Computational Electromagnetics
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Introduction to the Finite Element Method
Lecture – 10.8
1D Wave Equation: Formulation

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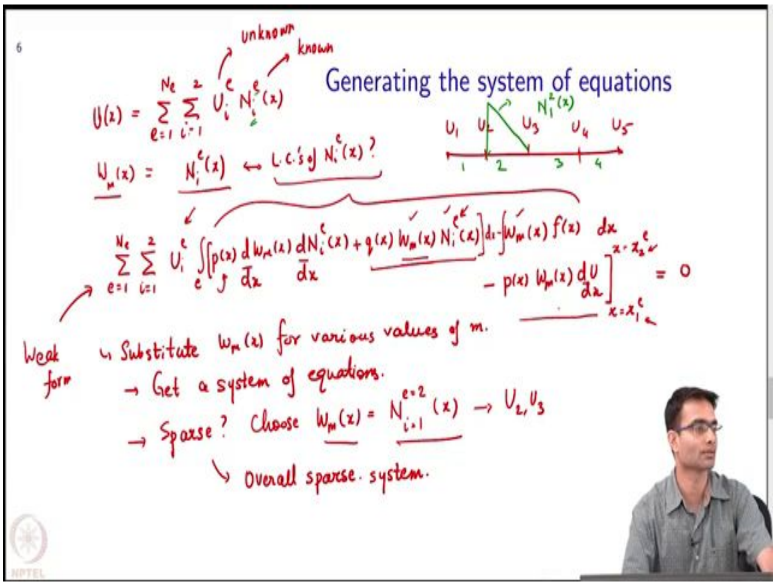

So, so far what we have done is we have sort of written this equation abstractly ok. So, we will take a quick look at a 1D problem ok.

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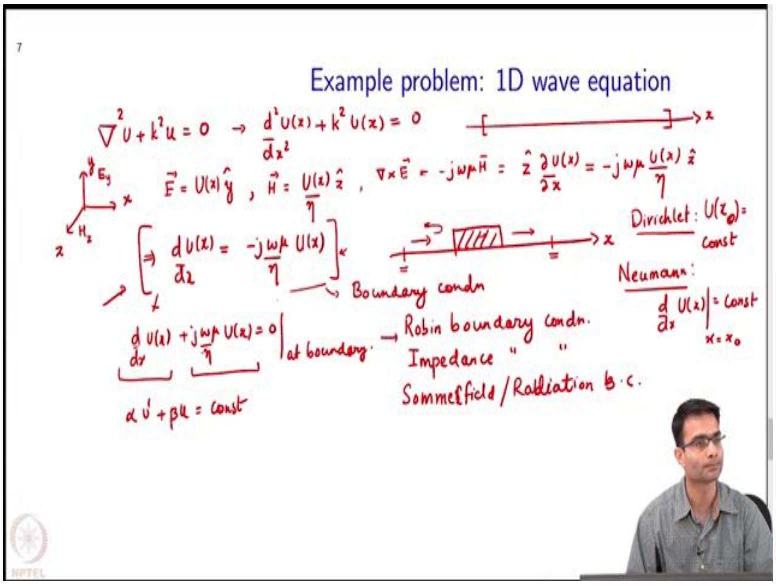

So, this 1D problem we will take the wave equation ok. So, the wave equation we all know is simply $\nabla^2 U + k^2 U = 0$ and this is a 1D equations. So, this is simply ok. So, in right now we would not go into actually solving the system of equations, because I want to tell you a little bit give you give you an example of a boundary condition. So, far we have not spoken about boundary conditions right.

So, this is the 1D problem; that means, the wave is going like this along the x direction right. So, let us take a coordinate system over here right. So, x and let us make this y and z and I am going to say that electric field is along the y axis right. So, which way if the wave is travelling along x, we will take H to be along which direction.

Student: z.

z right your right hand curl rule right. So, $E \times H$ is giving me the direction which the wave is moving ok. So, E I am going to take as my variable over here so, $U(x)\hat{y}$ fine. We already know what a plane wave looks like right. So, can I without any detailed calculations write down the form of H there? Is a relation between the magnitude of E and the magnitude of H right there is a η.

Student: (Refer Time: 02:15).

Impedance of free space right. So, $H = U(x)/\eta\hat{z}$ which is impedance of free space 377 on whatever $\sqrt{\mu\varepsilon}$ whatever here z hat right. So, this is my E this is my H ok. Now there is another way of writing this which is another way of getting E, I mean the original way of getting H was what take $\nabla \times E$ right and when I take $\nabla \times E = -j\omega\mu H$.

So, when I simplify this equation over here $\nabla \times E$ there is a only one term that survives which is a $\frac{dU}{dx}\hat{z} = -j\omega\mu(U/\eta)\hat{z}$. This is simple 1D wave equation we know all know how to solve it right you. So, from here what can we write? We can see it is all along $\hat{z}$. So, we can convert it into a scalar equation.

What do you get? $dU(x)/dx + j\omega\mu(U/\eta) = 0$. So, if I ask you: where all is this equation valid? It is valid everywhere take any point x this equation should be valid because basically it is giving me the relation between E and H right. if you look at the basic meaning of this equation is that the ratio of E to H should be η that is all.

Now this actually turns out to be a boundary condition because let us say that I have I want to get a wave equation the solution to the wave equation now not in free space, but I have a let us say a block of glass over here. My wave is travelling like this, some of it will get reflected some of it will go through multiple reflection will happen something will come out and I want to solve this numerically right. And so, what I will say I will make this the ends of my computational domain.

Now, this equation that I have written over here, it should be valid at this point also and at this point also right. So, this is in fact, the boundary condition on yeah it is a boundary condition I should not write boundary condition on boundary it is a boundary condition all right. Now you have seen so, far in previous courses you have seen Dirichlet boundary conditions what would Dirichlet boundary condition say? What is Dirichlet boundary condition?

Student: It is constant.

Constant right so, $U(x_0) = constant$ right what is the other boundary condition?

Student: Neumann.

Neumann what is that? $\frac{d}{dx}U(x)|_{x=x_0} = constant$ ok. What was this boundary condition look like? Is it Dirichlet is it Neumann if you want you can rewrite this as $dU(x)/dx + j\omega\mu(U/\eta) = 0$ at boundary is it a Neumann boundary condition?

Student: (Refer Time: 06:45).

Neumann boundary condition is would mean that this term is constant, but this if I can also call it a Dirichlet boundary condition because if I look at this term. So in fact, this is neither of the two this is a third kind of boundary condition its called a Robin boundary condition some people call it a another set of people call it a impedance boundary condition and another set of people will call it a.

Student: Sommerfeld.

Yeah, there are many names Sommerfeld some people call it radiation boundary and so, on, Every community rediscovers this boundary condition and gives their name to it ok. So, circuits people quality impedance boundary condition, scattering people call it radiation boundary condition, but the idea is because you have a linear combination of the function and its derivative being set to 0 right. So, in general what is it? Its $\alpha U' + \beta U = constant$ this is a kind of boundary condition that we have.

So, you notice if you recall over here this in the weak form the endpoints term has a dU/dx right that is why this is going to be useful I will substitute this dU/dx in terms of this other term over here we will see how it leads to simplifications. So, when we will continue subsequently with trying to solve this equation using the FEM framework clear so far.