Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Methods of Moments
Lecture – 8.13
Surface v/s Volume Integral Equations

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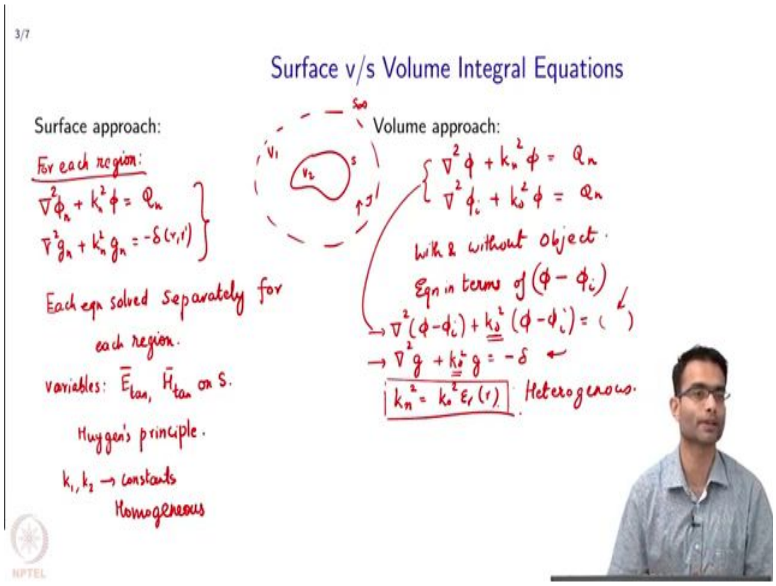

So, now let us go to sort of a comparison between the Surface and Volume Integral approaches right. So, the problem remains the same I had some object V_2 over here and some V_1 and this was my current source over here. So, we started with the surface approach first what? So, we are not going to redo all of it; obviously, we just want to see conceptually where does the paths diverge in the derivation.

So, what happened in the surface approach, what was the first thing that we did? We wrote down the Helmholtz equation for each region right. So, for each region, I wrote out so, let us call it

$$\nabla^2 \phi_n + k_n^2 \phi = Q_n$$

These are all function the space I am not writing the bracket r explicitly. What else did we do? Apart from writing this Helmholtz equation for each region the other thing that I needed was Green's function for each region alright; so,

$$\nabla^2 g_n + k_n^2\, g = -\delta(r, r')$$

And with these two equations what will I do then? I multiplied $1/g$ the other by ϕ subtracted them, but I never mixed the equations for region 1 of volume 1 with the equations for volume 2, as sort of solved them separately. And, then I applied some theorems of vector calculus which converted a volume integral into a surface integral that is how the surface S and S infinity came into the picture.

Integrals over S infinity vanish and I was left with the integral over S right. So, each equation solved separately for each region ok. That is what we did and our variables were the variables that we solved that we solved for the E tangential and H tangential on S right. And we finally, once we solve for it we use the Huygens principle right.

To find the field anywhere. So this is kind of like a revision of the surface integral approach right. So, I mean yeah this is not new. When we move to the volume integral approach what was the starting point, was the first step common to both? Right so, what we did there was we also wrote down a

$$\nabla^2 \phi_n + k_n^2\, \phi = Q_n$$

but here so, for example, I got two equations from each region. Here what was the difference how did I do it? What was the starting point itself became slightly different? What was the two cases? Beside with and without the object right, we did not consider a volume one and volume two, we consider considered with and without object. So, that became the

$$\nabla^2 \phi_i + k_0^2\, \phi_i = Q_n$$

So, this is with and without object and then what did I do? I eliminated the current source right and I got an equation in terms of $\phi - \phi_i$. Did I need to use the Green's function in this?

Student: Once I got.

Well once I got the differential equation and the solution 2 it was written in terms of Green's function as a convolution, but which Green's function did I have to use?

Student: 1.

The one where V_1 where there is no object right vacuum; if you remember that when I subtracted these two equations I kept it something like this, $\nabla^2(\phi - \phi_i)$ plus I kept k_0^2 over here. And then $\phi - \phi_i$ is equal to a whole bunch of terms over here. So, to solve this I use which Green's function the one with k_0^2, that is what I had used right. So, if I know the solution to this guy this I this solution becomes a convolution of the forcing function the right hand side with the Green's function right.

So, you notice the slight difference in approach is started write in the beginning, in the surface integral approach I went for each region separately solved separately subtracted. Here the entire volume is considered with and without object and the difference is taken and then I use my Green's function. So, Green's function is used in both cases. In the surface integral, I have two Green's function. So, one for V_1 one for V_2. In the volume integral approach I have just one Green's function alright ok. Any what else can we say about the differences?

So, one thing was that in the surface integral approach k_1 and k_2 they were constants right. So, they were for homogeneous objects. Whereas, did I have that limitation here? No right; here my k_n^2 was actually $k_n^2\varepsilon_r$ function of space. So, I could deal with heterogeneous these are the sort of the two contrasting differences between these two approaches; nothing new here we just summarizing what we have already done.

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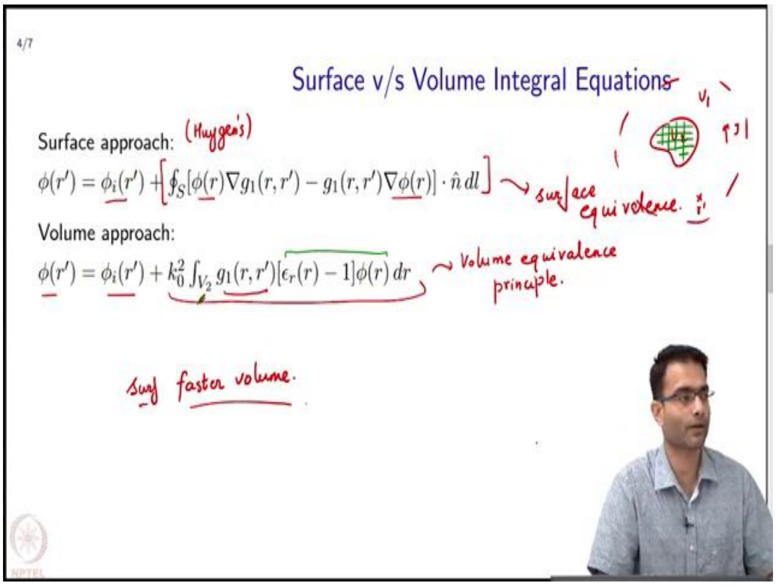

The differences do not end here, when I want to find out the field far away I use in the surface integral approach this is called my Huygens principle right. So, the field at some point over here r prime is written as a sum of the incident field and this term over here which is a surface integral again and that surface integral includes ϕ and $\nabla\phi$ which are $E_{tan}\, H_{tan}$ E tan H tan on the surface. So, these are my secondary sources in my Huygens principle alright. So, we studied two equivalence principles. Which equivalence principle is this? We studied surface equivalence principle and.

Student: Volume.

Volume equivalence principle; so, this is.

Student: So.

Clearly surface equivalence principle, because I have replaced V_2 by set of currents on the currents or tangential fields on the surface. So, it is a perfect example of a surface equivalence.

Right and in the case of the volume integral equation again I have the field at some point is incident field plus scattered field, now this integral is over V_2. That means, I am going into the object; so, what I have done to the object? I have replaced this object by grade of this sort, and at each grid point I have an equivalent current. So, this integral is saying sum over all of these currents what is the current strength over here?

Right so this is the perfect example of volume equivalence principle which we have started earlier. When we look at antenna problems later on in the course the reason why this is called an equivalent volume current will become clear ok. But you can see that there is something in the volume which is being summed over right. So, it is the volume equivalence principle that is used over here any other thing that sort of stands out for you over here? Which one we will take more or less time to evaluate?

Student: First one.

The first one obviously right; so surface is faster than volume, because every time you change your observer location r' I have to recalculate this whole integral right because when r prime changes this Green's function changes therefore, the whole integral also changes right. So, I have to evaluate over a surface whereas the first one I have to evaluate over a line right; So that is the reason the surface is faster than the volume.

But there are situations where a for example, you have a heterogeneous object you have no choice you have to reasons.

Student: Hello sir.

Yeah.

Student: Which Green's function you will be use?

This Green's function for k_0 vacuum.

Student: For vacuum.

Vacuum why because when I massage these two equations into this equation what I get is a k naught squared on the left hand side. So, this I should get the operator equation to be the exact same between what I have and Green's function. So, since its k_0 here its k_0

here. k_n we have already said is the function of space. So, this is going to be of not much use for me in calculating the Green's function right.

So, now, let us imagine that you know where you know in the business of calculating radar cross sections for aircraft ships and so on ok. So, you have got this code you have written which calculates the field at any point in space.