Computational Electromagnetics
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Method of Moments
Lecture – 8.6
Surface Integral Equations: Evaluating the Integrals – Part 2

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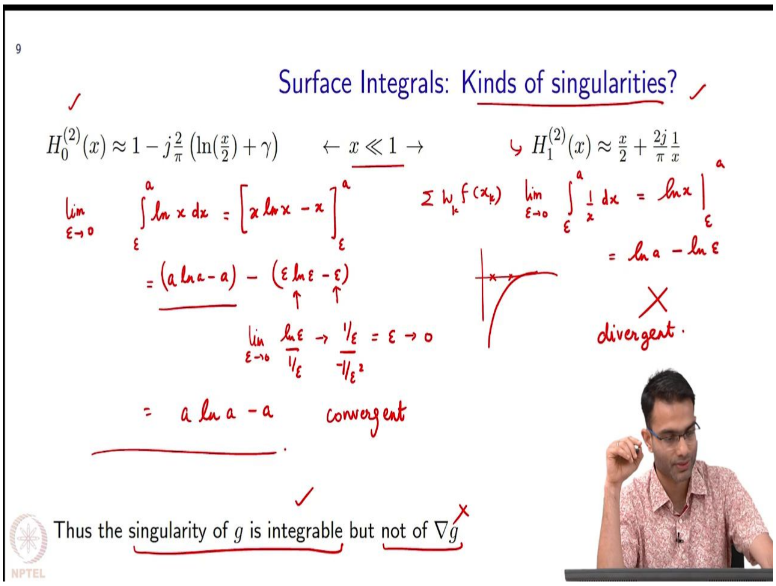

So now, let us see what we do about this.

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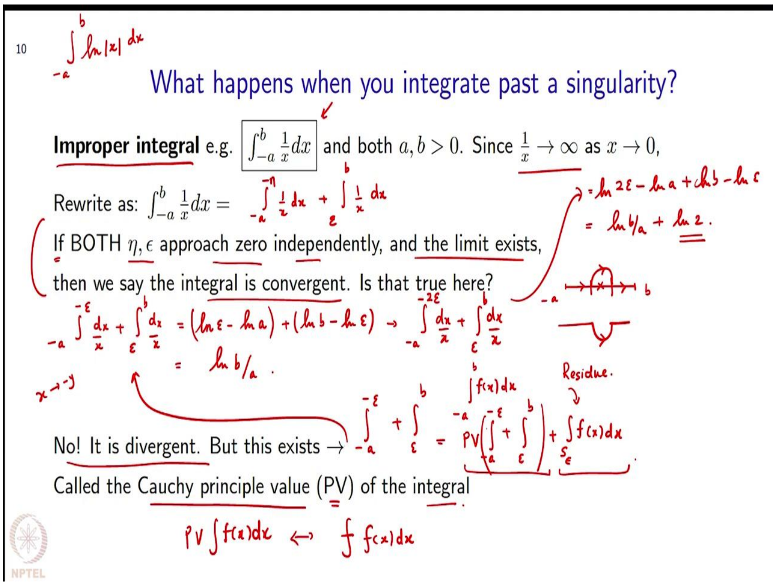

So, this kind of integral which I showed you is what it called a improper integral ok. So, this is the best example, I have an integration from minus a to b some function which is blowing up. And, since there is a point in between which blows up at infinity this is called an improper integral this is just the definition.

So, how do we I mean what I showed you previously both were sort of improper integrals, because both log x and 1/x they were blowing up. Do not worry so, much about the limits over here, previously I was going from ε to a, but I may as well have gone I could have written the previous example as something like this $\int_{-a}^{b} log(|x|)dx$ the same thing what happened right I would break it up into two parts.

So, in order for you to answer whether or not this integral is convergent or divergent, this is again a bit of revision from calculus what you should do is, you should break this up as $\int_{-a}^{-\eta}(1/x)dx + \int_{\varepsilon}^{b}(1/x)dx$ and is there a term that I am missing over here. So, no that is fine yeah. So, I break this up in this fashion and if it happens that as η and ε both approach 0 independently and the limit exists, then we say that the integral is convergent.

So, to understand this a little bit more sort of carefully what we can do is, we will take a we will take a counter I mean we will let us do something else let us do $\int_{-a}^{-\varepsilon}(1/x)dx+\int_{\varepsilon}^{b}(1/x)dx$. let us make η and ε to be the same. What can I write for this what will the first term give me? I can do a substitution x=-y right. So, there is no problem in taking log I mean it will not become log of a negative number. So, what will happen? $dx=-dy$ right I can do all of this what should I what will I get over here?

$(log(\varepsilon)-log(a))+(log(b)-log(\varepsilon))=log(b/a)$

$log(b/a)$; so, it seems that this integral exists, but what happened? First thing I did is I made $\eta=\varepsilon$. If I make $\eta=\varepsilon$ both are going to 0 at the same rate, because they are identical variables right. Now supposing I did a I played a little bit of a trick, I said that let me do the following ok.

Let me instead do this as $\int_{-a}^{-2\varepsilon}(1/x)dx+\int_{\varepsilon}^{b}(1/x)dx$. So, what I have done? I have called I have gone back to $\eta=2\varepsilon$. Now, η is approaching the approaching 0 faster or slower as ε tends to 0. $\eta=2\varepsilon$.

Student: (Refer Time: 04:21).

η has approaching faster because 2ε. So, ε is going to 0 at some rate and 2η going at twice the rate ok. Now when I evaluate this integral what do I get? So, I am going to get $(log(2\varepsilon)-log(a))+(log(b)-log(\varepsilon))=log(b/a)+log(2)$. So, what happened over here? I got $log(b/a)+log(2)$. So, what happened? So, now, you see appreciate the meaning of this definition if both of them independently approach 0 and the limit exists, then the integral is convergent. So, we saw that if these 2 are approaching at different rates then you got this funny log to appeared over here similarly any term if I made a 3ε I will get a log 3 over there.

So, this limit itself is not existing as both η and ε are approaching independently ok. So, this is the definition of a divergent integral you can try it out for the other case $log(x)$, you will find out that it is convergent and there is no problem therefore ok, but 1/x has this problem therefore, it is a divergent integral. But what existed was, this term existed right when they approach not independently, but in the same rate when they approach at the same rate it existed right. So, when they approach at the same rate and I get I mean the limit exists, it does not blow up right. So, when it is like this $\int_{-a}^{-\varepsilon}+\int_{\varepsilon}^{b}$ if it is of this form, and this exists this is called the Cauchy principle value of the integral ok.

So, the notation for it is PV. So, sort of to summarize I had a let us say segment over here -a to b, and there was there is a singularity sitting over here and what is happening is, I am going from here I am hitting the singularity right. So, what happens is, I will write down this integral $\int_{-a}^{b} f(x)dx$ I write it as. So, I am going to call this the principal value which is $\int_{-a}^{-\varepsilon}+\int_{\varepsilon}^{b}$ this integral and what I have left out? So, I have left out one segment in this right. So, I have to compensate for this segments among because the integral is from -a to b. So, the way around it is I have to somehow avoid the singularity by jumping ok. Jumping around it so, that becomes what is called I can write that as an integral over some contour that does not going to the singularity, that is how I write this. So, this is also called the residue. That is the total integral.

Now, again there is a little bit of ambiguity over here, how do I choose this deformation contour? Could I have chosen it like this? Yes, I could have chosen it right. So, that that piece of ambiguity survives how to evaluate the residue the physics of the problem will tell me which contour is allowed which contour is not allowed ok. So, this is something sort of new which you have encountered is that when you hit a singularity, then you have to do the integral in two parts. The first part is without the singularity and that is called the principle value, the Cauchy principal value of the integral right. So, the notation for it is as I have written over here you can write down you can just write down $PV\int_{-a}^{b} f(x)dx$ PV of f x dx right.

So, if you see this, it means that the singularity has been punctured out you are not including it. There is another slightly confusing shorthand notation for it which goes like this. There is a cut across the integral sign that cut tells you that the single singularity has been escaped ok. So, these are the things to keep in mind. So, the principal value is the first part over here and the second part is going to be the residue term which we will spend some more time on in trying to evaluate ok. So, should be go ahead then and calculate the integral $\nabla g.\hat{n}$ ok. Let us calculate this integral.

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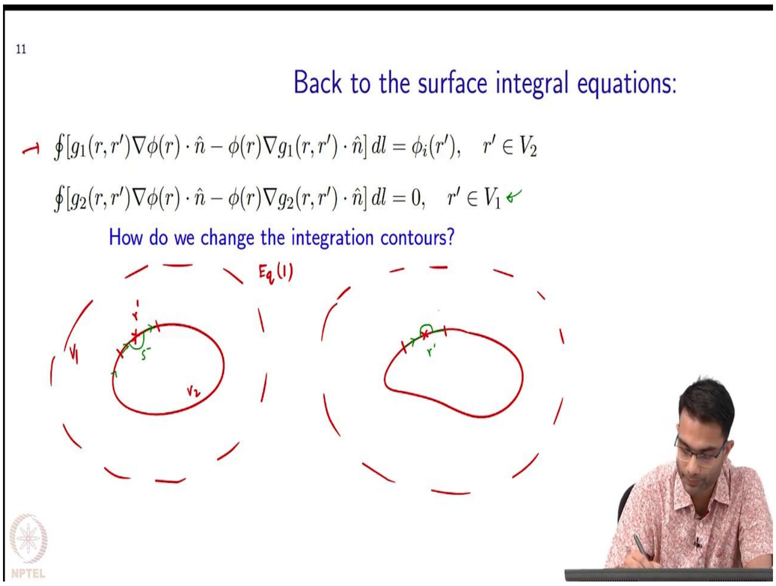

Well, before we get to the integration we have to see how do we modify our integration contour now right. So, if you look at this over here, either this or this any which way I have to escape the singularity right. So, when I am looking at let us say the first equation over here. So, I will draw it 2 times. So, this was my first integration first volume right this is my V_1 and this is my V_2 ok.

For the first equation r' is forced to be in V_2 ok. So, let us say that this is my r' I have fixed r' right now to be this segment the segment is from here to here r' is fixed I am trying to write down my one equation. So, r is going over this whole boundary that is my contour integration now it comes into this segment. So, it enters over here very good, now hits the singularity which way should I deform the contour.

Student: Above.

Above or below?

Student: Below.

It should go below right. So, it should go like this right that is why I use the word S_-. So, S_- tells me I am inside right. So, this is you can think of this as closer to the S_- term over here similarly wherever the location is wherever r' is I must bend around it that is going to be our strategy and for; so, this was for equation 1 and for equation 2 Im drawing it separately. So, there is no confusion. Again this is my segment over here and I am going about my integration this is my r' now r' in the second equation is forced to belong to.

Student: V_1.

V_1 right; so, what will happen is that is how I can do this integration. So, that will be the strategy that we will solve. So, we will bring it to we will bring this discussion to a close over here.