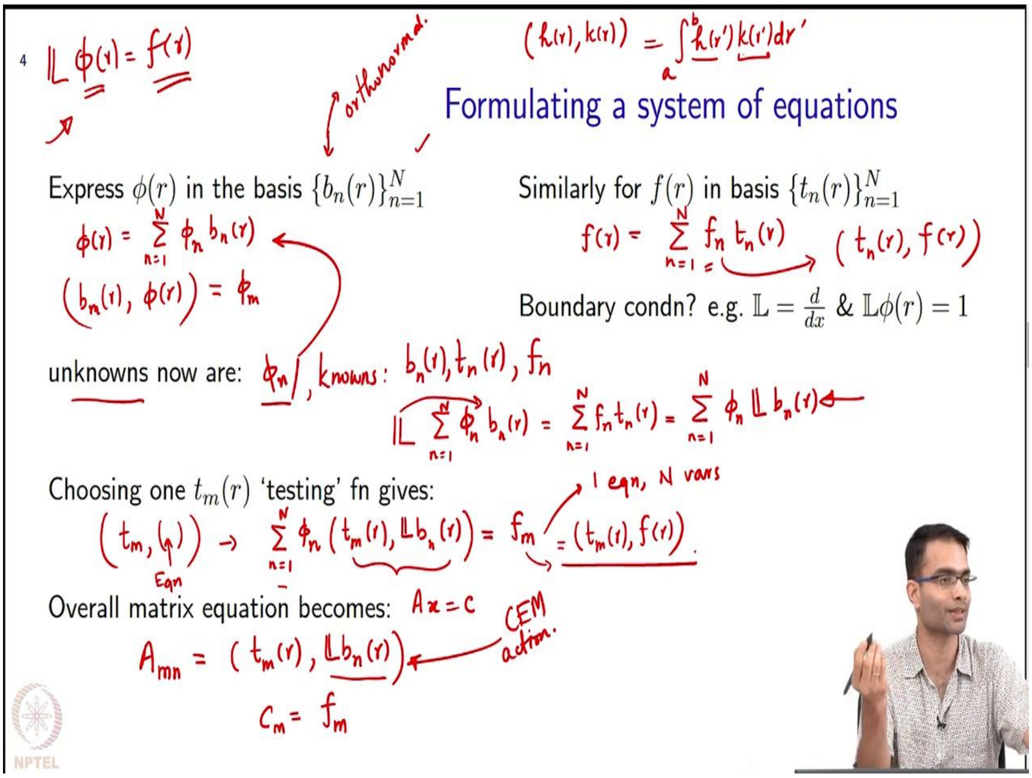
Method of Moments
Lecture – 8.3
Formulating Method of Moments

(Refer Slide Time: 00:15)



So, let us go ahead; let us actually formulate this Method of Moments ok.

(Refer Slide Time: 00:18)



So, remember my operator was $L\phi(r)$ is equal to $f(r)$ that was my operator equation right. So, L is the funny L right. So, first of all what I should do is, I have said that I am going to use the basis b_n for the domain. So, whatever function I am trying to find out ϕ, let me try to express it in the basis of b_n ok. So, that is the first thing I will do.

So, how do I actually write this? Let us further assume that these b_n 's let us keep life simple let us call the b_n 's and the t_n 's to be orthonormal ok. They need not be, but it makes math easier. So, the question is how do I write ϕ in terms of b right. So, it is some constants, but what are those constants? Inner products right. So, I will write $\phi(r)$, I can write it as some constants let us call them $\sum_{n=1}^{N} \phi_n \, b_n(r)$. Since it is a basis I should be able to express any function as a linear combination of these basis right and now I want to find out what is known and what is unknown here.

Student: b_n.

b_n is known.

Student: ϕ_n.

Phi n is not known right. So, from a function I have converted to a set of scalars ϕ_n right. So, to find out ϕ_n what should I do on both sides? Take an inner product with what?

Student: Basis function

Which basis function?

Student: (Refer Time: 01:50).

Some other one right; so, if I take it with b m r inner product on both sides right, since they are orthonormal which is the term that survives?

Student: Phi m (Refer Time: 02:06).

ϕ_m right; so, in other words what I am trying to say is that take my function ϕ and project it along each of these basis vectors and I want to find out the length of these projections that is what I have reduced my problem to right. So, the unknowns now are my ϕ_n. Similarly I can say that for f which is in the range I can write it in terms of the basis vectors for that space right. So, I can call it $f_n t_n(r)$ right and f_n is going to simply be given as $(f(r), t_n(r))$. So, far nothing very complicated is f_n known.

Student: f_n.

f_n had better be known because we said that $f(r)$ is known is given in the problem, it is a known right hand side. So, if f is known, then its projection on any known basis function is also known right. So, unknowns are.

Student: ϕ_n.

ϕ_n and knowns are we have which is known, we have t_n that is known and we have f_n that is known ok. So, we should be very clear what is known, what is unknown what are we trying to solve for right. So, in this problem what are we trying to solve for? ϕ_n s. Once I get ϕ_n s how do I get ϕ ? Put it back into this equation that gives me back my $\phi(r)$ which is what I am interested in right. My objective when I started with this operator equation was to find out the ϕ that satisfies this equation.

Now, how do we get a system of equations from my operator equation? That is the question ok. So, if I write down so far what I have done? So, I have my operator L, now phi I am going to write in terms of these guys. So, $\sum_{n=1}^{N} \phi_n b_n(r) = f(r)$ which is $\sum_{n=1}^{N} f_n t_n(r)$. Now, here is what the method of moments prescribes? Now what it says is that in order to solve this that is in order to get a system of equations for ϕ_n what I should do is, any hints? L anyway will go inside because ϕ_n is a number. So, this will go inside over here. So, this will become if you want to write it, I can write this as summation $\sum_{n=1}^{N} \phi_n Lb_n(r)$. L goes and now it is acting on a known basis function.

Now, but I want to get a system of equation so that I can solve this. This does not look like a system of equation so far right, do I have n equations in n variables? Not really I have n unknown for sure which are the ϕ_n 's, but I do not have n equations in them. So, how, what should I do inner product right? So, that is a very good suggestion. So, what is being suggested is that, in this equation what if I take a inner product on both sides with any t ok. So, let us take t_m . So, what does it do? So, what I am saying is now I am going to do t_m inner product of the whole equation right, the whole equation is over here. Which whole equation? This whole equation ok.

So, what is that become? So, the first term will become So, the summation will remain $\sum_{n=1}^{N} \phi_m t_m(r)$ inner product with $Lb_n(r)$ right is equal to. Now the right hand side comes in, what will survive from here. Only one term because the t 's are orthogonal in fact, right. So, now, what is this equation? There is a summation over here how many what will this is this known or unknown? Its fully known right, I know I chose t, b and L is given to me. So, this is fully known what is accompanying it? Something that is unknown that is the ϕ ; what about the right hand side?

Student: Known.

Known right so, what have I got? I have got one equation n variables right. How many times can I repeat this process? I can repeat this N times with each of which with each of the which with each of the ts right. So, that is why these are called testing functions, I am testing the equation with each one of them one after the other and I am getting n equations right. So, overall matrix equations supposing I get something like $Ax = b$ that is what I get. So, what will my A_{mn} be equal to? So, the rows, what are the rows coming by the different rows are being generated by choosing different what?

Student: (Refer Time: 07:48).

Every choosing a different testing function gives you a new equation.
So, every new equation is a new row or column.

Student: Row.

Row obviously, right; so, one question? $\phi(m)$ that is right good there should be $\phi(m)$ yeah right. So, A_{mn} the mth row will come when I choose the mth testing function right and what will give me the nth column?

Student: (Refer Time: 08:20).

The nth basis right. So in fact, the A_{mn} is already written for you on the slides. So, its $t_m(r)\, Lb_n(r)$ that is the mn^{th} element and what is the b_m? Simply f_m right. So, b is a column vector with f_1, f_2 all the way up to f_N right. Is this confusing?

Student: (Refer Time: 08:50).

Right yeah you are right; so, I should probably not call this b that is confusing let us call it $Ax = c$ right. So, we will say that $c_m = f_m$ yeah and this abuse of notation is the word you are looking for fine. So, I have got my matrix equation $Ax = c$ and we have done this ok. So, this is formally called the method of moments straight forward. Where do you think all the hard work will happen? What is so, I mean we have going to spent a long time discussing this. Where do you think a hard work happens?

Student: Finding transfer b.

Finding.

Student: Transforming b.

Transforming b and your answer.

Student: Finding b_n.

No.

Student: No.

b_n and t_n I will choose something simple I can choose pulse basis function also, if I am very lazy I can choose pulse basis function. So, b's and t's are easy; what is the.

Student: Solving the equation (Refer Time: 09:54).

Solving the equation is not difficult; once you form your system or equations MATLAB does $A\setminus b$ or done and you can do research in that, but sort of core work in computational electromagnetic, where is that effort going to come in.

Student: (Refer Time: 10:11).

This inner product over here because L is a complicated operator; so, you have to operate on $L(r)$ and then take the b_n and then take an inner product with t_n. So, it looks very compact over here it looks like this not much to do over here.

Student: Sir could (Refer Time: 10:31).

Well we will see ok. So, what are some of the things you have we have done one module on numerical integration right so for example, when I write something like $(h(r), k(r))$ when I write the inner product like this, what does that mean?

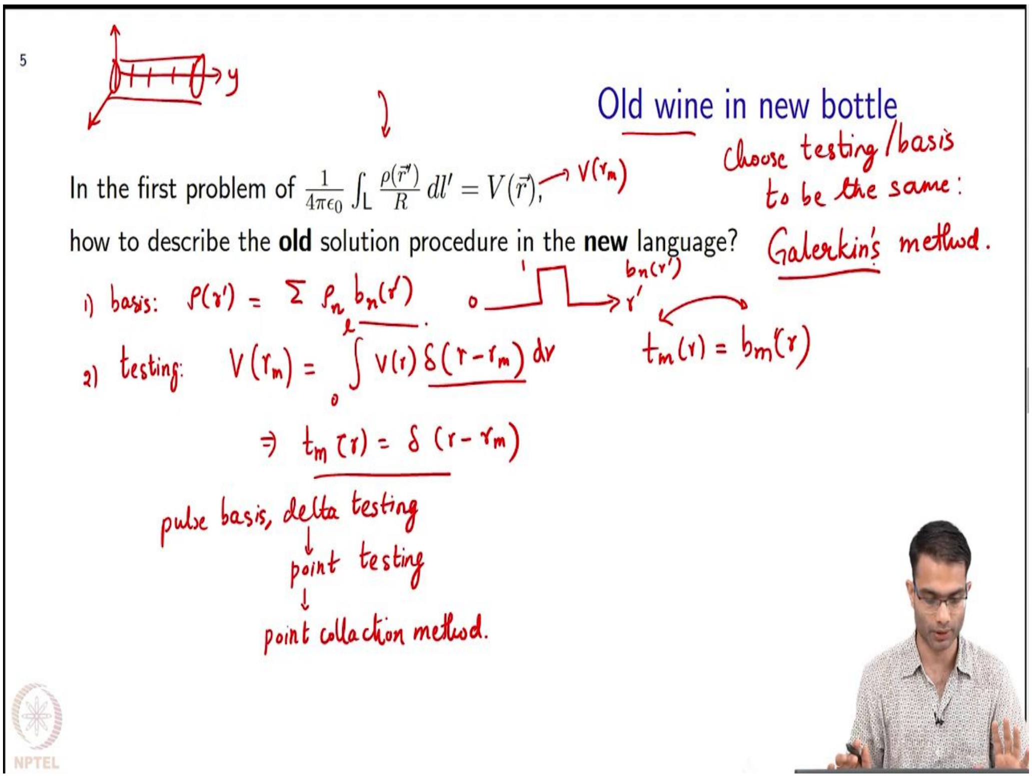
Student: Integral.

It is an integral right. So, it is going to be an integral of $h(r')\, k(r')dr'$ over some specified.

Student: Domain.

Domain right so what is going to happen? This guy and this guy they have to be integrated to get you to this inner product and this itself maybe an integral with differentiations inside it or whatever. So, this is where all the CEM actions happen ok. Another sort of I am getting ahead of myself, but I will tell you one other challenge that will happen over here is that your we have seen Green's functions right. Green's functions will appear inside here somewhere in the L operator or whatever, and these Green's functions has singularities. So, we have to be very careful of integrating in the presence of singularities ok. So, those are some of the challenges right.

(Refer Slide Time: 11:51)



So, let us as they say put old wine in new bottle alright. So, let us look at what we have already done, this was the equation that we had alright. Now I want to describe our naive approach that we did of discretizing the integral equation in the new language. So, first step was basis. So, what did I choose for the basis? I mean I my variable was ρ. So, I wrote of I wrote $\rho(r')$ in terms of the basis functions right I called it let us say some ρ_n, $b_n(r')$ right and b_n was something like this, this is my r' 1 0 this was my $b_n(r')$ right nth segment. So, that was step 1.

So, I did the basis function right, I did not really call it basis, I mean I did not call it method of moments and, but I chose a basis function right and then what would I do? Did I choose a testing function?

Student: But that is delta.

I did not actually choose a testing function right. I just evaluated the right hand side at different values along the along this y axis, the cylinder was here I just chose different discrete points over here. I did not choose a testing function right. So, the correct answer has come from the audience that the testing function was the analogue of a delta function ok. So, let us see how. So, what was the right hand side that we had in this equation? So, this f m that I wrote over here how did it actually come? It came when I took an inner product of $t_m(r)$ with $f(r)$; that is how I got my f right. Now, what am I saying testing function how do I how do I get $V(r_m)$?

Student: Integral $V(r)$ (Refer Time: 13:58).

Right exactly. So, I get $V(r_m)$ is equal to $\int_0^L V(r)\,(r - r_m)\,dr$. So, this is a inner product, if implies that my testing functions $t_m(r)$ is nothing, but a delta function ok. So, this is the easiest or the laziest thing you could do alright. So, this method that we did it is also called pulse basis and delta testing. Some places will not call it delta testing they will call it point testing.

Student: So, sir these are testing (Refer Time: 15:58) because delta (Refer Time: 15:00) because they always to go under (Refer Time: 15:01).

No that is what. So, discretizing the integral equation like what we did is equivalent in the language of method of moments to choose a delta function as the testing function.

Student: (Refer Time: 15:12).

Well ok. So, the problem is that here this is not actually a function this is a generalized function. So, I cannot talk of f_m exactly in this case. So, that is why I went back and I wrote another definition for f_m right because I cannot define the value of a delta function at some point, it is an undefined right. So, another word for this is called point collocation method, you can imagine this method has been around for such a long time everyone has come and given it a different name ok. So, you encounter any of these things you know they are talking about the same thing ok.

So, this is sort of what we did; if you want it to be more sophisticated going back to the very original question I asked the start of the module, what would you do instead what could you do instead if you want it more accuracy? Not choose delta testing function that is what we could do right? So, we could instead of doing this point testing, I could have chosen my t m over here. I could have chosen $t_m(r)$ also to be the pulse basis function I can say this is also of this form. I could have chosen this.

Student: Is a.

So again, question is that is there any advantage of keeping these two the same so first.

Student: Its actually (Refer Time: 16:48).

It makes the math somewhat easier it is not necessary that you have to do this, you can choose a different set of basis function where different set of testing functions.

Student: So, it will (Refer Time: 16:56) there.

Little bit easier I mean the simplification gets a little bit easier, but if you choose different, it is not that it becomes unsolvable or whatever. It is finally, you are not going to do this integration is in my hand you are going to code up something right. So, it is a one time effort anyways different choices of the testing in the basis functions give you different accuracy ok. One more term I am going to use getting ahead of myself because the question has been asked, when you choose the testing and the basis functions to be the same. This method is given is called by the name Galerkin's method, named after its person by the name Galerkin.
So, if you hear if you read the word anywhere with says Galerkin's method was applied, it means the testing function and the basis function whatever they maybe about the same. So, it is not necessary that they have to be pulse basis function they could have been trying well anything else ok. So, that is your, that is a Galerkin's method, which is yeah also very popular in finite element methods.