Computational Electromagnetics
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1D Finite Element Method
Lecture - 10.09
1D Wave Equation: Boundary Conditions

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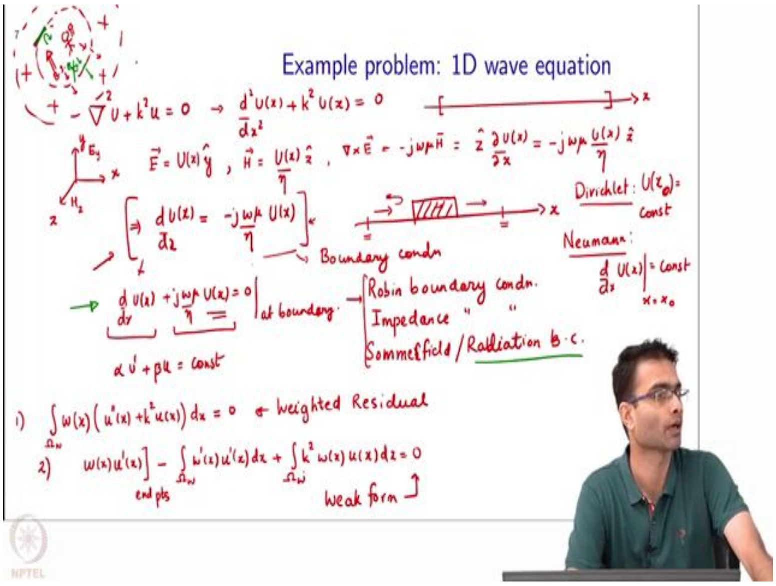

So continuing our discussion of the 1D Wave Equation, into which we applied the FEM. So far we have not come to the FEM we have just discussed the wave equation and shown you the boundary condition that is required right. And, we say that this is a new kind of boundary condition not Dirichlet not Neumann, but a combination of the two which is called the Robin boundary condition or Sommerfield radiation condition. So, this boundary condition is very important in almost all electromagnetic scattering problems. In 1D, it looks very simple. So, what is it saying? Derivative spatial derivative of the field is proportional to the field that is what it is saying right.

What will happen in 2D or 3D? This derivative will be get replaced by curl and I mean the del will appear over there right. So, we have to keep in mind that. So, do not get I mean you should not get overwhelmed when you see an expression in multiple dimensions, when you look at it in lower dimensions this is basically what it is gradient of field is proportional to the field itself. And why is this sort of very important is because, what is it saying this is the condition obeyed by a wave in free space any points in free space agree.

So, when we are so, take a general example let us say that I have an antenna let us say that is your WiFi antenna over here and you have some human being over here, you could even have a mobile phone in his hand his or her hand. And we are wanting to study a very important problem, all of us know that you know mobile phone radiation is a possible cause for concern health issues.

So, we want to simulate how much is the field that is entering the tissue, how much is the maximum power level and all of those things. So, when you go to solve try to solve this problem in a simulation you have to end the simulation boundary somewhere right. So, you can for example, say that this is going to be my simulation boundary ok. So, let us say in this problem you want to study two things; one is how close should you be to the base station to be safe; you know that base stations towers they put out a lot of power.

So, if you spend too much time in front of them there is a health risk which empirically has been observed. On the other hand someone carrying the mobile next to their hand and talking for extended periods of time; mobile produces less power, but if you keep it held for a long time that is also a possible cause for cancer. Now where do you to solve this simulation you need to terminate the boundary somewhere otherwise you I mean you can simulate infinite space. So, let us say you decided to terminate it here. Now this boundary that you have put over here it is; is it a real boundary or a hypothetical boundary?

It is a hypothetical boundary in the actual problem it is not there, but I had to put it there. So, whatever field is let us say emanating from this domain when it encounters this boundary what should happen in real life?

Student: It will keep going.

It will keep going this wave will keep going because this boundary does not actually exist, but mathematically when I end my boundary over here what will happen?

Student: (Refer Time: 03:27).

It should go to 0 or well if it goes to 0 there is a problem.

Student: (Refer Time: 03:32).

Because, for a field a field a physical field cannot abruptly go to 0.

Student: Exponentially decaying.

Exponentially decaying, you want the field to exponentially decay before the boundary?

Student: At the boundary

At the boundary you want a field go to 0 ok. So, let me give you an example this is a.

Student: Sir before the boundary it should (Refer Time: 03:51).

Before the boundary it should exponentially go to 0.

Student: When it reaches there in people.

So let us see the problem that let us say that this part of the boundary over here is made out of this green part. Supposing I made it out of perfect metal so, what will happen to a field when it hits there? It will bounce there by the field goes to 0 right which is what you wanted. So, it is that desirable or undesirable? Field went to 0 like you wanted at the boundary, but is it desirable or undesirable?

Student: So, it is going to be desirable.

So, it is undesirable because by putting a metal what you doing, even though the field at the boundary is going to 0 the way does it is by generating an equal and opposite field. So, what will happen is that math that by putting a metal over here that is going to be a reflection from there so, which was not there in the physical problem. So, setting the field to be 0 on the boundary is not the correct boundary condition. What is the correct boundary condition? Nothing should get reflected back in, if that happens and I am happy whatever be the magnitude as long as it is does not come back to me.

Student: Yes.

Then I am happy then I am recreating the real world problem as far as possible right. So, what I want is that this field should come over here and just go off; there should be nothing that is going back right. So, at this boundary over here if I impose this condition what does it recreating in some sense? This is the condition that is obeyed by a way of travelling unhindered unobstructed.

And, if I am able to impose this condition at that interface it will try to recreate the same situation that the field is because it looks at this is yeah the field is suppose to go on because that is the equation obeyed by a plane wave that is travelling unbound right. So, that is why this, that is why the name radiation boundary condition has come from the scattering people, because for them this is a radiation boundary condition meaning its simulating a radiation that is going off to infinity.

Circuit's people call it impedance boundary condition for different reasons ok, but the physical interpretation is this I do not want to field to come back from a hypothetical non-existence surface ok. So, this is the motivation will carry to 1D 2D 3D whatever how many of a dimensions we are solving a problem. So, your question is that is this boundary condition dependent on the boundary location?

Student: Then that its boundary.

So, does it look at the boundary condition does it depend on the location of the boundary why because?

Student: U of x.

Because it depends on $U(x)$; so, at whatever x I am I put this right. So, it is not independent I am not setting unlike the Dirichlet and Neumann boundary conditions where I am setting $U(x) = const.$ or $dU/dx = const.$ that seems to not depend on where I am. But on the other hand this boundary condition it adapts to where I am because $U(x)$ and dU/dx both are together being set to 0. So, you can impose it at 1 meter, you can impose it at 3 meters from the object. What is the disadvantage of imposing at a 3 meters?

Student: Extra.

Extra computation for no need right; so, supposing this was your boundary I can as well say oh I want to make this boundary sure do it no problem your for no good reason you are simulating all of this extra space. Extra space translates to extra physical space translates to larger computational domain, larger size of matrix, more computation time more RAM everything for no good reason there is no there is nothing, there is no exciting physics happening in this region why bother.

So in fact, one of the considerations in designing boundary conditions is how can I get my mathematical boundary as close as possible to the object to minimize computation time yet retaining accuracy. So, that is a very common requirement yeah. So, in 1D this is an exact boundary condition ok, it is exactly satisfied. We will see that this boundary condition and higher dimensions has an inaccuracy in it ok, but we will come to that alright. So, let us continue with are wave equation.

So, I have told you about the boundary condition and now what remains is to take a differential equation and convert it into the FEM form right. So, let us start that. So, what is the first thing I have to do? Take the differential equation and consider find its residual right weighted residuals. So, weight residue integrate ok. So, let us put the integration sign first ok. So, let us call the weighting function w and what is the residual in this case?

Student: (Refer Time: 08:51).

So let us just write it as $U^{''}(x)+k^2U(x)-0$ that is no right hand side right now ok. So, this is your this is just the definition of FEM take your waiting function, take your residual integrate. The w's are going to be the weighting functions for each segment I will define. I am I have not yet substituted the basis functions ok. Next step; step 2 we discuss this previously what do I do next?

Student: Converting to weak form.

Converting to weak form so that was basically integration by parts right. So, what do we do now, there is a double derivative term over there right. So, that can get converted how? So, one integration is going to help me with that. So, this will become

$$w(x)u'(x)|_{end} - \int w'(x)u'(x)dx + \int k^2 w(x)u(x)dx = 0$$

So, this was weighted residual and this is and as we discussed earlier this weak form is useful because it involves only first order derivatives. I have not put the limits of integration, but they correspond to the domain of w that is implicit ok. So, now let us proceed further, let us now put in now what does it that we have to choose? We have to choose w's and we have to choose u's correct.