Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture – 13.05
Absorbing Boundary Conditions - 1D

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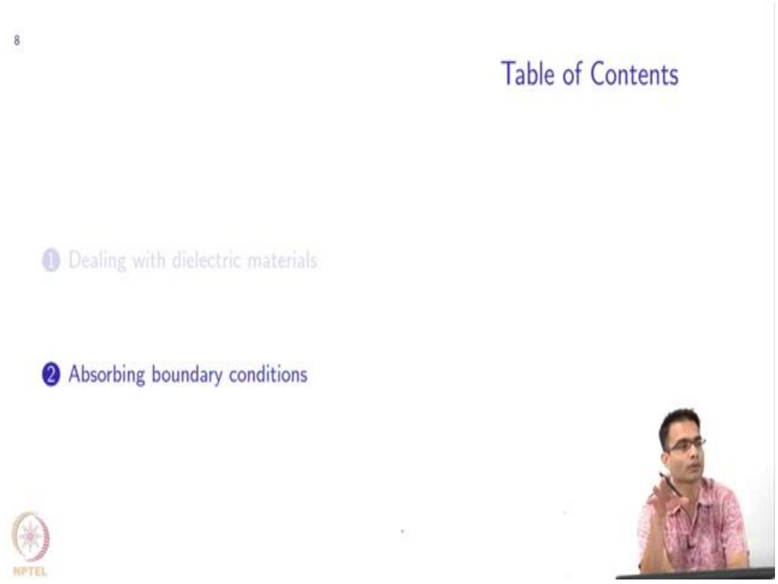

Alright so, now, let us move on the next module and we will look at is Absorbing Boundary Conditions ok. So, we are putting together all the building blocks, first we look at update equations space and time how do we update, then we put a material inside how do we handle a realistic material. Next thing is we need to terminate a computitional boundary somewhere, how do we terminate it right; so, absorbing boundary conditions.

These absorbing boundary conditions are the same as what we studied in the case of FEM ok, but now we have to put it in the language of FDTD ok. So, a little bit of this will be more like revision for those of you who have seen the FEM model ok.

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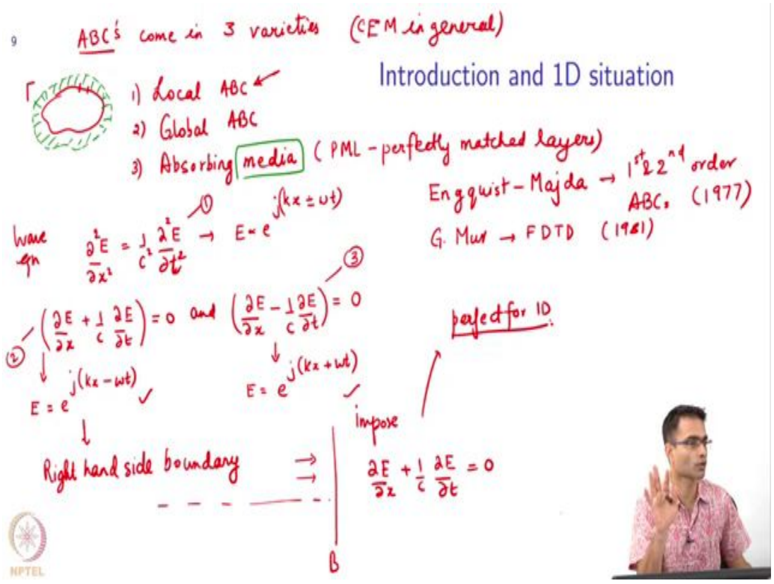

So, absorbing boundary conditions in general, so they come in so, we call them ABCs Absorbing Boundary Conditions ok. They come in three varieties and this is true of CEM not just FDTD ok. So, this is CEM in general ok. So, let us say that this is the computational boundary Γ. So, the first variety is what is what would be called local ABC ok. What is the meaning of a local ABC? Means that the value of I mean the way I implement the boundary condition lets say this part of the boundary depends on only the local values of the field; it does not depend on anything else ok, the word itself there is local ABC.

Then you saw that for example, in your FEM the special derivative dE/dx was replaced by jkE. So, the value of the field at that point only not anywhere else right, this is the simplest way to implement it. The second is a global ABC which means that to implement this boundary condition, I need to know the value of all the fields on the boundary actually you have already seen this. In the case of the integral equation method when you write a contour integral on the boundary, that one equation involves all the fields along the contour. So, that is also boundary condition because you are imposing it only on the boundary, but that is the global boundary conditions it depends on all of these right.

And, the third which is actually very popular in commercial software and all these are called absorbing media ok. So, the most popular of this is what is called PML: Perfectly Matched Layers which we will cover. So, here the philosophy is that you add another material over here, which has an absorbing property ok. So, I have to design this material which basically absorbs the wave and again it send it back so, no numerical reflection ok. So, these are the three things. So, what we will look.

So, we have actually seen local ABCs in FEM, you have seen global ABCs in integral equation methods and we will look at absorbing media little bit later, but the simplest ABC to implement is; obviously, local ABC this guy. So, we will once again implement this local ABC in FDTD and then we will go to absorbing media. So, few names a few historical name over here. So, Engquist and Majda are the names Engquist not the spelling right; Engquist and Majda. So, these guys are the scientists who gave the first and second order ABC's in the CEM literature ok. So, this is way back 1977 right and then you had a person by the name of Mur applied it to FDTD for the first time in 1981 ok.

So, FDTD is not that old a method also its only a it needed a upgrade in computing power to become popular right. So, its last 30 years that it has been (Refer Time: 05:19) and since then that has been developed more and more in a sophisticated ways ok. So, as before when we looked at this local ABC in the case of FEM what did we do? We had looked at the wave equation right.

So, we will again go back to our usual wave equation $\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$ and we know that the solution to this are of the form $E \alpha e^{j(kx \pm \omega t)}$ yeah we all know this is revision. Now let me try to write this equation or interpret this in different way ok. Supposing I write this equation as in the following way becomes the first order derivatives.

$$\frac{\partial E}{\partial x} + \frac{1}{c}\frac{\partial E}{\partial t} = 0 \qquad \frac{\partial E}{\partial x} - \frac{1}{c}\frac{\partial E}{\partial t} = 0$$

So, equation 1 was satisfied by both the solutions left going and right going wave right what about equations 2 and equations 3?

Do they also satisfied both left and right wave? No right supposing I take the first equation what E satisfies this? Right going wave right supposing, I take $e^{j(kx-\omega t)}$. The first derivative gives me plus jk the second derivative gives me $-j\omega/c$ and $\omega = ck$ so, I get it is satisfied right. So, this is satisfied and the second equation gives me $E = e^{j(kx+\omega t)}$ ok, but they do not satisfy the other equation ok. So, remember how we have derived the this wave equation I mean the absorbing boundary condition is in FEM.

We had taken we are taken up plane wave traveling in a certain direction and we had asked what is the condition satisfied by this wave equation, what was the relation between special derivatives and we had imposed that at the boundary? Saying that it should hold true for a wave traveling in vacuum, we will force it on wave traveling in any direction and we will suffer some error because of that. The same philosophy is going to be used now, let us say at a right hand side boundary of computational medium. So, right hand side boundary ok. So, there is that is a computational domain like this and this is my boundary over here ok. So, boundary and a wave travels like this and I impose the condition this one.

If I impose this condition and there is actually only vacuum over there, then right traveling wave will it see any numerical reflection? Here is a wave traveling like this. So, how is it traveling its traveling on the Yee cell on the Yee grid FDTD is updating every time I am updating moving the feels let us say a space and time, now hits this boundary and there is no computational domain to the right. So, I need to impose some boundary conditions and I impose this boundary conditions is that a good thing?

Yeah, but this is the right most boundary. At the right most boundary so, there are the scatterers are to always to a left of this boundary. So, what will happen here? Trivially good right if its 1 D problem there is no there is absolutely no issue over here, because this is the perfect boundary condition right. So, this is perfect for and that was also the case in FEM. In a 1 D problem it is perfect there is going to be no reflection because this is always true ok.