Computational Electromagnetics
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Radar Cross-Section
Lecture – 9.1
Definition of radar cross-section

So, now let us get into some standard terminology for defining this radar cross-section ok.

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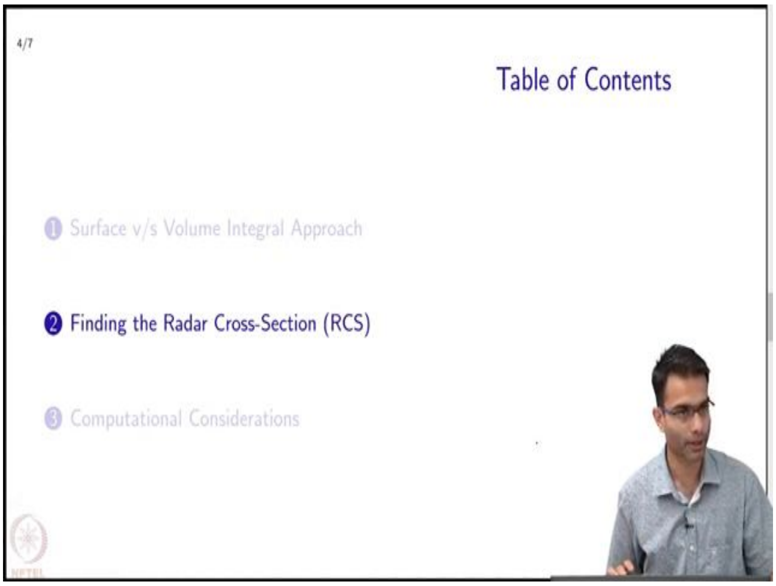

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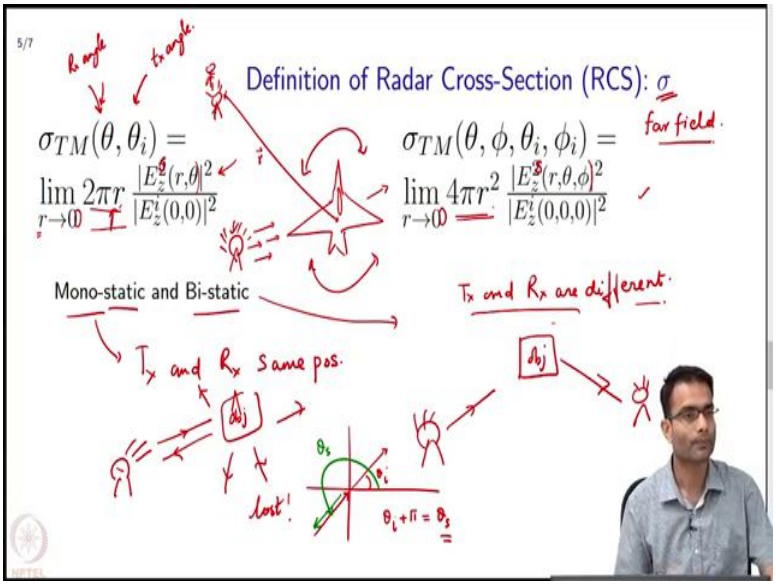

So, first of all the very definition; the symbol reserve for it is usually sigma and so, what I have over here there are different definitions in 2D and 3D and the definitions are very intuitive. What does it say? So, it is the limit that the observation point is very far away.

The field observed so, $E_z{}^2(r,\theta)$ right. So, there is a missing bracket here all right at some point normalized by the incident field at the origin ok; and you will see there is a $2\pi r$ over here and we will we will come to why the $2\pi r$ is over there ok. So, I mean the case is very simple. So, let us say that I have an aircraft like this alright and I am standing somewhere far over here r right, this is the distance r from the origin this is the value r this is the vector $\hat{r}$.

In the case of 2 D this is the definition, in the case of 3 D this is the definition; (I am missing a bracket here) inside of a $2\pi r$ I have a $4\pi r^2$, but the idea is the same find the field far away. So, this field far away is also called far field right. So, this is the far field intensity normalized by the incident field intensity at some point and the easiest point is the origin ok.

Now, so there are in the world of radar there are two different kinds of sort of quantities that are calculated; so, let us let us stick to this example which I have drawn over here. Let us say that this is one antenna over here, let us say this is you know air traffic control and this is another air traffic control over here. So, both of them have radars. So, the first guy is sending a wave with like this to the aircraft and this aircraft is going to when the radar wave bounces on the aircraft it is going to get reflected in which direction?

Student: Same direction.

One answer is same direction.

Student: It depends upon the.

It depends on the surface.

Student: Right.

Right, but given a general aircraft or a general ship, what direction will the wave gets scattered into? What is a common this is a common sense question.

Student: It should come back.

I should come back that is one answer any other answer?

Student: Away.

It will go everywhere and why depending on the shape of the object.

Right so, if there is some facet which is angled at a certain way the wave will go off at the same time. So, will there be any wave in this direction? There could be because think of this the spot size of the of the radar being when it hits let us say the tail finger something, some diffraction can happen it can go in basically all directions. So, the scattered field energy is going everywhere, in 2 D its going over 2 pi, in 3 D its going over the entire 4π you can say.

But what do you have that comes back to you? So, there are two different imaging modalities right. So, monostatic means transmitter and receiver same position; that means, this air traffic controller sends a wave and the object is over here let us just call it object, this receiver the; obviously, because its standing at that position it only gets only collects what is coming back to it.

So, everything else that is going what happens to it? It is lost. So, this is called a mono static mono means just one right; so, one transmitter and receiver at the same location. So, this is a monostatic radar. The other term for it is I mean the other kind of imaging modality is bi-static where you can have like I drew above over here. So, one transmitter object over here and let us say another receiver over here right.

So, I have Tx and Rx are different. Yes good catch. So, this should be scattered not squared the squared is already outside. So, this is scattered field far away either in 2D or 3D right. Question? But is the idea clear. So, these are just definitions I am giving you.

Student: So, is it r tending to 0.

Oh there is another typo here excel for field this should be $r \to \infty$ yeah. So, you notice that in the definition of the RCS the angle at which the receiver is here. So, this is the Rx angle and this is the Tx angle ok, so, just to test our understanding of the monostatic case. So, monostatic means I means the 2D case. So, let us say the incident wave is coming like this. So, what is the angle it makes? Let us say we will just continue this is θ_i all right. Now it reflects back from the radar and it goes into which direction the?

Direction is reversed. So, the reversed direction is going to be this right. So, what is the angle it makes? This is the angle it makes theta s let us call it scattered angle. So, is there a relation between θ_i and θ_s? Can I write that $\theta_i + \pi = \theta_s$ all right. So, this θ_s is what I will plug in over here into the receiver angle into the receiver part of the RCS definition ok. So, in a monostatic case it is already fixed.

I have theta $\theta_i + \pi$ and θ_i those are the 2 arguments to the RCS angle. Similarly in the 3D case I have to think about θ and π ok. So, this is just definition all right. So, should not be anything too complicated here.

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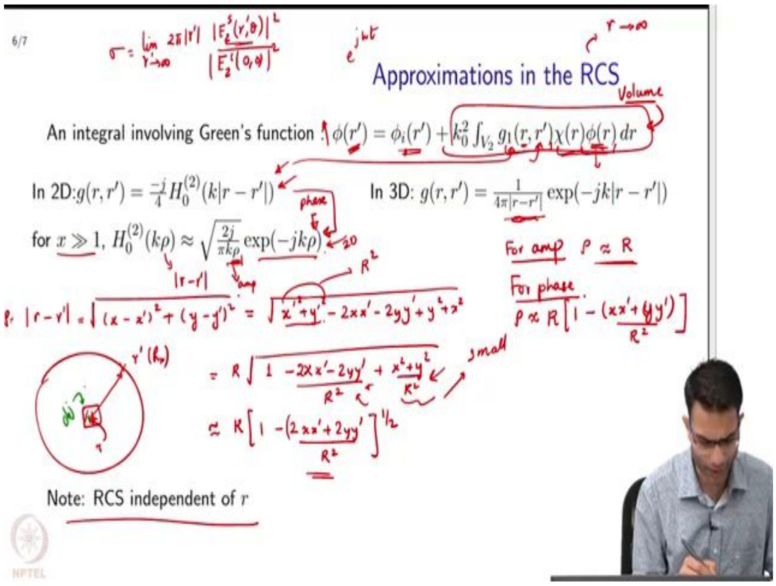

Now, let us let us get into some of the details ok. Now the details we can take either the surface integral or the volume integral approach it does not matter ok. So, I have just taken the volume integral approach here this is the field observed at a distance r'.

r' is some general distance right this is what I have. Now those of you who have programmed these Bessel functions Hankel Hankel functions will know; that to evaluate Hankel function or a Bessel function is slightly expensive operation. When you ask MATLAB to compute Bessel function Hankel function, it has to do an internal calculation to give you that number. It is not like asking the computer calculate x^2 or $2 \times x$ ok. Now when people start calculating the RCS of aircraft they do not want the value of the RCS at one angle.

Typically you are we want to know what is the overall behavior. So, in 2D you would calculate the RCS over and entire 2π radians ok. So, you have to calculate at various several different positions r' you have to put your receiver and calculate what is the scattered field at that point. So, there are some approximations that we can use; remember the RCS definition is $r \to \infty$ right now back in the very beginning of Green's function we had actually come across this approximation for very large x this Hankel function look like this.

It became like a plane wave in 2 D; because there is a $e^{-jk\rho}$ and this also what helped us to fix the which of the 2 incoming or outgoing waves we should keep remembering because my sign my time convention was $e^{j\omega t}$ combined with this I got an outward travelling wave right. So, we have seen this expression before now when I now let us just try to look at the challenges. Supposing I take this most general form of Hankel function and put inside over here.

What will happen at every time I change my receiver position r prime I have to evaluate this Hankel function; obviously, right and I am giving you further information that that is a slightly expensive operation. So, to reduce the cost of doing that, we should use this approximate form which is more at which is accurate as we go to larger and larger rho ok. So, rho y here is simply this distance $r - r'$ ok. So, if I plug this expression back into this that is I mean that is what will help me evaluating this is much cheaper than evaluating Hankel function right.

Because, this is an exponential and it is divided by root rho right; so, this is going to be much more computationally efficient without any real loss in accuracy. So, now, you can see one thing. This was in 2D. In 3D this was our Green's function with no approximations right. So, if you go back now to the definition of your RCS. So, there is a $2\pi r$ over here right and this first term over here is scattered squared. So, if I sort of you can imagine that this $\sqrt{\rho}$ is going to come out of this integral approximately and when I square it what will I get?

I will get ρ and ρ cancels of with this r over here as a result what we will get is that the RCS is independent of the distance at which I am it does not depend, it is not that it depends on the value of it is 1 kilometer or 2 kilometer I do not get different answers each time because that distance has cancelled from the numerator and denominator over here. So, that is the reason why the definition of RCS has this 2π over here and in the case of 3 D I have $1/r$ so to speak right. Remember r is going over the object whereas the r' is tending to infinity.

So, this term basically will approximately come out of the integral sign over here; when it comes out I have a $1/r$ in the denominator I have to square it take its mod squared I get a $1/R^2$, and that cancels of with this R^2 over here ok. So, this is actually what motivated the definition of RCS because you are reporting a quantity that does not strictly speaking depend on the distance to the object.

So, it is useful when I say this is the RCS of an aircraft you do not have to ask me in written, at what distance right it is more like a universal way to characterize this object. On the other hand if I had removed this $2\pi r$ or $2\pi r^2$, then I would have to every time tell you at what value of r and I will get different values of the RCS. This is one way of getting rid of the distance dependence ok.

So, one small sort of further simplification we can do over here which is a very common approximation in electromagnetics. So, this r minus r prime how do we write it? I can write it in terms of Cartesian coordinates like this ok. So remember, this integral r is over the object either surface of volume and r' is far away correct. So, I can further write this as. So, and this what I can assume is that since my object this is let us say my point r' right that is where I am evaluating this, is somewhere far away right that was the definition of RCS r trending to be very away from the object and my object is something over here ok.

So, this $x'^2 + y'^2$ I can take it to be equal to sum capital R squared let us say ok. So, I can write this as $\sqrt{R^2}$ now I can take this common from here. So, $2xx' - 2yy'$; divided by R + $x^2 + y^2$ divided by R^2 that is a.

Say again yeah this should be R.

Student: Coordinately taken from.

We took R. So, this term over here is equal to R^2 . So, I pulled out a R from this square root sign over here all right. So, this R came out from here and this is the expression that I got. Now from these terms which can you think I can ignore anyone term from here given that r lives over here and r' lives outside. So, from these 3 terms this is squared dependence on x and y and this is a linear dependence on x and y right. So, this term is actually going to be.

Student: Small.

Small right so I can approximate this as R and I can ignore this term which is going to be very small and alright ok. So, the common approximation that is done is for amplitude. So, I have this rho term over here this is rho is appearing in the denominator in the amplitude and in the phase the 2 places where it is appearing right.

Student: That is this is should go into the receiver.

Yeah that is the location of the receiver right. So, this is Rx yeah. For the amplitude term what do you do? It turns out that if you make a further approximation of ignoring this term also. So, you replace ρ as approximately equal to R ok. So, you ignore this term also and for the phase term you write $\rho \approx R$ into we can use the binomial expansion over here and you will get a. the twos cancelled off. So, this is a very common approximation that is used and that gives you very good accuracy also.

So, I mean the objective of doing all of this, it looks a little tedious to begin with, when you get the hang of it, it is basically to simplify evaluating this expression right. So, then what are the unknowns inside this? It will just be this χ_r and ϕ_r which you have calculated already which you know already and this g term becomes very simple it is just a plane wave term and this rho over here we were taking into be constant and when we square it ah. So, it becomes $\sqrt{R}$ and in the definition of the RCS there is an R sitting outside anyway. So, it cancels off right. So, this is so, these are approximations that we can do.

So, this is the amplitude term and this is the phase term. So, the approximations that we make in the amplitude term and the approximations we make in the phase term.

Student: Phase.

Yes, r' is the location of the receiver.

Student: So, we are definition r' should tend to infinity.

Yeah in if I have yeah. So, here I have chosen the prime coordinate. So, my definition of RCS will be.

Student: r' it will be.

Here it will be r' right 2π. So, we will make this to be.

Student: Why are we choosing the port machine into the same ρ?

You can, but it I am giving you even you can choose the same approximation for both, but it turns out that doing this gives you almost the same accuracy right. So, this is

$$\sigma = \lim_{r' \to \infty} 2\pi |r'| \, |E_z^s(r', \theta)|^2 / \ |E_z^i(0,0)|^2$$

That is because, I had the field here in terms of prime coordinates and this definition over here assumed it was in term of unprimed coordinates, just a small change in convection ok.

In the definition where does theta I come into the picture? While it will matter right because a scattered field that I calculate, will depend on the I mean when I solved these equations to get my ϕ, I needed the value of the incident field otherwise you going to a solved the system of equations.

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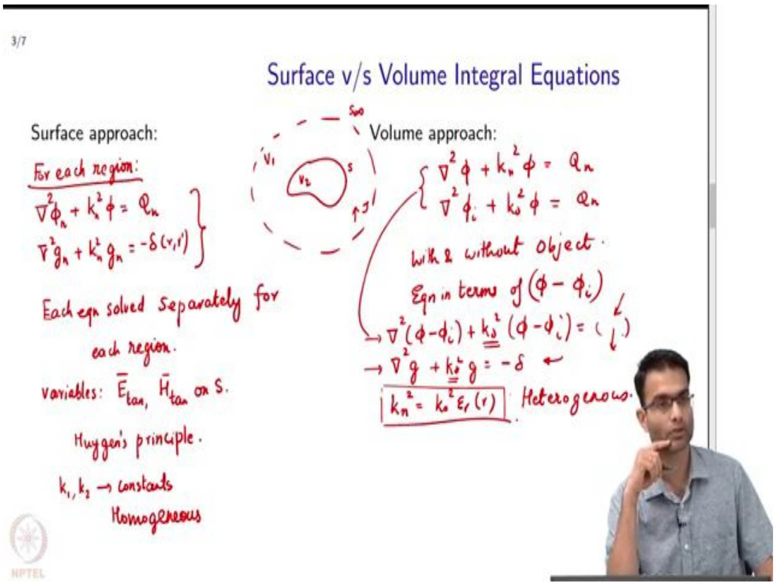

If you look back over here at are equation, the incident field appeared over here right. So, depending on the value of the incident field my current inside the expression got set. So, implicitly the solution that I get depends on which angle I am illuminating the aircraft form right. So, that is why it is a function of θ_i.

Student: Hello.

This is the. So, origin is here inside the object.

Student: At.

May there is an object which is centered on the origin it is approximately in that region.

Some yeah; so, this is the object.

Student: So, you would expect to the origin r does not go for bit r prime.

Yeah r does not go far because the finite object, r' is where my receiver is and the definition says take r' to infinity.