Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Method of Moments
Lecture – 8.7
Surface Integral Equations: Conclusion

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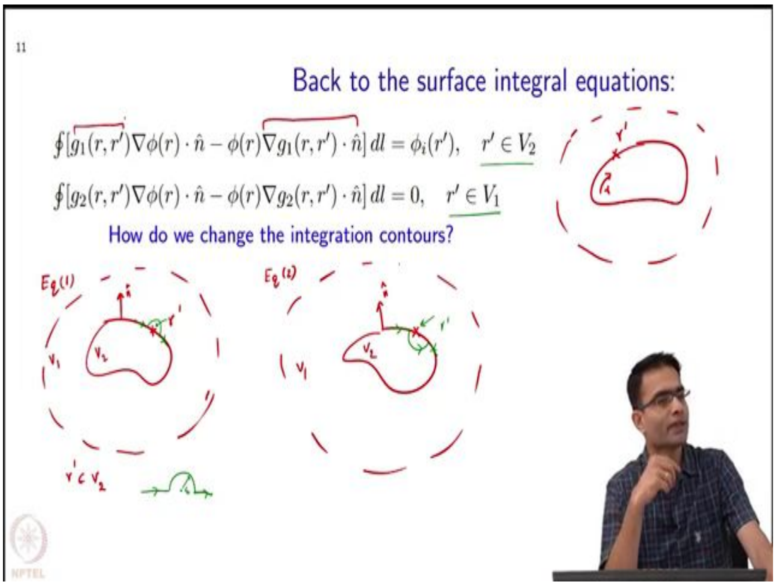

So, let us start with trying to put all these singular integrals to work over here.

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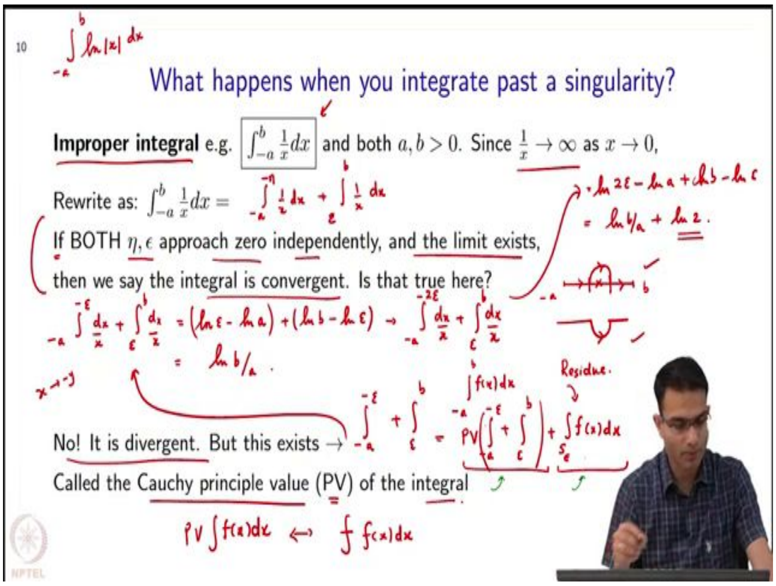

So, we have seen that when we have integral of the form $\int 1/x \, dx$, I have to do the integration little bit carefully. So, one is the principal value part and the second is the residue part right. So, these are our two equations over here and because of this grad g dot n hat there is going to be a problem right because in this contour integral of mine.

Let us say r prime is fixed over here and r is running over the entire contour. So, they will be some one point at which r hits this r' and that is the issue that we have to care about right. So, for each of these integrals, I have to modify the contour in the correct way ok. So, let us look at equation let us look at equation 1 ok. So, this is equation 1 this is my and my point here r prime I have put over here.

So, equation 1 is that $r' \in V_2$. So, I am coming along from here and I want to make sure that $r' \in V_2$ right. So, if you just write down what is V_1 and over here this is V_1 and V_2. So, which way should the contour bend it should bend in such a way that r prime still belongs to V_2. So, the way to make this small bend is now the point r' is inside V_2 because I have changed the surface just a little bit right and this I will zoom in and show you what I am doing is something like this.

It is a very small circle; the radius of which is ε and ε is tending to 0. So, I have in the limit I have not really change the surface. I am going to take the limit epsilon tending to 0. So, r' which is this point over here stays in V_2.

Student: (Refer Time: 02:41).

No r' is remaining; r' remaining at that point I am changing. The boundary around it; the boundary is the boundary between V_1 and V_2 and I have pushed the boundary little bit this way, but r prime is still there; r' is not moving I am moving the boundary a little bit.

So, I had started by saying I am putting r' on the boundary and boundary was because boundaries both in V_1 and V_2. Then we said that $\int 1/x \, dx$ has a problem so, I have to move the boundary a little bit. If I move the boundary the other way, then r' will fall into V_1. Now it is staying in V_2 right. So, that is the small boundary modification for equation 1. For equation 2, the same logic will give me a different contour right. So, this is again V_1 and V_2 and so, this is my point over here that is r'.

So, I must modify this in this way. I am exaggerating it to show you very clearly which way I am bending the boundary where; obviously, that is a it is a circle of radius ε which will go to 0. So, in this equation now r' which is this guy continuous to stay in V_1. So, that is that is how we will modify these boundaries ok. Now you might wonder I am going to take the limit $\lim_{\varepsilon \to 0}$ so, does it really matter how I bend it? Finally, I am going to take $\varepsilon \to 0$ whether the boundary bends downwards or upwards will you get different answers right.

So, it depends on the integrand we will see whether it depends right, we will see whether the integral matters one way or the other right. So, is this part is clear. So, this is this is a very very important part I have sort of given the answer of you already it matters which way you bend the contour you get different answers in both cases ok. So, let us see how to do this correctly alright.

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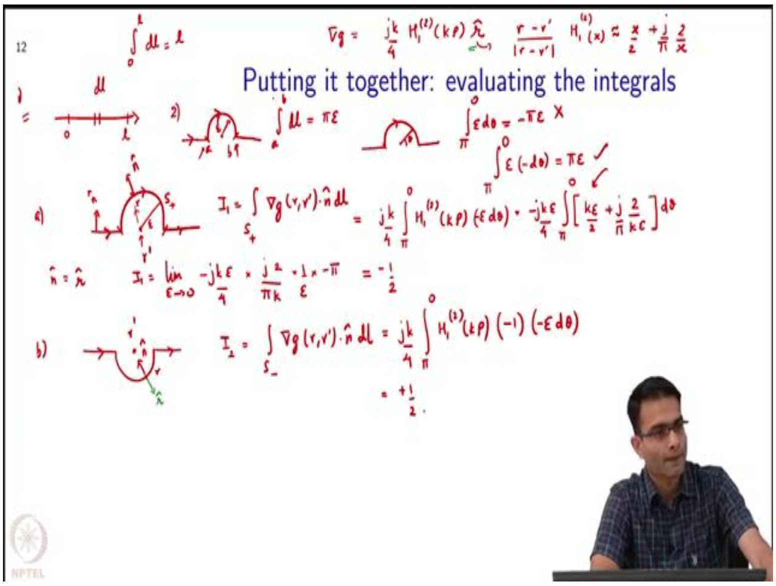

Now, let us now let us evaluate these integrals ok. So, before I show you the exact calculation there is some very basic mistakes that can happen in integration. So, to avoid those, I am going to show you some very simple integrations. So, case 1 is this. I am integrating from here to here and let us say my differential element is dl over here and I am going from 0 to l ok. So, what is the physical interpretation of this if I just have integral $\int_0^L dl$ what is the physical interpretation what is the answer first of all?

Student: L.

L right what am I doing? I have measured how to speak the length of the string between 0 and l right even a class eleven student will say answer is 0 to L right. Now let us do the same calculation in a slightly different way. So, supposing I have. So, I am going from here to here from this point over here to this point over here and I want to evaluate let us call this point a and let us call this b ok. So, I am going from point a to b along the contour and I want to find out integral dl ok.

I am staying and the integral is along this contour right. So, if this is a semicircle of radius ε coming from case 1 over here, what should what will you say this answer is? Length of the string which is $\pi\varepsilon$ that is correct. Now on the other hand if you decided to do this integral using let us say polar coordinates right. So, supposing you wrote this like this that this is your theta when you are integrating like this. So, what are the limits of integration?

Student: (Refer Time: 07:13).

From where to where.

Student: (Refer Time: 07:17).

No where am I starting from what is the original starting point?

Student: (Refer Time: 07:22).

$-\pi$ or π.

Student: $-\pi$.

Why $-\pi$? I could I could write π also?

Student: (Refer Time: 07:30).

And final integration final inter point is?

Student: 0.

0 and what is the differential element dl?

Student: $R\,dl$.

$R\,d\theta$ which is?

Student: $\varepsilon\,d\theta$.

$\varepsilon\,d\theta$ what you get? You get $-\pi\varepsilon$. So, what is the catch in this I should have got $\pi\varepsilon$ because I measuring the length of this π.

Student: (Refer Time: 08:03).

Right so, $d\theta$ is pointing in the $+d\theta$ is in which direction it is in the clockwise direction over here right anticlockwise direction right, but dl is in the clockwise direction right. So, dl is actually $-\varepsilon\, d\theta$. So, that is one mistake that you could do if you did not do this integral carefully ok. So, in fact, this is wrong. So, what we should be doing is let us say $\int_{\pi}^{0} \varepsilon(-d\theta)$ and then I get the correct answer $\pi\varepsilon$.

So, these are these are some of the things that you have to be careful about when we evaluate this integral. So, all right now let me have the simple case out of the way let us let us look at our the integral that we have. So, what is our integral? So, I have in one case. So, we look back over here one case it's going up and the other case is going down right.

So, let us take the up case first. So, that is the case so that is right what is our integrand over here? So, let us call this S_+ it is on the positive side. So, the $\int_{S_+} \nabla g(r, r') \cdot \hat{n}\, dl$, that is the term that we are trying to calculate. Which way is $\hat{n}$; which way is $\hat{n}$, upwards or downwards? Look back here which way was $\hat{n}$ over here? This was $\hat{n}$ and in this also this was $\hat{n}$.

So, $\hat{n}$ continues to be. So, for example, $\hat{n}$ is here like this and over from here it is readily out wards right. Now what is a $\nabla g \cdot \hat{n}$. So, ∇g is we had written it as minus $-jk\, 4H_1^{(2)}(k\rho)$ is that correct ot I am off by a minus sign.

(Refer Time: 10:35) we can just check back.

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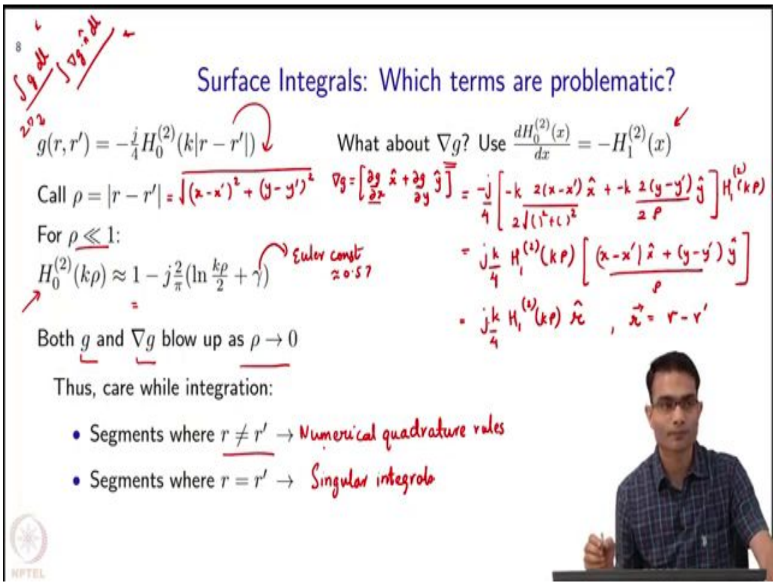

We have derived ∇g previously ∇g is a $+jk\,4H_1{}^{(2)}$ right. So, let us use the correct expression of here. So, there is no minus sign over here fine. Now what was r over here? This was the unit vector for $r-\hat{r}$ right that was my ok. So, now, let us look at this integral over here which $\hat{n}$ is .

So, this is this point over here this is my r' and let us say this is my point r. So, $r-\hat{r}$ which way is it pointing I mean $r-\hat{r}$ is my radially outward vector and along this circle s plus which way is $\hat{n}$? Same direction right.

So, here $\hat{n}$ is equal to $r-r'$ because r is here $r-r'$ is always going to be like this r' is fixed to be the centre of this semi circle that is the point that is fixed the testing point.

Student: R (Refer Time: 11:55).

r is the variable of integration. I am going from here over the circle and like this correct. So, what I am evaluating is only the residue part not the principal value; the principal value ends over here and starts over here right. So, this is the only the s plus integral is what I am calculating. So, $\hat{n}$ is equal to $\hat{r}$ right. So, when I evaluate. So, put everything inside over here this the direction of ∇g and $\hat{n}$ what should I get? Direction of ∇g is $\hat{r}$ normal is also $\hat{r}$ I get a 1 right.

So, I can write all the constants out $jk/4$ integral, now what should I write, what can I will write the limits of integration as π to 0 as we did in the simple example π to 0 and what is $H_1^{(2)}(k\rho)$ and what else?

Student: ε.

ε.

Student: Minus.

$-\varepsilon\, d\theta$ then I will get the correct interpretation. For example, if I put $H_1^{(1)}$ instead of $H_1^{(2)}$ I write 1 I should get the length of this string which I will get if I follow this recipe. Now is where I use the approximation for $H_1^{(2)}$ because I am going to take the $\lim_{\varepsilon \to 0}$. So, I can I should use that approximation right. So, that approximation which we have seen before that is the part that we are going to use.

So, we are going to when we take the $\lim_{\varepsilon \to 0}$ ok. So, in this case over here what is let us just actually write it all out. So, its $-jk\varepsilon/4 \int_{\pi}^{0}$ and. So, I just substitute so, I will have $k\varepsilon/2\, j/\pi\; 2/k\; \varepsilon\; d\theta$. Is that correct? I am just substituted everything that I know and I know that finally, I am going to take the limit let us call this I_1 so, I am going to take the $\lim_{\varepsilon \to 0}$.

So, without doing evaluating this integral completely what can you say about the real part in the limit? I will get ε^2 and the length of the circle right semicircle, so, that term goes to 0. The term that is left is what is the term that is left only second term right. So, I am going to get $-jk\varepsilon/4\, j2/\pi k\; 1/\varepsilon$.

Student: into $-\pi$.

In to $-\pi$.

Student: $1/2$.

Right that is that is the only thing that is left. So, what has happened to the ε? ε have gone away. One k has also gone right there is a $-j \times j$ that is $+1$ and left with the -1. So, the answer is?

Student: $1/2$.

$1/2$ or $-1/2$.

Student: $-1/2$.

$-1/2$ right the minus goes away and $j \times j$ is -1 I get a $-r$ yeah fairly clear. Now let us come to the other case; the other case is case b where I have a very similar, but an oppositely deform contour this is how it is. This is my point r' this is my point r which way is $\hat{n}$? Same as before it has to be out wards. So, on this contour this is my $\hat{n}$ right which way is my this vector r upwards or downwards.

Student: Downwards.

Downwards right because its $r - r'$ rather. So, this is my r hat. So, in this integral now I_2 I can write as s minus grad g r r prime dot n hat d l what happens. So, I will write j k by 4. Now I will use the same limits pi to 0 because I should get the length whether I go upwards or downwards I should get the length of the string ok.

So, let us not complicate that let us just keep it as $\int_{\pi}^{0}$ and $H_1^{(2)}(k\rho)$. Now is the term which I have to be careful of $\hat{r} \cdot \hat{n}$ which is going to be give me now.

Student: Minus sign.

Minus sign right. So, there is a -1 and I have a $-\varepsilon\, d\theta$ right that is all that is what I have left. So, its identical to the term above except for.

Student: Minus.

One extra minus sign right. So, what should I get?

Student: (Refer Time: 18:06) 1 by 2.

$+1/2$ right; so, it did matter how I did the integration, in one case I get a $-r$ in the other case I get a $+r$. So, it is suggesting that ∇g has a discontinuity across the surface right if I approach from bottom or if I approach from the top there is a difference.

So, when we look back at all the terms that we had in the integration right this was the only problematic term when I have. When I looked at the term involving integral of g_1 even if there was a singularity I had it was the integral of $\log x$ that turned out to be finite. So, there was no problem.

This was the only problematic term and this is how you sort of do it carefully and this is the characteristic feature of all surface formulations where there is a point at which the singularity and in the case of 2D you had g and ∇g. ∇g was not integrable. Also in the case of 3D you will have g and ∇g; ∇g will not be integrable.

So, you have to deform the surface correspondingly, but in 3D it will not be a line that you are deforming its a surface that you are putting a hemispherical bowl around. So, that integration becomes a little bit more challenging ok, but the idea is the same there is going to be a discontinuity of one on both sides of the thing.

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So, that brings us to the end of the part of surface integral equations. So, before we go any further, is this part clear how we evaluated $\nabla g \cdot \hat{n}$ or any other questions on it.