Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture - 13.06
Absorbing Boundary Conditions – 2D

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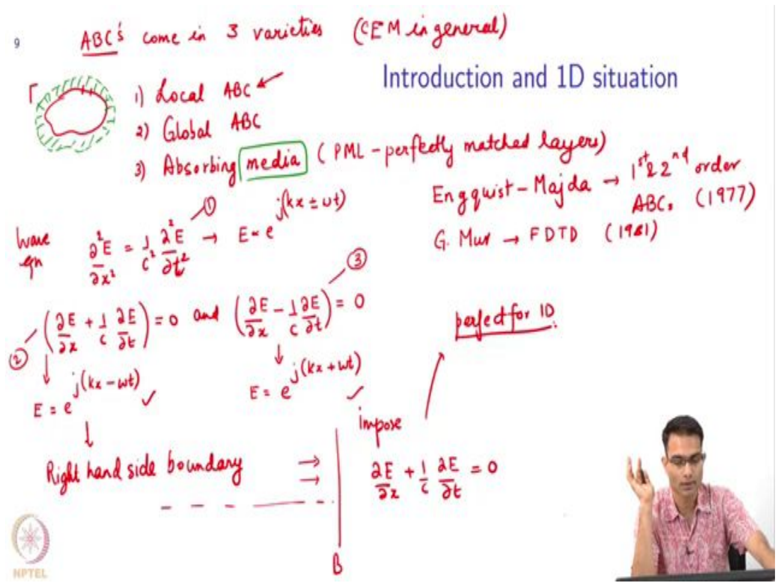

So this is the motivation for it.

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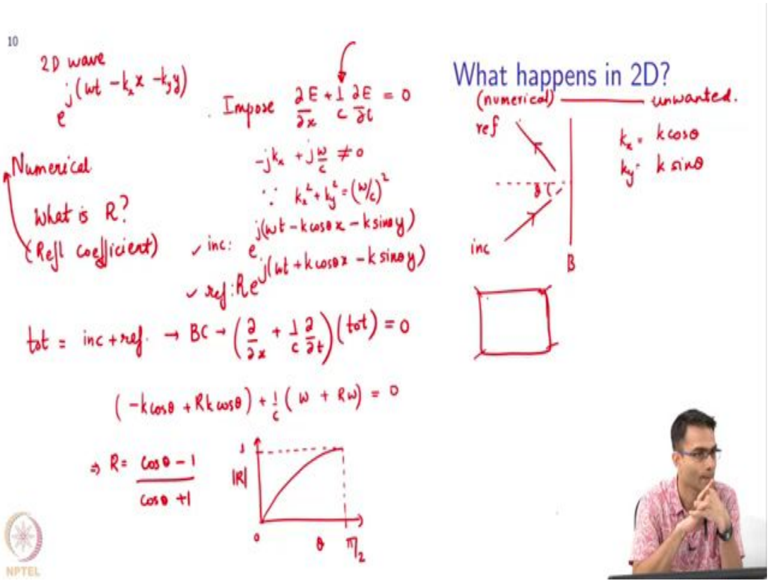

Now, let us see what happens in 2D, we already know what will happen actually, it will not be a perfect boundary condition but, let us make the let us take a discussion little bit further than last time and see and do you have some control over it ok. So, what happens in 2D right so, again it is something like this, I have some boundary over here and let us draw the an axis over here and let us say that there is a wave that is travelling like this.

What is the equation for such a wave? There will be a k_x and a k_y wave vector right. So, I can say that for this 2D wave, I will write $e^{j(\omega t - k_x x - k_y y)}$ right and now at this boundary I am going to impose, this boundary condition right, which was $\frac{\partial E}{\partial x} + \frac{1}{c}\frac{\partial E}{\partial t} = 0$. So, what happens now, is it going to be satisfied? So, the first derivative what will it give me, it will give me a $-jk_x$ and the second derivative gives me a $+j\omega/c$ right and this is actually not equal to 0 why.

Student: (Refer Time: 02:02).

Because, ${k_x}^2 + {k_y}^2 = (\omega/c)^2$. So, this equation is not valid. So, what happens to this wave, anyway this reflected there will be a numerical reflection right. So, the wave hits over here and this is unphysical numerical ok. So, this is a I mean unwanted reflection. So, so far what we had done at least in the case of the FEM discussion, we have said we live with it, this is what you have get right. This is going to be I mean it will give me 0 reflection as long as $\theta = 0$, any nonzero θ I am going to get a reflection, that is what we have considered ok.

But, now what we should try to do is actually calculate what is a value of this reflection, I should know right how bad is my error. So, let us see. So, let us ask what is R right. So, R is the reflection coefficient and this is remember the numerical reflection coefficient right. So, I can also write down k_x can be written in terms of cos and sin. So, what is it.

$k\cos\theta$ and $k_y = k\sin\theta$ ok. So, I have an incident wave over here and a reflected wave over here ok. So, what is the incident field, incident field is a wave travelling from bottom left to top right. So, I can write this as $e^{j(\omega t - k\cos\theta x - k\sin\theta y)}$, the reflected wave is going to be $Re^{j(\omega t + k\cos\theta x - k\sin\theta y)}$ ok. The total field is going to be equal to incident plus reflected right. The reflection coefficient is R, which I do not know I want to find it.

So, the boundary condition is going to say that apply this, this is what my FDTD will do, it will say ok. Whatever electric field is coming, I am going to impose $(\frac{\partial E}{\partial x} + \frac{1}{c}\frac{\partial E}{\partial t})(total) = 0$ and we know that this is not 0, as when the wave is travelling away from the normal right. So, so now, this gives this actually allows us to calculate what R is as a function of θ.

So, let us substitute it and see. So, there is a x derivative, there is a t derivative, there is no y derivative correct. So, in this exponential I mean this incident and reflected fields, the y term will just cancel off right yeah.

Student: It was actually (Refer Time: 06:11). So, that should be (Refer Time: 06:16).

The.

Student: (Refer Time: 06:20).

No. So, this incident yeah you are ok. So, slight confusion over here, this incident field do not think of it as the field that is coming out of an antenna or something, I mean it is not it is the field that is being incident on the boundary, which I should observe. So, this incident is actually a scattered field only and if this a boundary condition was perfect, there should not be any reflected field. So, that is what I am trying to say. Previously, supposing I took a wave travelling in this way right. So, then R was zero. So, this whole thing the whole sum was 0. Now, we want to use this to find out, what is the reflected field.

Student: That is the (Refer Time: 07:07).

No in FDTD again there are two formulations, total field formulation and scattered field formulation and depending on the formulation, we will apply the boundary condition to only the scattered field or the total field. So, do not mix up total and scattered field here with the incident and reflected, this is not a physical reflection, I am not put a material over there right, this is a numerical reflection ok. So, let us just apply this and see what happens, what will the first term give me so, incident plus yeah.

Student: (Refer Time: 07:49).

Yeah. So, this is actually not what the FDTD is going to implement, this is an analysis of the FDTD yeah. So, in the so, far what we have seen in the FDTD update equations, the E and H are total fields right. So, if for example, there is no there is there is scatter here and my boundaries over here. So, whatever fields are following on it are scattered fields right.

So, I am and I am going to impose this boundary condition on that but, what we are doing now is an analysis of what is the reflection due to this in perfect boundary condition. So, first term what does it give me. So, I am going to get $(-k\cos\theta + Rk\cos\theta) + \frac{1}{c}(\omega + R\omega) = 0$ ok.

So, this only variable here is R, which I do not know right. So, I can write down R from here,

$$R = \frac{\cos\theta - 1}{\cos\theta + 1}$$

So, if I plot this over here, if I plot $|R|$ over here as a function of θ ok, this is θ is equal to 0. So, what happens at $\theta = 0$? 0 starts from here, what happens that $\theta = 90$ yeah. So, $|R|$ will become 1.

So, this allows us to see ah, I mean this analysis actually does not depend on FDTD or anything, it is a general analysis, I have not included any update equations or whatever right, I am what I am asking in this, what is the effect of imposing this boundary condition on a 2D wave that is all, you could have done this in the FEM discussion as well. So, does this give you some insight into how to design a boundary condition, here the reflection is becoming 0, at $\theta=0$. Can I change that angle? If I wanted to minimize the reflection at some other angle is it possible, you have to change the boundary condition right.

So, in changing the boundary condition, what will have to what will happen? So, I mean we will do this later but, basically if I introduce some new term over here, some $\cos\theta_0$ or something over here, in such a way that this expression over here, cancels off at some desired θ_0 then, I can control this boundary condition. Supposing I know in that my simulation most of my waves are going to come at $\theta = 30^0$ for whatever reason then, I can change this boundary condition such that 30^0 gets absorbed perfectly, the rest will have some also, yeah.

Student: (Refer Time: 12:15).

Yeah you similarly you have to change it for the left.

Student: But (Refer Time: 12:20) left only.

Yeah no so, what.

And this derivation is correct for a in wave that is incident on the right boundary. Now, if I were talking about the left boundary, my boundary condition that I impose will not be this one right, it will be other boundary condition, the other boundary condition meaning this guy, equation 3. Will be equation 3 and I will do a similar analysis, to find out what is the reflection coefficient? You will get the same thing right. So, in general if my computational domain is something like this, I have different boundary conditions on each of these segments, if I use our right handed boundary condition on the left boundary, I will it is, I will not get even 0 reflection at $\theta=0$. So, do not make that mistake.

Student: (Refer Time: 13:14).

The reflection will get added and it will be equal to this.