Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Introduction to the Finite Element Method
Lecture - 10.06
Weak form of 1D-FEM Part-2

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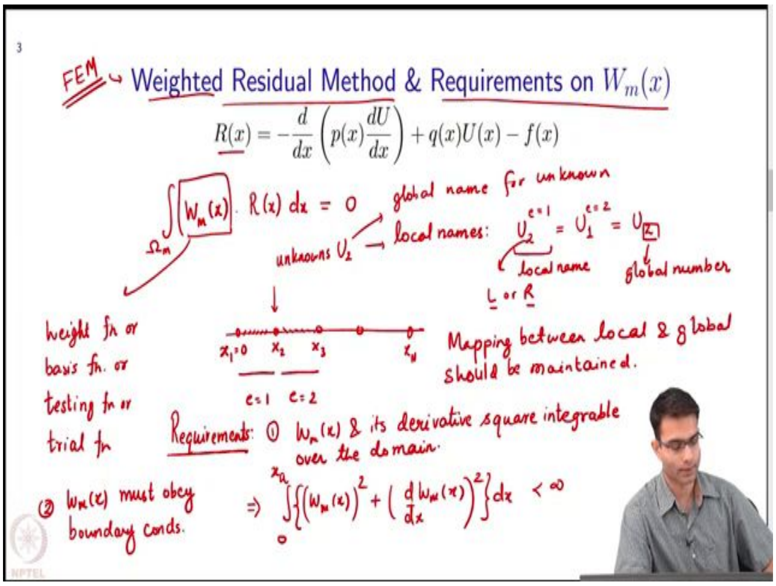

So we will continue our discussion of the FEM. So, what we have seen so far is this idea that FEM is a weighted residual method and we discussed some very simple requirements on the weighting functions; basically that the functions and their derivative should be square integrable over the domain and the other is that they must obey the boundary conditions ok.

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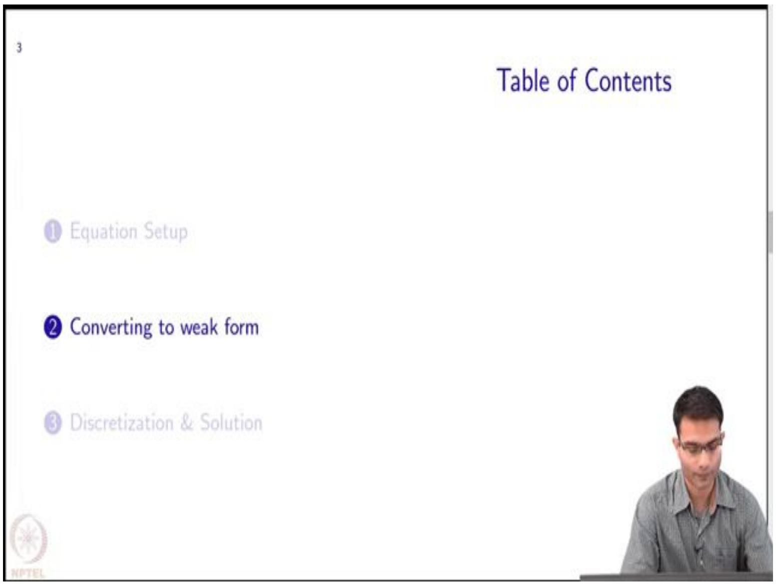

So, with now with this having being established let us look at one of the very key steps of this FEM which is converting to what is called a weak form.

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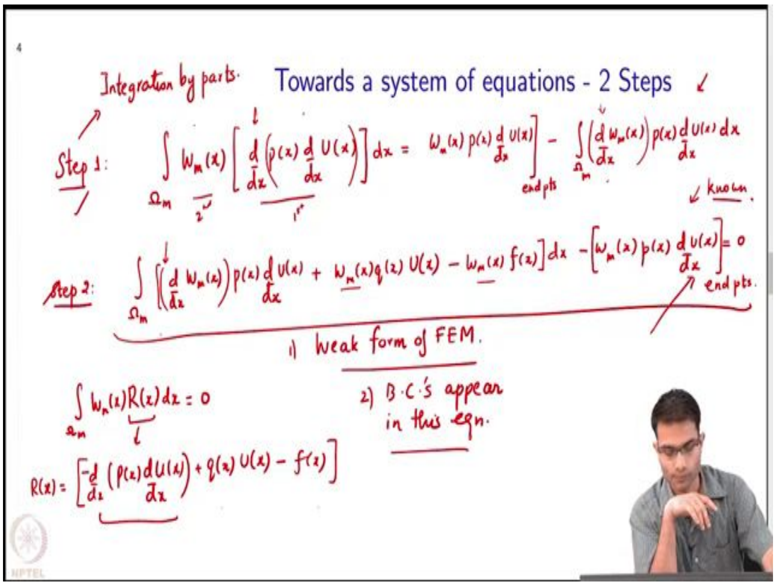

So, what we will do is we will we will take consider 2 steps which will help us to accomplish this thing ok. So, let us go back to our equation. So, if you notice our equation over here. Let us look at the this one term over here. So, there is a it seems to be a double derivative right and there is a W function next to it right. So, let us look at step 1 ok. So, I have a

$$\int_{\Omega_m} W_m(x)[\tfrac{d}{dx}(p(x)\tfrac{d}{dx}U(x))]dx = W_m(x)p(x)\tfrac{d}{dx}U(x)|_{end\ points} - \int_{\Omega_m} (\tfrac{d}{dx}W_m(x))p(x)\tfrac{d}{dx}U(x)dx$$

Second derivatives of course, given the choice you would rather not had deal with higher order derivatives better you deal with lower order derivative.

So, it is d/dx of this whole thing. So, is there any tricks that comes your mind what about integration by parts. If I do integration by parts I have to integrate one function and I already have the differential inside there. So, this seems to be tailor made for integration by parts with this guy as the first function and this guy as the second. So, what happens of this term?

So, integration by parts is what has given as a simplification; now this looks very simple to do in 1D its actually very simple to do in higher dimensions also except that instead of a d/dx will have a ∇ operator that is the only difference and we look at that in 2D when we come to 2D. But, notice apart from ease of integration what has actually done you have in some sense if you look at this expression you have transferred one derivative from U onto W right.

This derivative which was acting here on $p(x)$ and $U'(x)$ has now been transferred onto W and that gives a nice kind of symmetry because you know before we even get into you know that in FEM W_m and U_m there going to be chosen from the same family right its Galerkin's method the testing function and the basis function is the same. So, I have to only worry about W_m and its derivatives no higher order term is coming over here.

Because only du/dx not d^2u/dx^2 ok. So, this was step 1 and the step 2 is very simple substitute this into the original equation whatever trick we did integration by parts we substituted back in. So, when we substitute this we can write this as let us look back at our original form over here. So, first term we have dealt with now I have the second term over here and this third term over here ok, all of it weighted by the weighing function W_m ok.

So, let us write this how. So,

$$\int_{\Omega_m} [(\tfrac{d}{dx}W_m(x))p(x)\tfrac{d}{dx}U(x) + W_m(x)q(x)U(x) - W_m(x)f(x)]dx - [W_m(x)p(x)\tfrac{d}{dx}U(x)]_{end\ points} = 0$$

All the minus signs are fine now right and this I should write a little bit more carefully this is at the end points, the first term in integration by parts

So, everyone has followed the trick that we have done is just integration by parts that is all. Now this longest looking question that we have written over here this is what is called the weak form of FEM why is it called the weak form any guesses what is weak about it?

Student: The order of a.

Not the order of derivatives it is simply weak because its I am not enforcing at every point in the domain I am enforcing it in a weighted sense over some interval. So, it is called the weak form that is all. But this is the form that we will use to convert into a numerical solution the once I have transfer the second derivative into a first derivative ok. So, that is a first thing this is the weak form that is a first thing I want to mention.

The second thing is notice that this one equation has inbuilt into it information about the entire problem not just the differential equation, but the boundary conditions also how is that?

Student: The end points.

The endpoints right; so, I have so, the boundary conditions appear in this equation how do they appear? Because I have to substitute the value I mean I need to know: what is the behavior of U or dU/dx at the endpoints.

And that get substitute over here. So, typically this is a term which is known it has to be. Boundary conditions in the 1D case simply means end points x_0 and x_n whatever right. So, this that is another sort of nice aspect about this equation that the differential equation and the boundary conditions both have appeared into this in one shot fine.