Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture – 13.11
PML - Phase Matching

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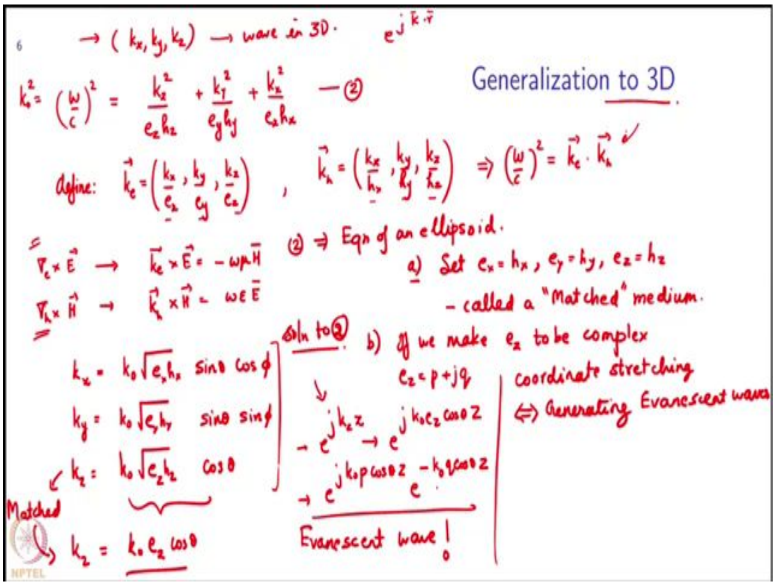

So, we continue with our development of the PML, and the concept that we have to keep in mind is that coordinate stretching is mathematically the same as generating a anisotropic medium therefore, it absorbs right it has evanescent waves ok.

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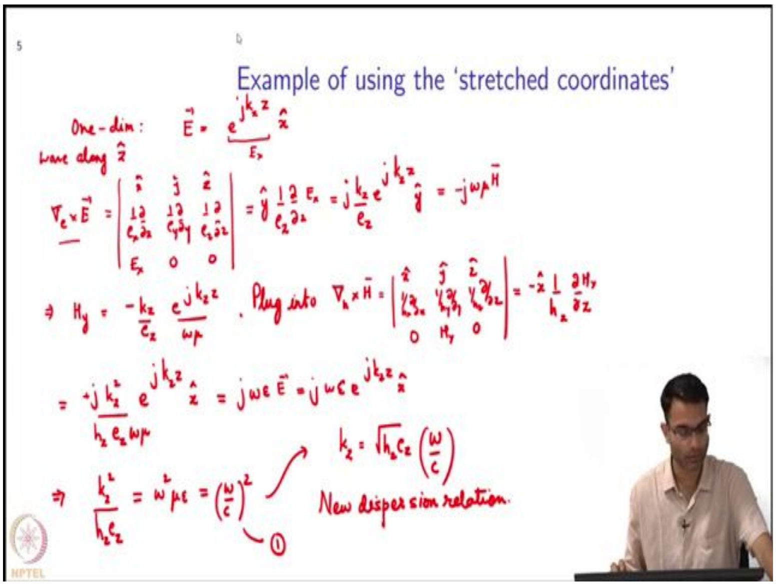

And the key idea was as we already mentioned over here, we have generalized this ∇_e and ∇_h operator, to now have a $1/e_x, 1/e_y, 1/e_z$ term, similarly for the ∇_h, $1/h_x, 1/h_y, 1/h_z$ ok. As a result of which I got a new dispersion relation which is over here and Maxwell's equations got redefined a little bit, the moment I put $e_x = e_y = e_z = h_x = h_y = h_z = 1$ I get back vacuum ok. So, that is the condition.

So, so far what we spoke about was just a simple one medium right in which it has some values of e_x, h_x, but what was our original motivation? We wanted to put in an absorbing layer in our computational domain such that the waves did not get reflected back. So for that we have to consider the interface between two media.

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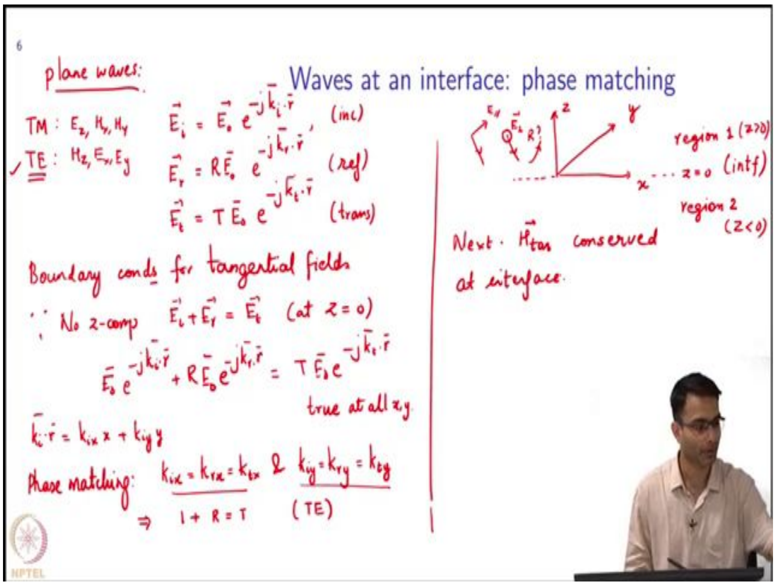

So, that is what we will look at next, so an interface between two mediums ok. So, how will we do this is let us say that I have a xy plane is denoted over here, z=0 is an interface ok. So, this over here on top is region 1, this is z=0 and then I have region 2 ok. And, we will do a simple thing we will consider a plane wave that is you know falling on to from medium 1 on to medium 2 ok.

So, for example, region 1 could be our computational domain, region 2 could be the PML layer ok. So, that is the set up and I have some wave falling like this getting reflected over here and my goal is that this reflection coefficient should be 0 in the worst case right that is what I want to design.

Now so, we will deal with plane waves ok; so now, when you looked at this kind of a problem I mean this is something that we have been seeing from high school, plane a planar interface and a wave falling over there that is when we have derived the Fresnel reflection coefficients for two different polarizations. We had called it back then we had called them parallel polarization and perpendicular polarization depending on whether the electric field vectors in the plane or perpendicular to the plane right.

So, I know that any arbitrary plane wave can be broken up into two orthogonal polarizations, but how I choose those orthogonal polarization that is up to me, one convention is parallel perpendicular, but that is not the only way, another way could be TE and TM. Those are also two orthogonal polarizations into which any arbitrary plane wave could be written I need not do perpendicular and parallel not exactly right. So, what was the difference between; so, for example, this was.

Student: Sir.

Ok; so, this is E which is coming out of the board.

So we will call this perpendicular polarization right, but and the other case E will be like this.

Right; so now, the there is yeah they are actually the same. So, there is a non-zero E_z or not that is yeah that is basically the definition of TM and TE whether non-zero E_z or not fine. So, it makes life simple also this is the two cases, but we will not use this perpendicular and parallel notation, we will just use TE TM ok. So, in TM what was our situation I had H_z.

Student: (Refer Time: 04:29).

H_z was 0 right; so, I had E_z.

Student: (Refer Time: 04:33).

E_z, H_x, H_y and then TE I had H_z, E_x, E_y and reality is a linear combination of these two fine ok. So, what we will do is we will start with TE and derive something, and generalize I mean the together TM is actually even easier o, so will start with this one TE polarization. So, plane waves in two regions; I will start with a plane wave incident from region 1 what kind of waves do I expect? A reflected wave, transmitted wave those are the waves that I expect right; so, this (Refer Time: 05:18).

So, $\vec{E}_i = \vec{E}_0 e^{-j\vec{k}_i.\vec{r}}$ ok, and $\vec{E}_0$ in this case is going to be a vector in the x y plane right because its E_x, E_y right. So, whatever it is we do not need to specify it. Similarly, I will have the reflected wave ok, so E reflected ok, so this is going to be. So, can I simplify the form of the reflected wave so, the wave vector I mean the electric field for the reflected wave, I can write it as $\vec{E}_r = R\vec{E}_0 e^{-j\vec{k}_r.\vec{r}}$.

They should be in the same plane right.

Student: Yeah.

$\vec{k}_r$ is the reflected wave vector there will be a simple relation between $\vec{k}_i$ and $\vec{k}_r$ which we will derive ok, and so this is reflected so incident. And then similarly I have a E transmitted again the way the plane should be the same, $\vec{E}_t = T\vec{E}_0 e^{-j\vec{k}_t.\vec{r}}$ is the transmitted wave. Now so, when I try to use these equations the what is the one thing to enforce given these equations how do I proceed further? Systematically what can we use, what is the principle? Tangential boundary conditions, which states that?

Student: (Refer Time: 07:31) E r will be.

Because, I mean in general that will not be true, but here it is true because.

Student: No (Refer Time: 07:37).

No not no current sources are of course not there, but remember the boundary conditions are for tangential fields.

Student: (Refer Time: 07:44).

Yeah exactly, so boundary conditions for tangential fields.

My fields are already tangential because there is no z component; right, if there were a z component that would be a normal component right. So, since no z component then I can write $\vec{E}_i + \vec{E}_r = \vec{E}_t \quad (at\, z = 0)$. So, then what happens next, so we will substitute these expressions inside. So, I am going to get,

$$\vec{E}_0 e^{-j\vec{k}_i.\vec{r}} + R\vec{E}_0 e^{-j\vec{k}_r.\vec{r}} = T\vec{E}_0 e^{-j\vec{k}_t.\vec{r}}$$

So, what will these I mean I can cancel out the $\vec{E}_0$ from here from all the, from both sides, what else what another further simplifications can I say from here. This is a bit of a revision of what you would have done in the your undergraduate electromagnetics course. What can I say from here, the propagation constant?

Student: What (Refer Time: 10:01) there can be (Refer Time: 10:03).

But, can I say that while looking at this equation. So for example, this equation; so, $\vec{k}_i$ in general will have a x,y,z component, this interface is happening at z=0, so $\vec{k}_i.\vec{r} = k_{ix}x + k_{iy}y$ So, I cannot actually say anything about the z components ok. What about the x and y components, what can I say? Because I mean what is the mathematical argument or reasoning from here; I mean you can interpret these as phasors.

Student: (Refer Time: 10:36).

These are phasors that are rotating.

Student: (Refer Time: 10:38).

So, right so, this should be true at z=0, but at every at if I take any point (x, y) it should be true right, because the choice of origin is arbitrary it is a plane interface that the wave hitting over here right, and the wave is not a it is not a point right its hitting the entire interface. So, this should be true at all x,y right, and you notice that this $\vec{k}_i.\vec{r}$ this is going to have terms like $k_{ix}x + k_{iy}y$ and these phasors are rotating in whatever in phase space and this expression should be true always.

So, what is the only possibility for these phasors to rotate at the same speed in some sense? So, visualize this right the first is a phasor, second is a phasor, third is a phasor right, as I change the values of x and y what will happen? These phasors will change in phase, now I am adding 2 phasors on the left hand side and equating it to another phasor on the right hand side.

And this relation should be true for all values of rotation, when will that be possible, when all the phasors are rotating at the same speed otherwise sometimes they will be in phase, sometimes they will not be in phase and this relation will not be true. So, when for the phase for the phase speed in some sense should be the same what can I say?

Student: (Refer Time: 12:04) theory.

All these k_{ix}'s and all they should all be the same right. So, that is what is called in optics the phase matching condition, this is actually the rigorous way of deriving yours Snell's laws ok. So, phase matching it is called more like a commonsense thing, so $k_{ix} = k_{rx} = k_{tx}; \quad k_{iy} = k_{ry} = k_{ty}$. If I ensure this then no matter what x and y are plug into this equation? The phasors will rotate at the same speed once this is done, what do I have to I mean the immediate conclusion from here is because the phasors are equal they can get cancelled off right right. So, this is at TE polarization right. So, it is clear what we did right.

Now, next is; so, I have got one equation in two variables R and T right what do I do next, but wait I mean one thing before that you notice that this these relations over here these taken together basically give you your Snell's law. Once you write your these k vectors in terms of angles you will get your $n_1 \sin\theta_1 = n_2 \sin\theta_2$ n 1 sin theta equal to n 2 sin theta all of that comes from here, but we are not going that route. Next is next what do we need to do?

Student: Normal (Refer Time: 13:50).

Normal components is that a wise idea, when we derive our reflection coefficients from a planar interface we did in the undergraduate course what did we do? We use tangential boundary conditions for both E and H, not normal boundary conditions. So, I have used tangential boundary conditions for E next what it remains?

Student: (Refer Time: 14:13).

H tangential right, so next is going to be H tangential conserved at this is actually how you derived your R for TE and TM polarization right, if the second equation comes from here and you just enforce it.

Student: (Refer Time: 14:34) tangential field (Refer Time: 14:36).

Yeah, because z=0 is the tangent to the surface.

Student: In the surface we have (Refer Time: 14:44).

Well, it is in my hand how I design the surface right.

Student: (Refer Time: 14:50).

Because I am designing of the layer, so I am going to I make sure that is planar ok, but if it is weird then it is weird then it will only be a local condition approximately too.

Student: Sir.

Yeah.

Student: The direction of (Refer Time: 15:00) different from the (Refer Time: 15:01).

The direction of yeah the.

Student: (Refer Time: 15:09).

Yeah.

Student: That when is we need the components are same.

Yeah.

Student: Our product will be different.

So, the directions of the incident and the reflected wave vector are different and that difference is primarily coming because of which dimension x, y or z.

Student: (Refer Time: 15:25).

z right incident is going in -z reflected is going in +z.

Student: (Refer Time: 15:33).

Right so, z is the guy who is making the difference, but z term vanished from this phase matching condition.

Student: Only at the interface?.

Only at the interface; I mean boundary condition are anyway true only at the interface ok. The z component of the wave vectors will be very crucial we will come to it subsequently, but that is a good point I mean you have a wave vector that is going like this if I flip the sign of the z component it will just go up right that is was.