Computational Electromagnetics
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Introduction to the Finite Element Method
Lecture - 10.03
ID Basis Functions

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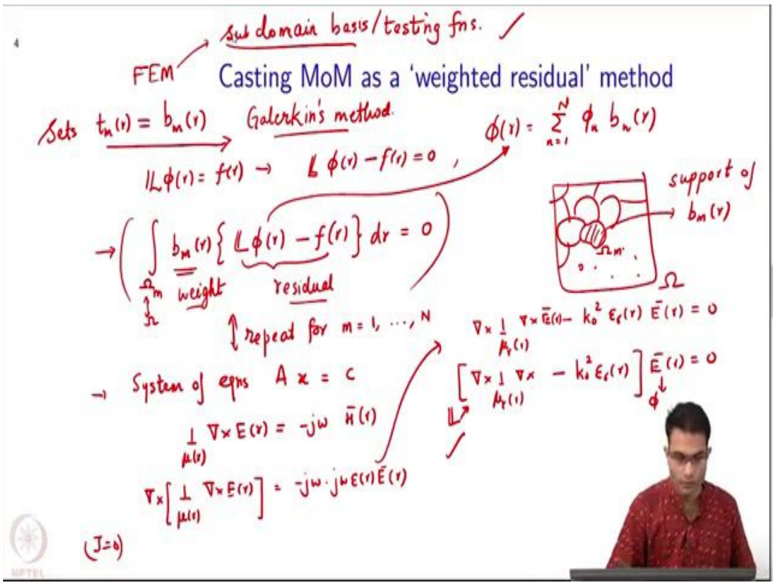

So, we will continue our discussion of the Finite Element Method which is the new method which have been studying and the main idea that we want to sort of start building on is this weighted residual idea right. So, residual was the difference was you know the difference between $L\phi$ and f. Ideally, we want it to be 0 and the testing function which is or the weighing function we are in an average sense enforcing it to 0 over the domain and we repeating this over all of the little-little sub domains ok. So, now like we will have a brief discussion of what are the kinds of basis functions.

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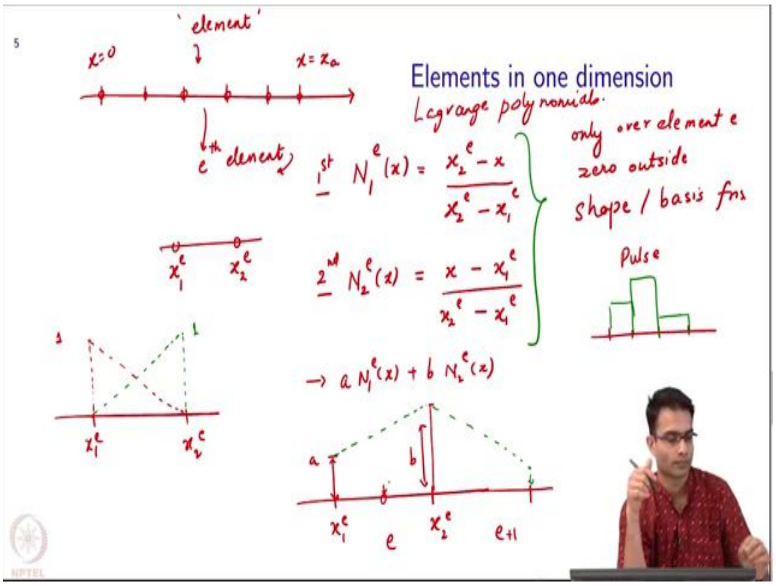

So, now coming to one dimensional basis functions so, this is really the easy part. What is the most simplest way of discretizing a 1D problem?

Student: Line segment.

Line segments right. So, that is exactly what will do we have a line segment language like this and we discretize it like this. Some x is equal to 0 to x something like that right and we have to decide what are the unknowns for this problem ok. So, the unknowns that we will say for example, can be the value of the function at these nodes.

Now, let us take one segment over here, so, this segment over here. So, this in FEM we do not call it a segment we call it an element ok, but when you hear the word element in 1D you should just think of line segment. So, we will call this element. So, that is how hence the word finite element method lots on lots of finite elements make up this whole method. So, let us zoom this guy over here right.

Now, for each element I will define some kind of basis functions ok. So, this is going to be slightly different from what you have seen so far. For example, when we were discussing finite difference method is just the unknown are the values of the function at various nodes and we form our equations out of those right. Now what we will do is, those unknowns are still the same, the value of the nodes, but we will create a kind of a smooth function that take that interpolates between these node values.

So, let us define something which you have already seen before ok. So, N_1^e ok so, each element has two basis functions and let us see what they look like. So, N_1^e I am going to write as $x_2^e - x$. So, this is x_1^e, x_2^e ok. So, this is the eth segment or the eth element which I am zooming in and then showing over here and each element in 1D will have how many nodes? 2 nodes: a left node and a right node ok.

So, those the left node we are calling as with subscript 1 and the right node I am calling subscript 2. So, if I take any element so, there is a kind of a local numbering every element has a local 1 and 2 left and right. So, this function if you look at this is the first basis function.

So, this is defined only over element e and it is zero outside. So, you should not use this function outside the element itself, this function is defined only over the element. So, this is N_1^e, what about N_2^e? It is sort of an anti symmetric definition

Student: There are two basis functions.

There are two basis functions for an element and these are those two basis functions. So, let us see why do we need 2? So, what is so, this is the first one.

Student: This is one element consists of the area segment as well as those points will.

An element consists of the segment and the segment I mean element is a segment and that segment has 2 nodes on it and the element also has two basis functions defined on it. In the FEM literature these basis functions are also called shape functions. So, you will hear this word being use alternative. So, if you read shape function do not get confused it means basis function. So, shape or basis functions means the same thing.

Now we will get into why we are using two of them it is a very beautiful geometric property only. So, if I plot the first basis function what does it look like? So, let us say that this is my x_1^e over here this is my x_2^e over here. So, this first shape function what is its value at x equal to x_1^e at this node what is its value?

Student: 1.

1 right. So, its value is let us say 1 over here and at x_2^e?

Student: 0.

0 right. So, it is a and it is a straight line in between because it is linear right. So, this is what the first shape function looks like what about the second shape function? The other way. So, it is 0 at x_1^e and then goes to a value of 1 at x_2^e right. Now if I tell you now if I take $aN_1^e(x) + bN_2^e(x)$ what will this look like?

Student: Trapezium.

It will look like a trapezium right. So, first of all this function over here what degree of polynomial is it?

Student: First, first.

First degree.

Right both of them has a linear x term. So, overall it is linear and what are what are the values of this function at x_1^e, the value of this function at x at x_1^e what its value?

Student: a.

a right. So, this is a what is it is value at x_2^e?

Student: b.

b right. So, if this is b and we know it is linear right. So, this function is basically right. So, by constructing by using these two shape functions as your basis functions what you are allowing? You are allowing your unknown function which you will express in terms of these guys to be a smoothly varying linear function between the 2 nodes right.

So, the advantage of this is that for example, if I let me remove this b from here and put it on the inside adjacent to this element. So, if this is element e this is element e plus 1, adjacent to it is going to be I mean the next element next to it. This element will also have two shape functions and it will; obviously, since 1 node is common to it right. So, the next kind of if the function can go like this let us say depending on the value at this point correct whatever it is value is.

So, by constructing this it is piecewise linear and what is the nice property that you can see coming out of this, the function given by the green curve is.

Student: Continuous.

At least continuous; previously if you remember when we had done the pulse functions what will what did we do? We broke it up like this again into segments and here we said pulse 1 has some value. So, I wrote it like this pulse 2?

Student: Some other value.

Some other value so, even though the function is continuous by forcing it to take to be on a pulse basis I am forcing it to be become discontinuous. So, I am introducing a numerical error into my in the way I am expressing the function itself; Instead if I use these kind of functions I am doing a little bit better.

Student: Sir, this is the only thing we did not try with our function.

Exactly so, this is what we studied earlier in the case of triangular pulse right that is right. So, FEM does not consider pulse basis at all whereas, you could use pulse basis in the earlier methods and you got a little bit you got higher errors. So, like I said this is nothing new. In fact, these polynomials we all know what are they called?

Student: Lagrange.

Lagrange polynomials; so, Lagrange polynomials are central to FEM in as many dimensions as you want; everything is sort of built out of these Lagrange polynomials. You can even go fancier than this, why stop at linear you can also?

Student: Go to quadratic.

Go to quadratic, but what you need more there I mean what more do you need then? Will your unknowns increase? What about derivative? So, this function that I have shown here is not difference is not I mean it is.

Student: Differentiable.

Differentiable at the node point that is fine, but in between it is (Refer Time: 10:16).

Student: One more point (Refer Time: 00:17).

Exactly so, if I want you to introduce a quadratic approximation to this then I would need one more node over here. So, your number of unknowns?

Student: Increase.

Increases ok, but the element stays the same. The element is this, now you can say in this element I want how many shape functions, how many unknowns? Right those are called higher order elements. So, this is the most basic some people call it first order element some people call it zeroth order element depending they have different conventions. So, we will call this a first order element.

Student: We increase the number of (Refer Time: 10:49).

Yeah so, if I introduce ok. So, the question is if I introduce one more node over here I can make my element smaller and have linear piecewise linear between them or I can keep the element size the same and now use a quadratic in between. Both possibilities exist and both will give you different quality of solutions ok. You get by forcing the function to be what do you call quadratic you may be able to enforce better properties on the solution apart from continuity you may also be able to get differentiability right.

So, those are the tradeoffs. The math gets a little bit more complicated the higher order of polynomials you impose over here ok. So, I mean this was something that you have already seen before not very surprising ok. So, any questions on this before we go to two dimensions, if you have two nodes can you say that again if you have 2 nodes.

Student: Polynomial can still be.

Can you use with 2 nodes you can only fit a line through it. What you are thinking of is Gauss quadrature which is the integral of a function over the given interval, here we are not considering a integration; we just talking about given these 2 nodes how many degrees of freedom are there? Right so, if I have two points I can only fit a line through it I cannot do anything better fine ok. So, this is in one dimension.