Computational Electromagnetics
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FDTD: Materials and Boundary Conditions
Lecture -13.18
Sources in FDTD - Part 2

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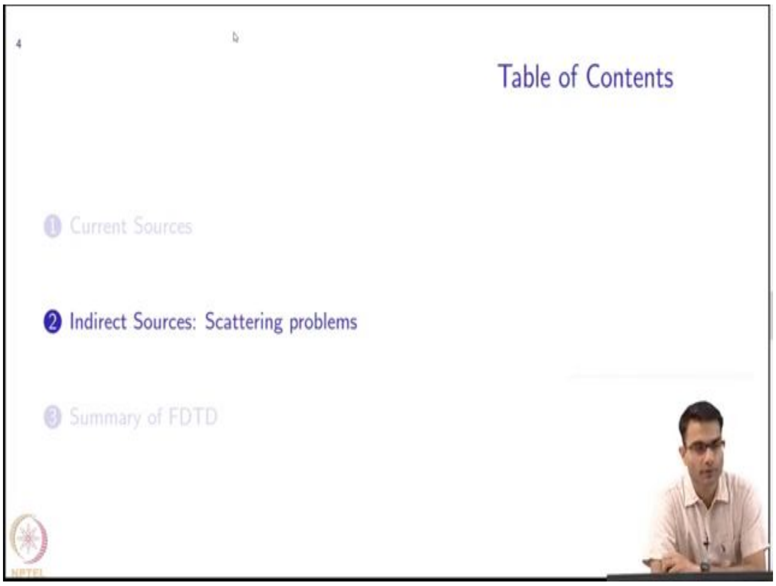

So we were previously talking about the kind how to incorporate sources into FDTD and what we looked at how was how to incorporate current sources.

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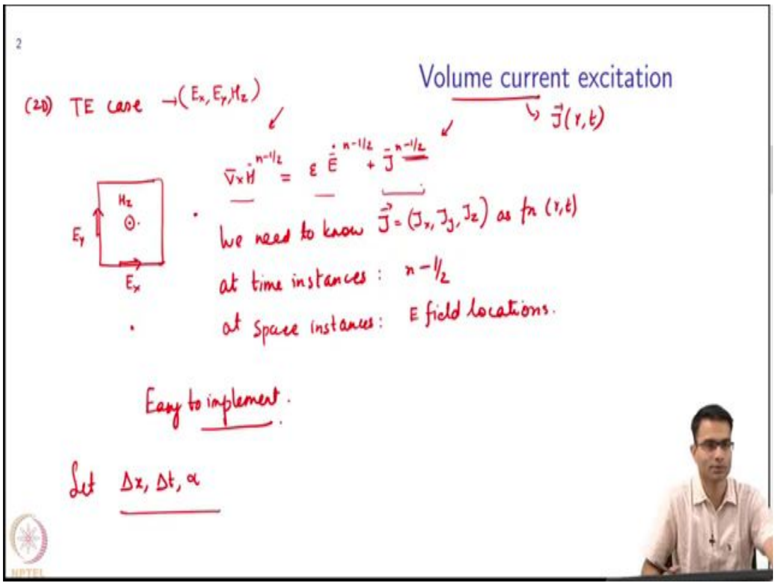

And current sources simply introducing the $\vec{J}(r,t)$ term and we looked at how to implemented in FDTD pretty straight forward.

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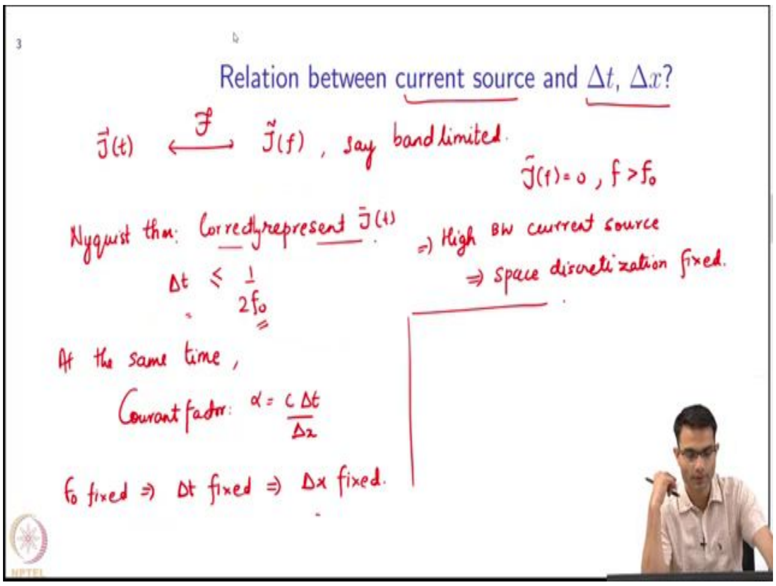

Then we looked at some considerations on the current source bandwidth and how that the maximum bandwidth gets indirectly related to the time step and therefore, the space step from the courant parameter ok.

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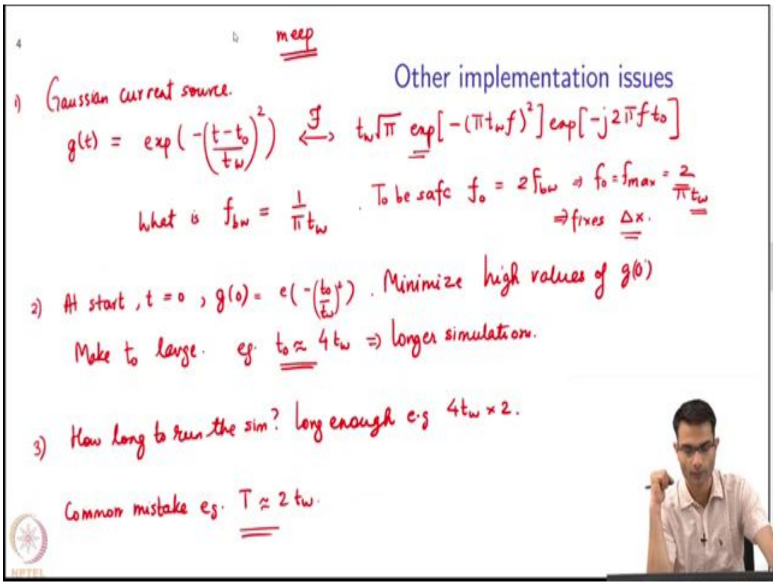

So, this is about what are called direct sources.

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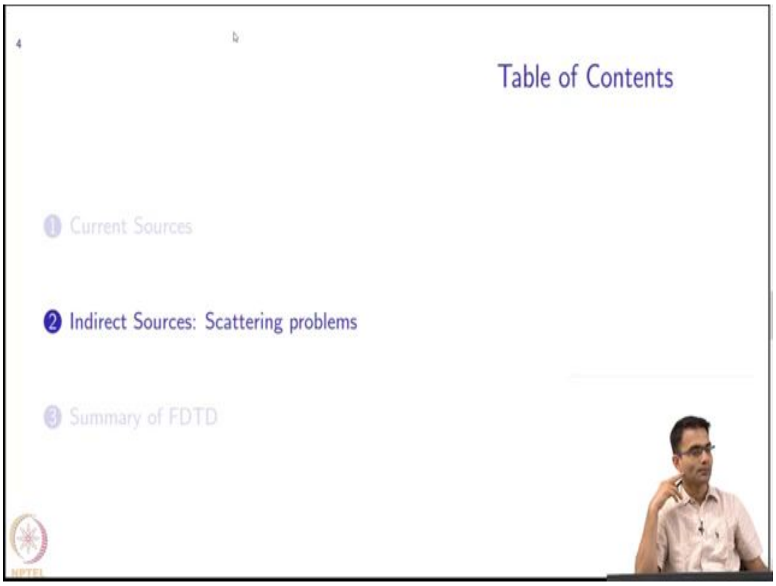

Now, we will look at what are called indirect sources and this is what happens in scattering problems ok. As I mentioned in the scattering problem, the current source is far away. So, I cannot include a J term over here. So, what can we do? So, before giving you the answer what do you think is the way? How will I incorporate a source in a scattering problem?

Student: (Refer Time: 01:18).

E incident it has to come somehow from E incident, but how will it come from E incident? In which set the value of the electric just manually set the value of electric field incident electric field? Now look at think of your FDTD update equations, you have $\nabla \times E$ is dB/dt, $\nabla \times H$ is dD/dt and if you have variable is let us say total field, where will you introduce the incident field? Boundary conditions we have seen for example, ABC and all or PML. So, where is the scope to introduce incident field? Like we did in volume integral how did we?

Student: (Refer Time: 02:04).

With and without the object any other?

Student: (Refer Time: 02:13).

Subtract the 2 any other?

Student: (Refer Time: 02:17).

At the interface.

Student: (Refer Time: 02:18).

So yeah the hint is that at the at some interface between the object and free space is possibly where I can introduce this, and we have seen one example of this in the FEM ok.

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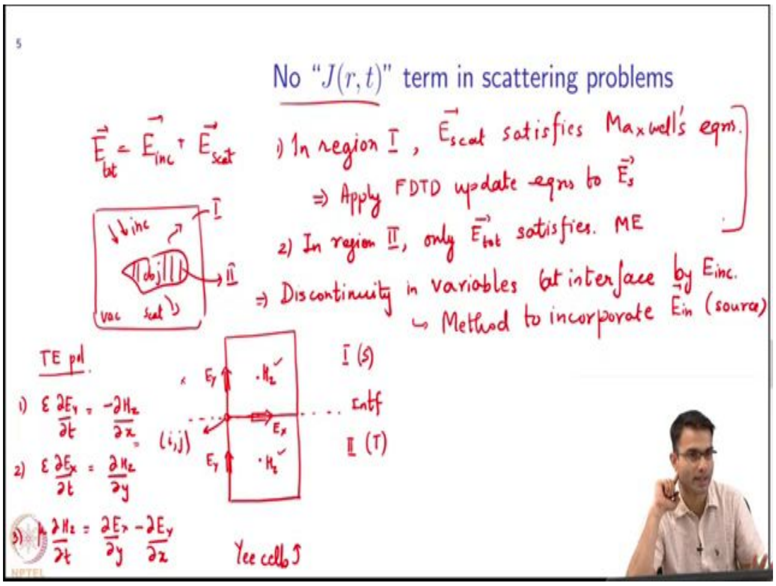

So, what is that form what is that trick is what is called the total field and scattered field formulation right. So, that is going to be the direction that we will take at ok. So, this is; obviously, clear that there is no J term in scattering problem, but what we can do is we can write down my electric field as $\vec{E}_{tot} = \vec{E}_{inc} + \vec{E}_{scat}$. So, let us also draw computational domain over here. So, let us say this is my object right some incident field is falling on it, and something is getting scattered field this is scattered this is total.

Now, if I consider the object well before I come to object let us look at vacuum. Let us assume that the incident field is in vacuum. In vacuum does the incident field satisfy Maxwell’s equations? Yes does the total field satisfy Maxwell’s equations? Yes. So, also does the scattered field satisfy the Maxwell’s equations right. So, in let us call this region I and let us call this region II. So, in region I scattered field satisfies Maxwell’s equations.

So, what is the implication? So, what no I can apply FDTD update equations to scattered field right if it obeys Maxwell’s equations I can apply FDTD to it in region II what satisfies Maxwell’s equations? Only total field; so, I and II together imply that there is a discontinuity that will happen if I make my variable to be E scattered in vacuum and E total in the object, there is there will be a discontinuity of the variables, there is no physical discontinuity the tangential field will be of course, be satisfied, but there is a discontinuity in the variables. If I assume this scattered field here and total field over here. So, so; that means, there is a discontinuity in the variables and what is the magnitude of that discontinuity not current incident itself right?

Student: (Refer Time: 06:01).

Incident field is it is the a magnitude of the discontinuity at interface by E incident ok. So, this is going to be the sort of method to get E incident into the FDTD method to incorporate right which is our source ok.

So, if let us take for example, our TE equations TE polarization in 2D what were our variables in TE?

H_z, E_x, E_y right; so, the three update equations that I can write.

$$\varepsilon \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x} \quad ; \quad \varepsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} \quad ; \quad \mu \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

So, these are the update equations that we need to take into account and what will be the two interesting kind of Yee cells that I have to take? One I mean the interface one Yee cell above one Yee cell below. So, that is what we will do. So, let us take this as the interface ok. So, this is interface.

So, this is for example, this is my region II this is my region I. In region I my variable is the what is my variable scattered total or incident. So, this is scattered this is total ok. So, these are my Yee cells where do I record H_z typically? Center of the cells right; so, my H_z is over here H_z is over here then I have a E_x that is at the boundary right at the middle over here, then I have a E_y we take this point to be (i,j).

So, let us take let us take these update equations and see what happens to them ok.

Student: Variables (Refer Time: 09:49).

Variables will be same in.

Student: (Refer Time: 09:52).

Huh. So, that is what the variables cannot be the same right I have drawn these two Yee cells they are overlapping at the interface, but I have to choose right I mean is my way in region I the variables is scattered field in the region II the variable is total field ok, but in my equations I am going to have only one variable right. So, so we will yeah I mean we will exactly see how this is done.

So, let us look at the first equation. So, that is giving me E_y correct rate of change of E_y in time and H_z in space right. So, that is what I want. So, first equation so, let us write down over here.

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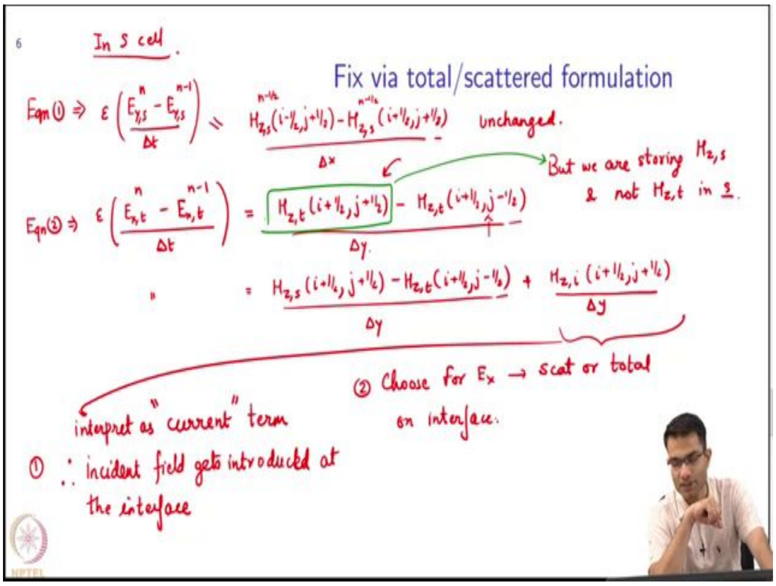

So, equation 1, we are going to write the update equations now ok. So, equation 1 is

$$\varepsilon\left(\frac{E^{n}{}_{y,s}-E^{n-1}{}_{y,s}}{\Delta t}\right)=\frac{H^{n-1/2}{}_{z,s}(i-1/2,j+1/2)-H^{n-1/2}{}_{z,s}(i+1/2,j+1/2)}{\Delta x}$$

Anything changed here? Nothing changed here because it is all clean there is no overlap with the boundary right the Hs are above the boundary the E_y s are above the boundary. So, this is my unchanged then we go to equation 2 again writing in the s cell. So, equation 2 is where it will become a little interesting because I have partial derivative of H with respect to y. So, I have to straddle the boundary right. So, the left hand side is going to be easy.

$$\varepsilon\left(\frac{E^{n}{}_{x,s}-E^{n-1}{}_{x,s}}{\Delta t}\right)=\frac{H^{n-1/2}{}_{z,t}(i+1/2,j+1/2)-H^{n-1/2}{}_{z,t}(i+1/2,j-1/2)}{\Delta y}$$

$$=\frac{H^{n-1/2}{}_{z,s}(i+1/2,j+1/2)-H^{n-1/2}{}_{z,t}(i+1/2,j-1/2)}{\Delta y}+\frac{H^{n-1/2}{}_{z,s}(i+1/2,j+1/2)}{\Delta y}$$ (Not storing scatterred field in s cell)

We are applying only to yeah, but the point is that I want to store for every edge only one variable I do not want to store two variables for an edge.

Student: (Refer Time: 14:11) decide.

Decide it does not matter I decide which one I want; I can either choose that this common variable be for corresponds to scattered field or total field its my choice I just have to be consistent that is all.

Student: (Refer Time: 15:22).

Let us just write down what Maxwell's equation say Maxwell's equation say.

Student: (Refer Time: 15:31).

In the region I, I am storing $H_{z,s}$, but not $H_{z,t}$ right.

Student: (Refer Time: 17:34).

Can I replace this term you are saying should we replace that term also.

Student: (Refer Time: 17:39).

No, but the first term corresponds to upper cell, where the variable is s and the lower one the variable cannot be s because it is region II; region II is the object where scattered field is not a variable.

Student: (Refer Time: 17:56).

Correct. So, I should leave it as total right what is the point you are saying in you are saying that for this second term also I should replace it by incident plus scattered?

Student: (Refer Time: 18:10).

No, because I am making this region I mean because of these two considerations right over here. In region I Maxwell’s equations is satisfied by both. So, I can choose the total field or the scattered field, but in region II I should not make my variable scattered field, because your update equations and I mean Maxwell’s equation scattered field does not obey Maxwell’s equations in the region II. So, I should not be trying to write update equations for scattered field.

Student: (Refer Time: 18:46).

Yeah, because it’s not physically valid variable inside. So, I will keep it as total inside over here ok. So, this is remains as before over here ok. So, this term over here what does it remind you of? Looking at it what can you say? So, how does this equation differ from my usual Maxwell’s equations or update equations?

Student: (Refer Time: 19:19).

This is like the J term you have a question no yeah.

This is exactly like the J term over here. So, I have my update equations in the variables of my choice plus one term over here which I can interpret as a current, but it is coming exactly as the value of incident field over here. So, this second term; so, at every Yee cell on the interface I will have this replacement of total field by scattered plus incident and the incident field therefore, gets introduced at the interface.

Student: (Refer Time: 20:19).

Yeah incident field is known.

Student: That is (Refer Time: 20:22).

Analytical form is known.

Student: (Refer Time: 20:24).

Yeah.

Student: (Refer Time: 20:25).

Yeah. So, what, if I know incident field in the electric if I know the incident electric field take the curl you get the magnetic field right. So, you know radar problem for example, the simplest way of incident field is a plane wave travelling at some angle. So, H_i, E_i these are trivial to write down the in closed form. So, these are known right gets introduced at the interface ok.

So, that is that is all there is. Now we could also write the third equation, but I mean you will got the basic idea for now if I look at the third equation I have rate of change let us say I am writing it for the region s. So, first term on the left hand side any problem? No right because its rate of change of H with respect to time. So, if I am in s then the variable is s, then is first term on the right hand side $\frac{\partial E_x}{\partial y}$ that is where I have to choose make a change and in $\frac{\partial E_y}{\partial x}$ I do not need to make a change right.

So, this is where the incident field makes it's appearance and you have to choose that is that is one, the second thing is choose for E_x at scattered or total on interface. If you are going to choose your variable as the scattered field on the interface, then make sure that you add incident to get the total and vice versa ok. So, those are the two ways in which we get the incident field yeah.

Student: (Refer Time: 22:31).

Yeah out of the plane.

Student: Out of the plane (Refer Time: 22:42).

Normal.

Student: (Refer Time: 22:43).

Yeah.

Student: (Refer Time: 2246).

No, it is interpreted as a current because no do not think of it as a physical current it is not a it is like a the comparison is with.

Student: (Refer Time: 22:57).

With this over here look at this equation over here. I had a some space difference in H, some time derivative in E and an additional term.

Student: (Refer Time: 23:07).

So, that interpretation is like this is just like a we know how to implement it right it is like adding a current and what is the value of this equivalent current? The space derivative of h that is all it is not a physical current ok.