Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering (EE)
Indian Institute of Technology, Madras

Finite Difference Time Domain Methods
Lecture – 11.12
Introduction to FDTD

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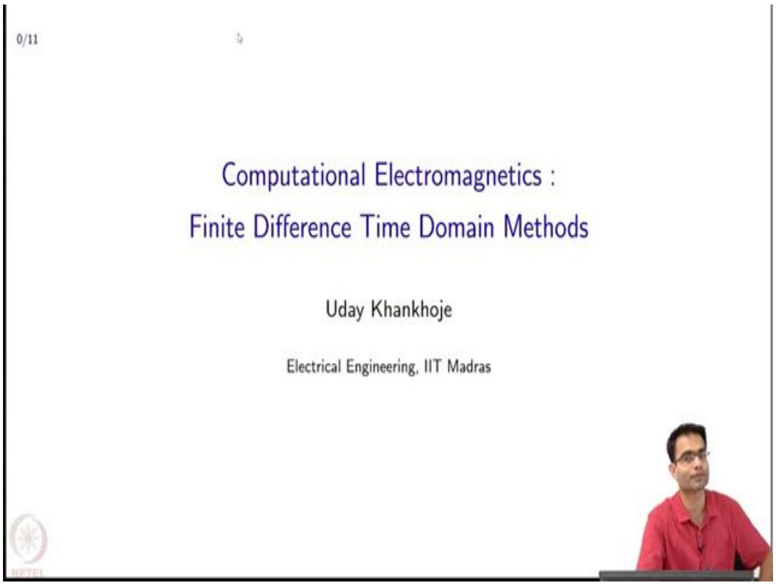

What we will start now is what is called the Finite Difference Time Domain Method alright; many of you may have heard this word in the past right. It is in fact, one of the most popular methods in Computational Electromagnetics right.

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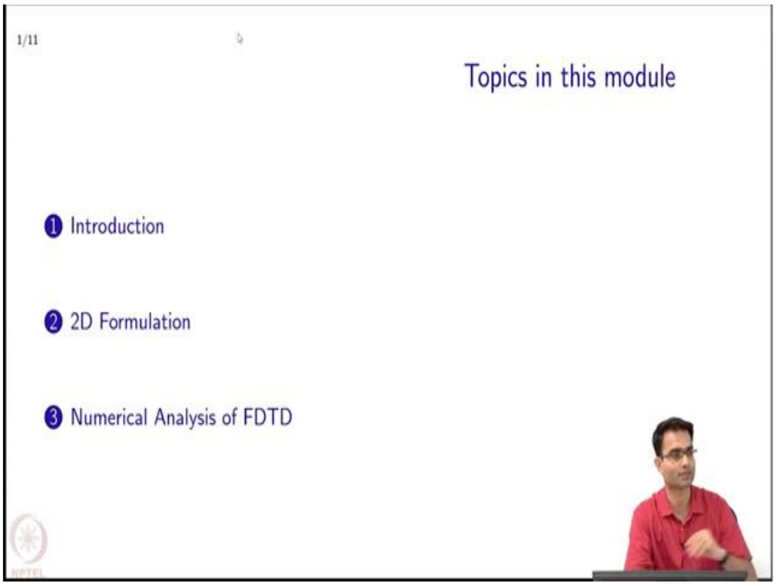

So, this is what we are going to roughly do. Let us get motivated and give some history about this problem and we will deal with the 2D formulation and then look at various aspects/numerical analysis of this method ok.

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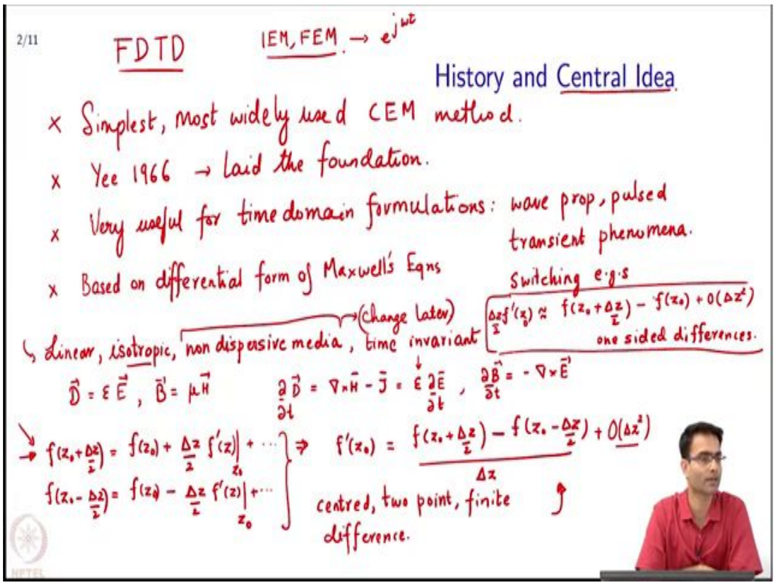

So, let us start with introduction. So, as before, we would like to start with a little bit of sort of contacts and history. So, FDTD that is the short form that we will be using throughout, Finite Difference Time Domain ok. So, we will see why we called that finite difference time domain.

So, a few very general points first. So, it is the simplest and also the most what can we say, most widely used not just in the electromagnetics community, but even in the optics community this method is used extensively ok. So, if you ask when was the original formulation right. So, there was a landmark paper by the person called Yee 1966 which laid the foundation.

And in fact when you look at this method and the paper as formulated by Yee you would think; oh I could have done this, you know it is really that simple. In fact, the central idea of FDTD can be told to you in this one slide and that is it the rest of it are all just details ok. So, before we come to that what we have seen.

So, far before we came to FDTD was we looked at integral equation methods and then we looked at FEM methods. So, you can ask one question now why do we need yet another method. In integral equation methods and finite element methods what was how did we deal with time? Did we deal with time at all? Right so, in these methods what did we assume? e to the j omega t right. So, we can say harmonic form of the field was assumed.

If I wanted to have some time-dependence on the problem, what would I need to do? Supposing my source my let us my radar sources not single frequency, but it's ultra wideband thing and I wanted to use IEM or FEM then what would I do?

Student: Solve for each frequency.

Solve for each frequency and then use Fourier transforms to get answer right. Now, if my system electromagnetic system actually is very wideband lots and lots of frequencies are there then you can see that this both IEM and FEM will become very tedious right because not just you have to do computational n you also have to do then Fourier transforms and all and over very large data sets. So, that becomes tedious. So, instead this FDTD is directly formulated in the time domain ok. So, very useful for time domain formulations, can you think of some examples where you might where the problem is naturally post in the time domain?

Student: (Refer Time: 04:09).

So, if I want to see how a wave propagates; so, if you so, a wave propagating like $e^{j\omega t}$.

Student: Thing like a from a source.

From a pulse right so, like you want to pulse is traveling and you would see how it goes in rights that can be one thing right. So, wave propagation, but we want pulsed form right because continues waveform is already $e^{j\omega t}$ right. So, pulse from. So, like a radar pulse has a finite time duration right. So, it cannot actually be approximated as $e^{j\omega t}$. So, that is one case. So, you want to see in real time, how is the wave propagating any other situation?

Student: Transients.

Trans.

Student: Transient phenomena.

Transient phenomena. Yes transient phenomena is very important. Example? What is the most transient electromagnetic phenomena that you can think of natural phenomena?

Student: Switching

Student: Switching.

Switching yeah switches yeah ok. So, that is the more down to earth example, switching. There is some circuit, there is some switching happening. So, that is your EMI EMC as a quality electromagnetic interference problems. There is some you know gate that is operating at some switching rate and because of the switching it is producing some electromagnetic waves that we something naturally in the time domain.

Natural phenomena lightning right very sudden burst of electromagnetic energy in terms of frequency content it will be all frequencies, but in time domain is localized a very small point and time. So, that might be one example where time where time domain formulation is better. So, you can say switching in general ok. So, as it turns out the number of applications where time domain is the more natural domain is actually more than frequency domain.

So, that is why this FDTD is a more widely used than the other two. It is also a little bit of a brute force method as we will see. So, let us see what is the basic idea. So, unlike the integral equation methods the FDTD is based on the partial differential equation form of Maxwell's equations ok. So, it is based on differential form, not integral form. So, let us make the assumption we will start with a very simple assumption of a medium and as we go in subsequent modules we will generalized it ok. So, I am going to assume a linear medium linear medium means.

Student: (Refer Time: 07:18).

Well that is what you said was about isotropic. So, it is an isotropic medium.

Student: (Refer Time: 07:27) epsilon (Refer Time: 07:29).

Epsilon is equal to?

Student: Some epsilon is scalar.

Is a scalar, right. The permittivity and permeability are scalars. Let us make one further assumption that it is a non dispersive media. What is non dispersive, what is the word non dispersive tell you?

The permeability permittivity rather that does not depend on frequency if you take this feature away you cannot have a rainbow right. So, we are making some very strong assumptions, but we will relax these as we go we want to just give the central idea first then we can generalize it.

So, over here under this assumption; so, I can write down that my displacement field can be written as just a scalar times electric field and B also can be written as $\vec{B} = \mu\vec{H}$ in particular this assumption we will change later, but it takes a lot of work. Now with this having been said what happens to our Maxwell's equations in we have seen all of this before right.

So, the first equation will become curl of this $\nabla \times \vec{H}$ write it in this form first let us write it. So, that becomes that is simply $\nabla \times \vec{H} - \vec{J}$ and that simply becomes $\varepsilon\partial\vec{E}/\partial t$.

$$\partial \vec{D}/\partial t = \nabla \times \vec{H} - \vec{J} = \varepsilon\, \partial \vec{E}/\partial t$$

That is your first equation and then the second equation is:

$$\partial \vec{B}/\partial t =- \nabla \times \vec{E} = \mu\, \partial \vec{H}/\partial t$$

Student: So, what this isotropic mathematically?

Isotropic means good question. So, what is isotropic mathematically means isotropic means that.

ε so, physically what does it mean that no matter which direction you go the wave experiences the same medium.

Student: Yes.

Ok; that means, ε is a scalar number ok, but if it is anisotropic then the wave experiences different property let us say different propagation speed in this direction that direction. So, epsilon over there becomes a tensor that.

Student: So, epsilon what like what the linear different.

How is linear different ok? So, linear means that epsilon does not depend on the strength of the field itself that would make it a nonlinear medium for example, does the refractive index depends on the intensity of light in general know right if I take a glass it has a refractive index. So, might a what beam as sent through it, but in optics particularly they are there are non-linearity. So, where the refractive index begins to depend on high over powers of electric field then you call that a nonlinear medium ok. So, all of what I said. So, linear anisotropic means I can write ε as a number.

Student: So, I thought isotropic meant like ε would not be a function of x y.

Student: So, that it varies with space.

That is heterogeneous or homogeneous. If ε becomes a function of space then we call it a heterogeneous medium ok, but I can have a homogeneous medium that is anisotropic. And the most common example in the most common use of this is in optics what is called birefringent media. They have different propagation constants in different directions even though. So, bulk medium and that comes from the atomic structure of the material there is an asymmetry in the atomic structure.

Student: So, maybe in the displacement like $\vec{D}$ one direction of that.

The yes, that is also possible yes. So, what is saying is that the displacement field can depend on electric field in different directions. So, that will that is taken into account by making ε to be a tensor ok. So, those are all sorts of complications. So, one more thing I should add over here will be time invariant why should I add that such that ε can come out of the d/dt right.

So, we are assuming that the medium is static. So, when we began to list out the assumptions we can see that there are so many over here right all right. So, lest actually write this equation on the right hand side make some space again. So, we had $\partial\vec{B}/\partial t = -\nabla \times \vec{E}$ ok. So, so far I have only told about the differential form and now what is the what is the central ideas that you use in finite differences time domain it is basically built on Taylor's theorem ok.

So, Taylor's theorem will tell us that you know $f(z_0 + \Delta z/2)$. Let us take this we all know this f of z naught is a first term next term is $\Delta z/2\, f'(z_0)$ right and higher order terms. What is the, in the Maxwell's equations that you have seen, what is the order of derivatives that are coming in space and time?

Student: First order.

First order right. So, we will just stick retain or discussion to first order for now and by symmetry this will be become $-\Delta z/2$ right.

So, if I wanted an approximation for the first derivative using Taylor's theorem, what can I say? What will Taylor's theorem can Taylor's theorem be used to give an approximation for first derivative of a function yes what should I do?

Student: Subtract.

Subtract these two equations. We subtract these two equations, what happens? $f(z_0)$ cancels off right and then I am left with $\Delta z/2$ is common and its their 2 times right. So, that becomes

$$\Delta z\, f'(z_0) = f(z_0 + \Delta z/2) - f(z_0 - \Delta z/2)$$

So, this implies that $f'(z)$ now I will just write it as a $f'(z_0)$, but you should know that this means $f'(z)$ evaluated at $z = z_0$ right. It's understood by now. So, this is your $f'(z_0 + \Delta z/2)$. What is the error term over here? There is an error term over here which I am ignoring when I subtract. What order will it be?

Student: Square.

Square. It won't be cube because there is a, I am dividing everywhere about Δz right subtract these two equations and then divide by Δz. So, I am left with because if you see the second term on both of these is going to have a $(\Delta z)^2$ which will get cancelled off the next term will be delta z cube when I divide by Δz I am left with the error term of order $(\Delta z)^2$.

So, this is these sort of the central idea of finite difference time domain replace derivatives by differences what kind of differences are there? Finite differences so, differences between finite values in space. So, the z can be space z can be time which is up to me right. So, this is the technical name given to this kind of a difference. So, it is centered two point finite difference ok. So, this is the basic idea. So, Maxwell's equations Taylor's theorem this is all that you need to develop the entire FDTD method. There are other concepts here about how to deal with, you know, anisotropic media and heterogeneous media and all of that right, but those are all details the main conceptual work is going to come from this part over here. So, notice one thing over here. Supposing you solve a problem with a certain discretization. If I half the discretization, what happens to the error?

Student: Increases.

I have a problem I have solved this using the details we will see, but I have solved at using replacing these finite differences and now I say I want to more accurate solution. So, when I am reduce make Δz make it $\Delta z/2$ what happens to my error? Increases or decreases?

Student: Decreases by a factor of.

Decreases by a factor of?

Student: 4.

4 right. So, that is in that is the reason why you use this centered two point difference because if I just look at the first equation over here. If you look at the first equation that alone gives you an approximation for the first order derivative right I can write from here the first order derivative in terms of $f(z_0 + \Delta z/2) - f(z_0)$. I can do that. But what happens to the error term?

Student: Order delta order delta.

Order Δz right. Do you follow? Ok, so let us see. So, let us maybe we will write it over here. So, I can also right $f'(z_0)$ approximately as just using the first equation alone. So, $f(z_0 + \Delta z/2) - f(z_0)$ and what is next? So, we will keep this $\Delta z/2$ by here ok. So, I am just rearranging the term and what are the other terms over here order of $(\Delta z/2)^2$. Now I can divide this expression on both sides by?

Student: (Refer Time: 18:56) order del q right.

No I have just rewritten this first equation. The first equation had second order terms and so on which I am ignoring. So, when I divide this equation by $\Delta z/2$ what happens the order becomes I mean the error becomes of order Δz right. So, what, so the approximation now if I make my discretized if I reduce my discretization by a factor of 3 my error also reduces by a factor of 2. So, the other scheme which I derived over here is centered two point difference is better because halving the discretization makes the error go down by a factor of 4 right; so, this.

Student: The error.

The error term is what I am talking about how does the error behave as a function of the discretization.

Student: So, the error is order of delta z square.

Yes, the error is an order $(\Delta z/2)^2$ in a centered difference, what I have written over here is called a one sided difference ok.

Student: (Refer Time: 20:03) why do get the delta z by 2.

Divide by delta z by 2. See I am trying to approximate $f'(z_0)$.

Student: Yeah. So, there right.

No start with this equation Taylor's theorem plain and simple.

Student: Yeah.

I am just rewriting here bringing the $f'(z_0)$ term on one side everything else on the other side I am not done anything so.

Student: Ok fine.

Now, I have divided by Δz ok. So, these inside the computational domain everywhere I use centered two point difference, but I am forced to use this one sided differences sometimes at the end of the computational domain because there is no point outside. So, there I may be forced to use a one sided difference ok. So, these are some of the compromises or tricks you can use given some differential equation.

Also, you notice that there is nothing very secret about Maxwell's equations here given any system of partial differential equations I can use this technique right. Nothing special about Maxwell's equations right. So this is the basic idea Maxwell's equations finite differences.

Student: (Refer Time: 21:09).

Yeah.

Student: You are going to replace E with all these (Refer Time: 21:12) etcetera.

$\vec{E}$, $\vec{H}$ wherever I see a derivative in space or time, I am going to be replace it by a finite difference that is the philosophy.