**Transmission lines and electromagnetic waves**
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**Lecture -17**
**Polarization of an Electromagnetic Wave**

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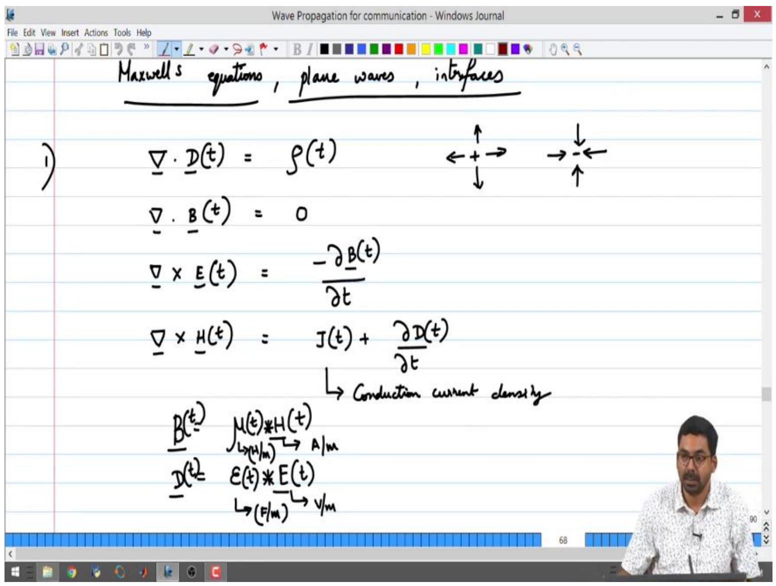


We will get started, ok. So, previously we have seen how to write Maxwell's equations and time domain, all right. And we also saw some similarities between Maxwell's equations and Telegrapher's equations. We classified the materials into different types.

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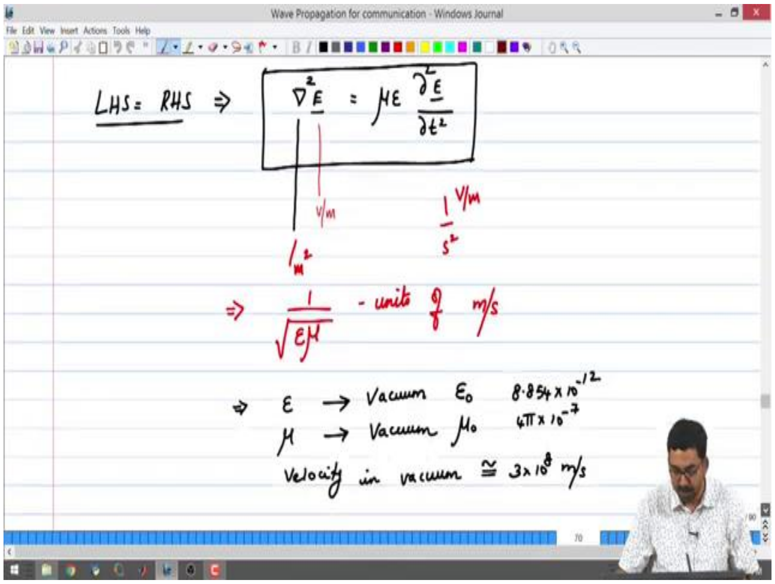


We also derived the wave equation which looks very very similar to the transmission line wave equation, right.

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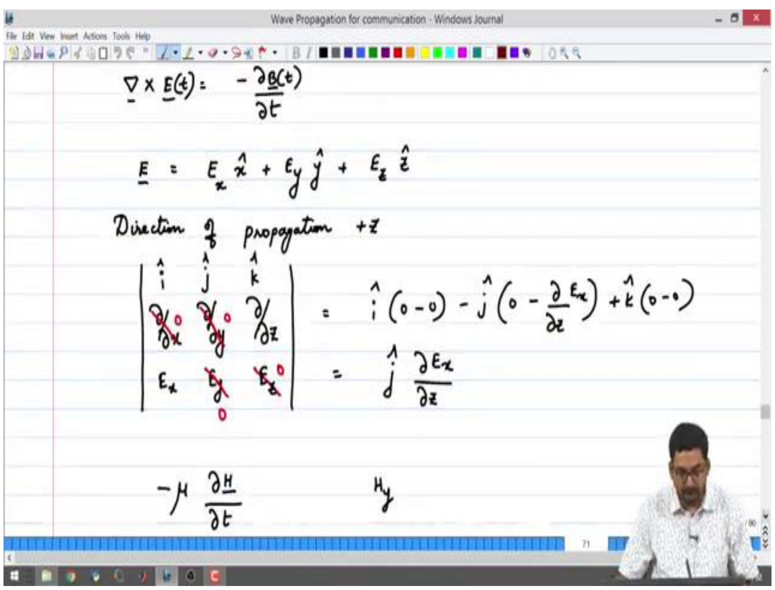


And we saw about this direction of propagation, electric field being a vector, magnetic field also being a vector. Previously voltage versus scalar quantity, now we are getting used to this vector notation and then we are also saying that the direction of propagation is z that means that your electric field cannot have a component in z. Similarly, your magnetic field cannot have a component in Z, so you have to get used to these things, right.

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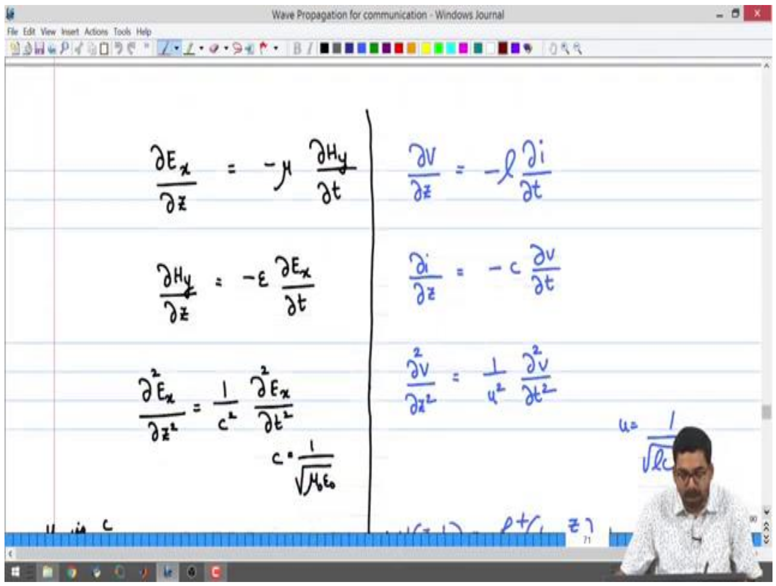


And then we drew a table where there was a similarity between Maxwell's equations and a telegrapher's equation. We saw that there is a one to one correspondence, all right between the equations, the quantities, the solutions, all right, and also the interpretations, ok.

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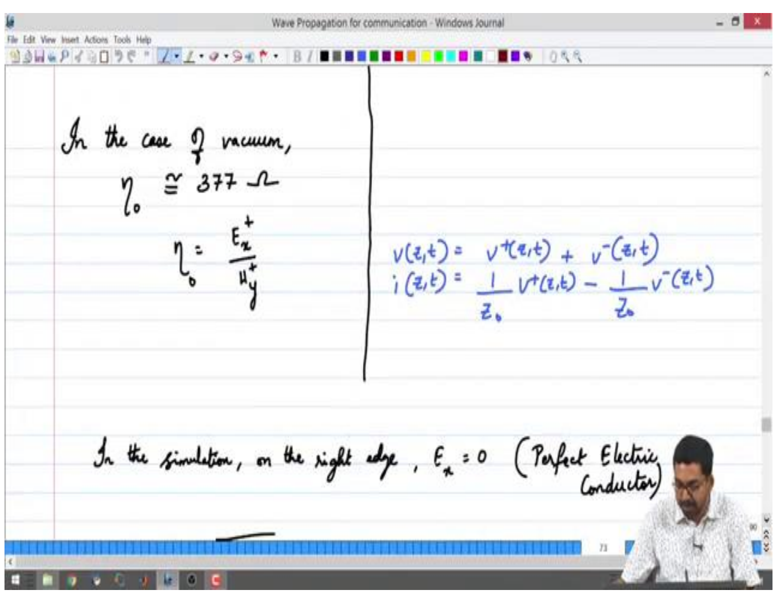

We also then modified the program. All the modification that we did was just modifying the variable names, we ran and it ran fine, all right.

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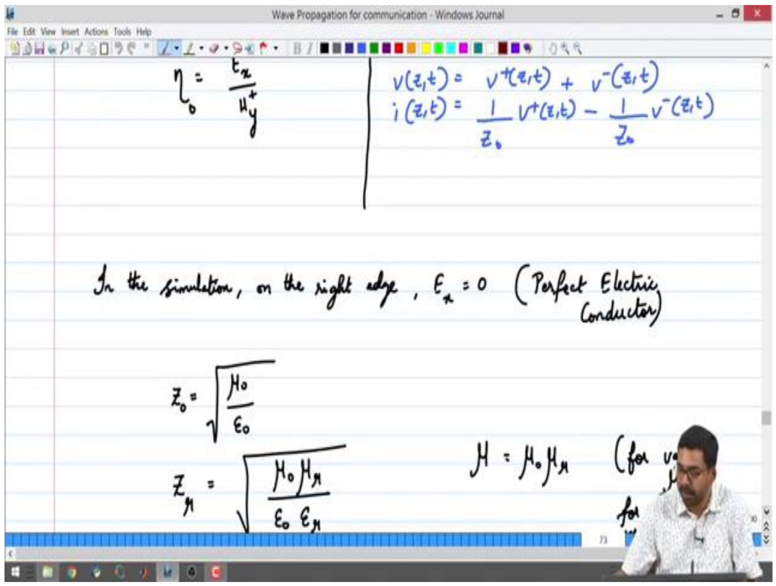


But the interpretations were slightly different. For example, in the case of a transmission line simulation, you would have voltage equal to 0 being a short circuit, in this case you will have to deal with an electric field being equal to 0, that is a perfect electric conductor, all right. So, the slight changes in interpretations we touched upon them, all right. And we also briefly saw about a characteristic impedance and or intrinsic impedance of a medium and the velocity. And then we'll begin where we left off, ok.

So, in this class the idea is to introduce the concepts which are particularly different with respect to the electromagnetic waves rather than the transmission lines and one of the things that we are going to be introducing is a polarization, ok. Previously, we saw that the one of the key differences was electric and magnetic fields being vectors; that was one difference, ok. Now, we are going to go ahead and show what polarization actually means, ok. So, we will begin where we left off and then proceed systematically towards the concept of polarization, ok.

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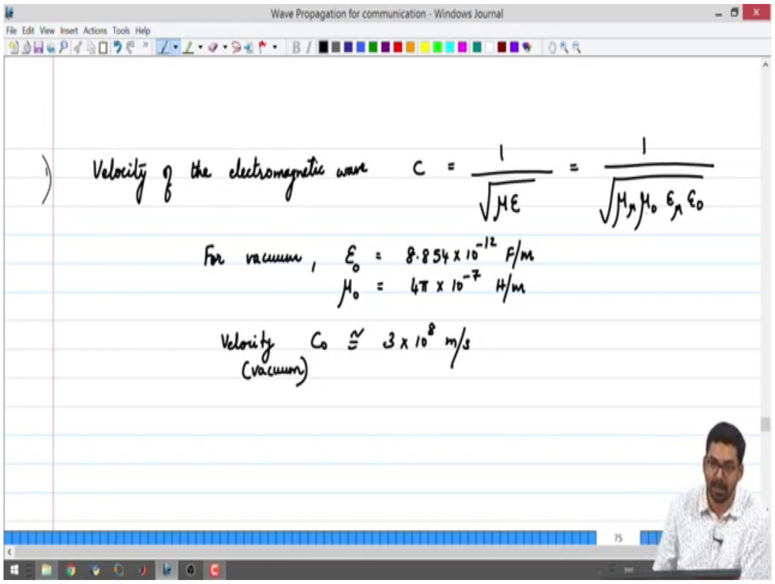


Where we left off was writing the velocity of the electromagnetic wave, ok, ok. And it is a denoted by the letter C. Previously, we had used u for the transmission line, just to differentiate that we are talking about an electromagnetic wave, we just switched to a different variable, all right and it turned out to be

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

And $\mu$ is for the medium, $\epsilon$ is for the medium.

In case a we are talking about vacuum, ok we represent it with the suffix of 0, right, $\epsilon_0$. So, that is going to be equal to $8.854 * 10^{-12}\ F/m$. And the value of $\mu_0$, ok it is going to be $4\pi * 10^{-7}\ H/m$. If one has a look at the units you will find out that this looks similar to L and C the distributed L and C, ok, H/m and F/m, all right.

So, there is a one to one correspondence and for the case of vacuum $c_0$ is, ok is approximately $3 * 10^8\ m/s$. This sets the upper bound for the velocity of an electromagnetic wave in the course that we are going to be studying, all right. In any other medium we already discussed that in the denominator for the velocity, you will be expanding it for a homogeneous isotropic frequency independent medium, all right.

We will be just writing this as a product of $\mu_r\mu_0\epsilon_r\epsilon_0$ all of these are constants for a particular medium. And then, we notice that a $\epsilon_r$ and $\mu_r$ for any material medium are going to be greater than or equal to 1, which means that the velocity in any medium other than vacuum is going to be smaller than the velocity in vacuum itself, ok. So, the vacuum velocity sets an upper bound which means that there is a maximum velocity, the velocity cannot be infinite once again just like in transmission lines we talked about, ok.

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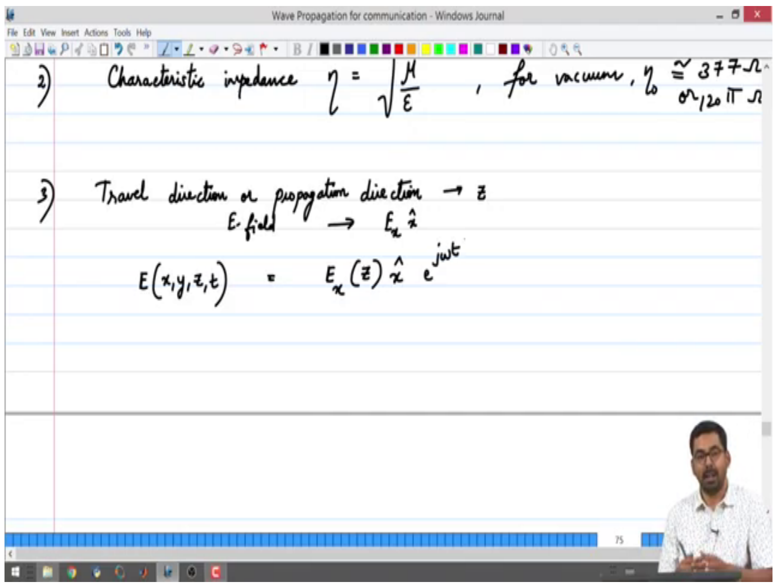


We also briefly talked about the impedance, ok. The characteristic impedance $\eta$ that we wrote down in the previous class was $\mu$ by $\epsilon$ for any medium. You can expand those to

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

all right. And in the case of vacuum 377 Ohms, you can also remember this as 120 pi, ok, ok. These are things that we have already seen towards the end of the previous lecture.

We also had a short discussion about if the electric field is 1 volt per meter. The magnetic field is going to look like a smaller value in amperes per meter, all right, but that does not mean that the effects are going to be, I mean far lower or anything depending on the material. And we said that this is one of the reasons why we ran the programs with the relative units of you know $\mu_r$ and $\epsilon_r$ rather than $\mu_0$ and $\epsilon_0$ to avoid some 0 approximation errors concerning the magnetic field. This is what we had talked about at the end of last class. So, we will begin where we left off, ok.

The first thing that we can start with is now that there is a one to one correspondence. We can start to look at the ac excitation of transmission line analogous in this case. So, if you have an electric and a magnetic field that has a frequency dependence, all right and that is what we mean to say is our constant frequency source of electric and magnetic fields, ok. Previously we had a constant a frequency source for a voltage that we will be considering as an AC source, equivalently here we will be having a constant frequency source of an electric and the magnetic field, ok. So, we will begin with that. The previous class already we were discussing about travel direction of propagation direction being z, we will retain the same, ok, ok.

And in the previous class we had seen that the electric field cannot have a component in the z direction because it will not create a time varying magnetic field that has to be Ex or Ey and the previous class we are taking Ex to begin with. And we will retain the same over here, we will say that the field has an x component, all right and x component only, ok, ok.

If this is the case, the electric field, ok in general, is a function of both space and time just like your voltage is a function of space and time. Previously, we had written V(z, t), all right. Here just to be clear because it is a vector quantity, you can always start with a general description of E and say that it depends on E (x, y, z, t) just to say that it depends upon space and time, all right. However, we know that a with respect to x and y there is no change in the value of electric field that is how we had written the $\nabla \times E$ in the previous lecture. We had crossed out $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, we retain only $\frac{\partial}{\partial z}$. So, E is a function of only (z, t), the manner in which we have considered this problem.

So, you can always write this as it has an Ex component which is a function of z, ok pointing towards x direction, right. On top of that we are introducing a time dependence which is going to be periodic, right. So, the time dependence we just write it down as $e^{j\omega t}$, this will have $cos(\omega t) + jsin(\omega t)$. We can always write this down as $e^{j\omega t}$. Later on we can include only the real part for our analysis, and as we had discussed before the advantage of using exponential expression for the trigonometric quantities is derivatives become easy to calculate, ok.

So, first what we can do is we can start by writing down the wave question in the frequency domain, all right or we can start to write the wave question for the alternating source of electric field, right, similar to what we had written a for the voltage, ok.

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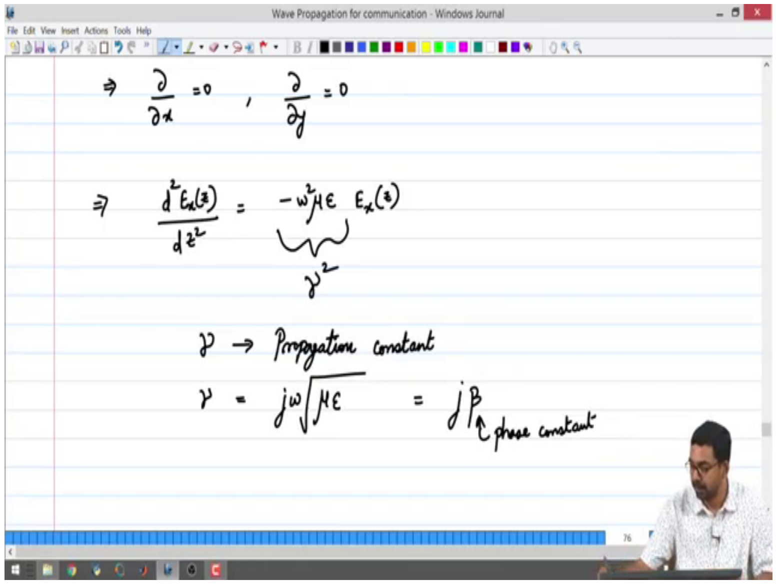


So, here once again to emphasize $\frac{\partial}{\partial x}$ = 0, $\frac{\partial}{\partial y}$ = 0. So, I am very well known now already in the class, ok.

So, we can start to write down the wave equation as.

$$\frac{d^2E_x(z)}{dz^2} = -\omega^2\mu\epsilon E_x(z)$$

assuming that there is a variation with respect to time that is periodic I have not written the $e^{j\omega t}$ over here explicitly, but Ex(z) should involve that also. So, if you want to be more specific you can always write this down left hand side and right hand side can be multiplied with $e^{j\omega t}$ and you will get the exact E(z,t) on both the sides, ok.

In the case of voltage, we had not considered this because with respect to time it is only periodic, ok. If you know the time period, you can always calculate what is happening at any instant of time at a point in space. So, this would be the equivalent expression for a wave question, all right, for an alternating source.

And just like in the voltage case for transmission lines we will call this gamma to be a propagation constant, ok. Now, the variable names are also not changing so much, ok, ok and in this particular case this omega is $j\omega\sqrt{u\,\epsilon}$. Previously, in the case of transmission line you had $j\omega\sqrt{LC}$, so, we are just replacing L with $\mu$, C with $\epsilon$,

$$\gamma = j\omega\sqrt{u\,\epsilon}$$

We already know from the transmission line case that this corresponded to a lossless transmission line there was no r and g involved in that. So, in this case also it is the same. This is a lossless transmission line with no wires, we are dealing with fields that are all, all right. And equivalently we can say that this has

$$\gamma = j\omega\sqrt{u\,\epsilon} = j\beta$$

where this is the phase constant. So, there is really a one to one correspondence, not much is changing other than changing variables with respect to transmission lines, ok. Having written this, ok it is an easy way to write down the solution.

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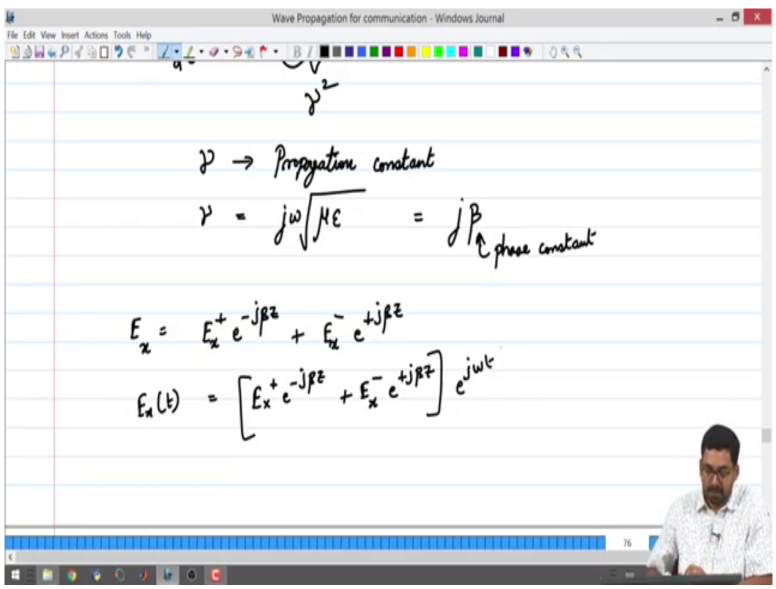


Now, looking at our previous notes we can just write this to be

$$E_x = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

ok. This is with respect to space. If you want to include the effect of time, all right then you will just say that, ok. So, if you want to find out the instantaneous value of the electric field you can always multiplied with $e^{j\omega t}$, ok.

$$E_x(t) = [E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}]e^{j\omega t}$$

Once again as we proceed we see that the equivalence only is increasing and increasing, there is no big deviation between what we saw in the case of transmission lines and electromagnetic fields at all, all right.

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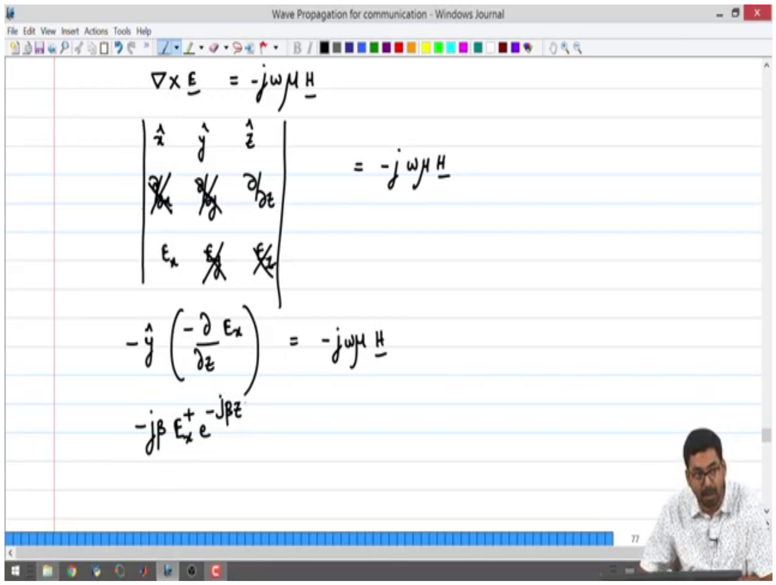


Now, we have talked in length about this electric field, ok. Let us uh write down the equation for no $\nabla \times E$, right, ok. We are considering a time harmonic case or a case where the source is periodic in time, all right, which means that a you can also write down the right hand side of your equation to

$$\nabla \times E = -j\omega\, H$$

Now, we can write down the left hand side. So,

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

We already know that a we can cross out $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ because the propagation direction is along z and there is no way you can have Ez cross that out. Ey we have assumed it to be 0, we are having only Ex, ok. So, the condition that we have post, ok and the right hand side is $-j\omega\mu H$, ok.

This means that I can write down the left hand side, all right to be

$$-\hat{y}\left(-\frac{\partial E_x}{\partial z}\right) = -j\omega\mu H$$

which already tells you that the direction of the magnetic field can only be y because on the left hand side I have the like unit vector for y direction.

We already know that the magnetic field is going to have only a y component, all right. And we can write down an expression, because now we have a solution for Ex, ok we can always plug that solution over here. The general solution is what we will plug in over here to find out what the general solution for H is going to look like, ok.

So, we can just write this down as

$$-\hat{y}\left(-\frac{\partial E_z}{\partial x}\right) = -j\beta E_x^+ e^{-j\beta z}$$

The general solution that we have written is

$$-j\beta E_x^+ e^{-j\beta z} + j\beta E_x^- e^{j\beta z} = -j\omega\mu H_y$$

I have dropped the vector terms because I am just equating y component to y component, ok.

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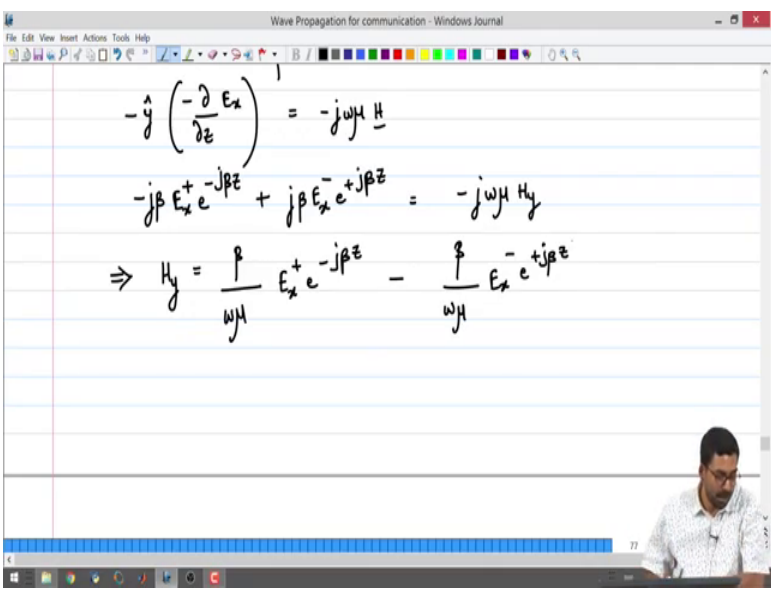


So, I just want to equate the y component to y component. So, I am just taking the coefficient of the vector unit vectors and I am just equating, whatever is corresponding to $\hat{y}$ on the left hand side should be corresponding to $\hat{y}$ on the right hand side, ok. Thus, we can make some rearrangement and write down the expression for the magnetic field, right? It is going to look like, so, minus j can be canceled on all the left and the right hand side and you can start to write down an expression for Hy saying that it will have

$$H_y = \frac{\beta}{\omega\mu} E_X^+ e^{-j\beta z} - \frac{\beta}{\omega\mu} E_X^- e^{+j\beta z}$$

We again notice the similarity only keeps adding up and adding up and adding up, when we had the expression for the current in the case of transmission line, the forward current was having a positive sign, the backward current was having a negative sign.

Once again we are dealing with a magnetic field, the unit is ampere per meter, the forward magnetic field seems to be having a positive sign, the backward magnetic field is having a negative sign. So, the correspondence is only increasing and increasing and increasing, all right. And we can now write down that there is going to be a forward characteristic impedance and a backward characteristic impedance.

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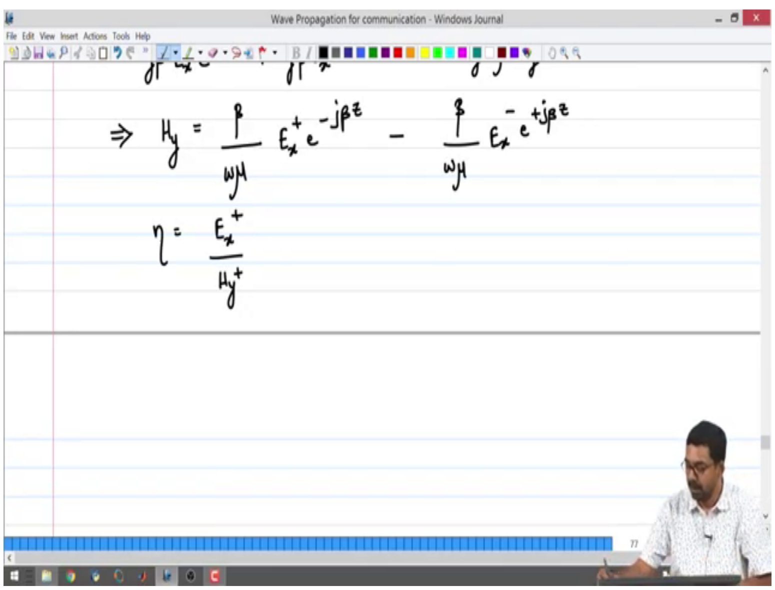


So, you can take H, the characteristic impedance the way we have to now define is

$$\eta = E_x^+ / H_Y^+$$

ok. You can always substitute for $E_x^+ / H_Y^+$, ok and you can see what is the value that you are getting, all right.

For the backward case, all right, you will end up getting a negative sign identical to the case with transmission lines. The negative sign does not mean that you are having a negative intrinsic impedance, just means that the direction of travel is reversed, ok. So, the similarity keeps on increasing, ok.

Now, at the beginning of the class we said that we are going to look into an aspect which is not very similar. So far we are just strengthening the similarity too much, ok. What is the dissimilarity and what are we going to do about it? Right.

So, this configuration where you have electric field x component, magnetic field y component, and the direction of travel to be z component resembles the fingers in your right hand, all right where you hold your thumb index and middle fingers orthogonal to each other, ok. The direction of travel is given by a thumb, that direction of your electric field is the index, all right and the direction of the magnetic field is the middle finger, ok. And you can point it to one direction and say that this is the z direction. So, for all practical cases we will say that we will use x to be like this, we are used to drawing xy graphs where x is like this, y is like this, and the z is pointing out of the plane that is how we always look at it.

Now, one of the things that you can easily figure out is as you rotate your wrist, all right while keeping the direction that your thumb is pointing to be the same way, you can notice that you can rotate the electric field, the magnetic field will rotate itself by the same angle, but the direction of travel remains the same, ok.

This actually means that if you fixed the horizontal axis to be x, the vertical axis to be y, it is not mandatory that you should have propagation in z direction only for Ex and Hy, you can also have anything in between. You can have a component of electric field along x, component of electric field along y, correspondingly your magnetic field will have a component along x and y. And you can keep rotating and this means that your direction of travel is still going to be the same. This is something which is different from your transmission lines, ok.

So, you could have an electric field having components in x and y. But remember that electric fields cannot have a component in z, ok. Under no circumstance your finger is going to point towards this; you are rotating everything in the plane. The electric and the magnetic field lie in the plane perpendicular to the direction of travel and one of the terms that is used for these waves are plane waves, ok.

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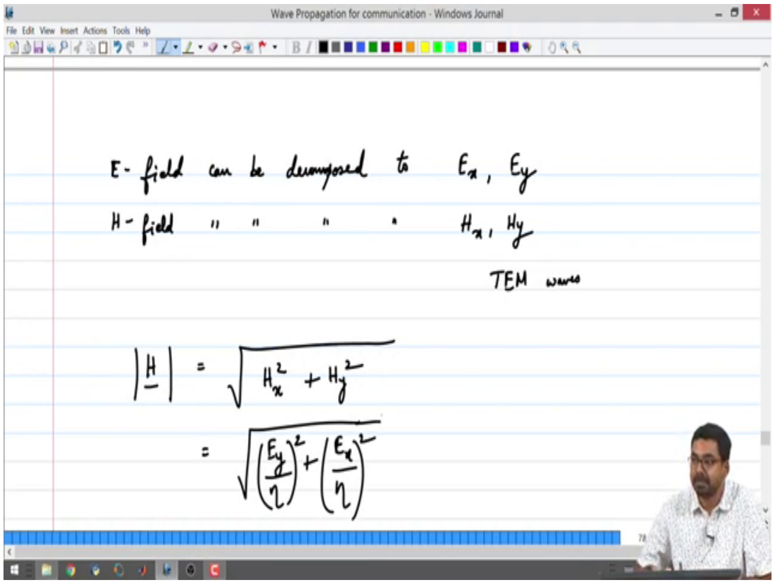


So, here we can say that in the most general case, right, the electric field can be decomposed to Ex and Ey. When you do that, that means that the equivalent is rotating your hand by keeping the direction of travel fixed, when you move away from the horizontal axis you can always take a projection of the index finger on the horizontal axis. Projection of this on the vertical axis you will get Ex and Ey. Correspondingly the magnetic field can be decomposed to Hx and Hy, ok.

But you will notice that you will always be decomposing them into Ex, Ey, Hx, Hy extra, but never have a component in z direction, ok. Now, the direction of travel is referred to as the longitudinal direction, ok that is the direction of length, ok. If your point a is here, point b is here the distance, I mean the line that is connecting them easier, if it is your direction of travel we call that as the longitudinal direction.

What you can do is if you have a source on one place and receiver on the other place you can point your thumb towards from the source to the receiver, and you will notice that the electric and the magnetic fields are going to be perpendicular to this direction of travel or perpendicular to the longitudinal direction, ok. So, the electric and the magnetic fields are going to be having only transverse components, ok. So, the term that we use transverse is perpendicular to the direction of travel. It will always lie in the plane perpendicular to the direction of travel.

So, these kinds of waves where the electric field is perpendicular, magnetic field is perpendicular to each other and to the direction of travel are known as transverse electromagnetic waves or simply put TEM waves, ok. So, ok more generally one can also say that you know if I know the value of the electric field. For example, if I know that electric field is having an x and y component, all right, one can quickly calculate at least the magnitude, ok of the magnetic field let us say

$$|H| = \sqrt{H_x^2 + H_y^2}$$

$$= \sqrt{\left(\frac{E_y}{\eta}\right)^2 + \left(\frac{E_x}{\eta}\right)^2} = \frac{|E|}{\eta}$$

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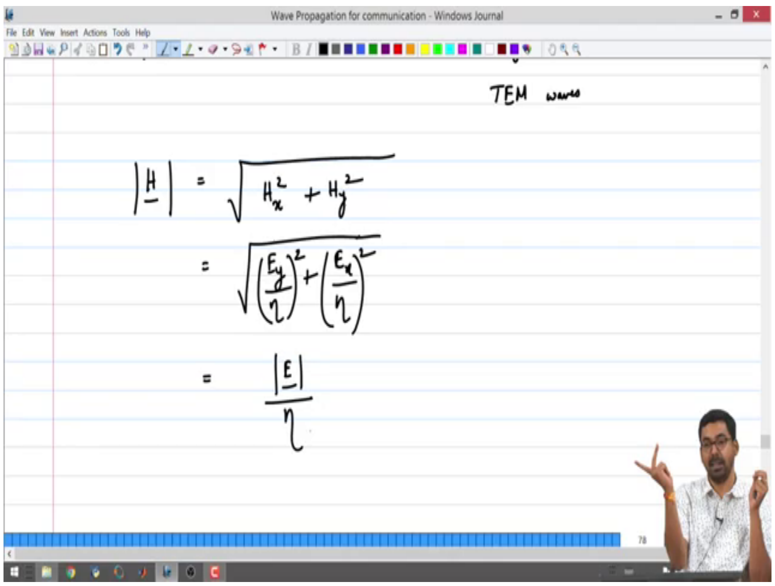


If one writes like this you can just say that the magnitude of the magnetic field is going to be the magnitude of the electric field divided by that characteristic impedance, which is similar to the magnitude of the current being equal to the magnitude of the voltage divided by the characteristic impedance, ok. This gives you the magnitude, ok. Now, we are going to talk about the direction, ok.

Now, there are many cases, right. You can have the electric and the magnetic field pointed in this same way, and have all points in space, ok. You could also have for example, with respect to time in a particular point in space, you could observe different things happen. It is quite possible that at some instant of time you are having Ex, I mean Ex, Hy and Ez like this, all right or you know like this is how we usually are. And maybe at another instant of time it is like this, maybe at another instant of time it is like this, maybe at another instant of time it is like this all of these are valid as long as your direction of propagation is still pointing to the same side.

So, it is quite possible that at a given point in space with respect to time the fields cannot go along the plane, right. They can rotate about the plane, right. So, it is quite possible. And if it is possible, all right what do we call that. Now, how do we distinguish between and at some instant of time? If the wave is like this, all right and at all instants of time suppose it is like this, we have to distinguish that from another wave where the direction of travel is like this, but it is doing this periodically with respect to time we need to distinguish. So, what we do is we use the term known as polarization, ok.

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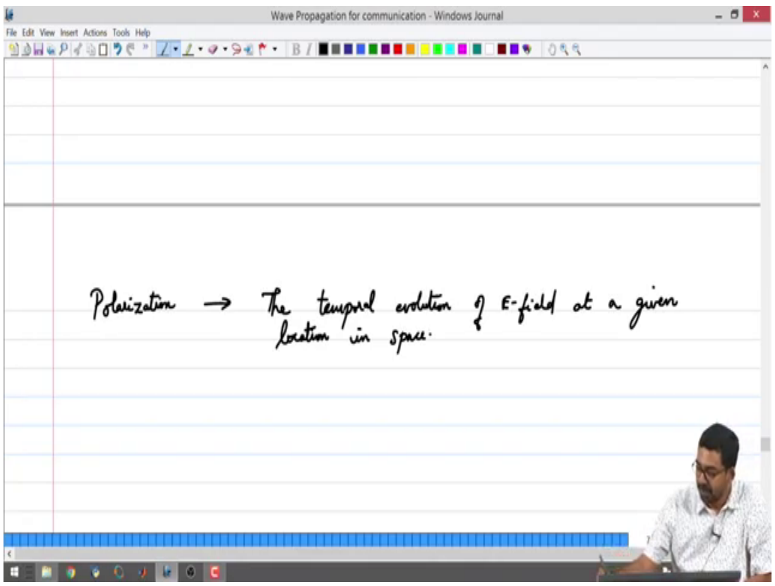


So, broadly it is defined as a temporal evolution, all right or the time evolution of the E field, had a given location in space, had a given location in space with respect to time. What happens to the direction of the electric field is called polarization, ok.

Now, we need to look at this a little bit more closely because this is something very different from the transmission line case. Most of them were similar. Most of the properties are similar until now, but now there is a new property that is coming into the picture, ok.

In the case of transmission lines, we just had what is known as polarity; we never had polarization, right. So, here we are having polarization and we are also saying that at a given location in space with respect to time it could be doing this, but still the direction of travel is pointing in the same direction, right. So, what are all the possibilities, all right and what are they called is what we are going to be seeing now, ok.

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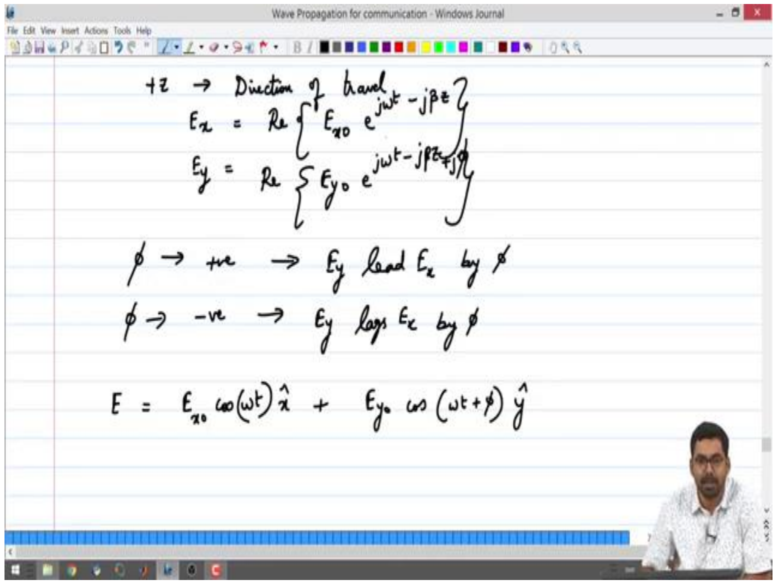


So, we will start with the general case and then make it more specific, ok. Once again we are always assuming here plus z to be the direction of travel, ok. We are having a very you know general case where you could have Ex, where you could have Ey also, ok. because you are breaking down the electric field into some two components in the plane perpendicular to the direction of travel. So, in the most general case you will have some competent Ex and Ey, ok. So, you can just write these down, all right to be periodic.

You can say that this is say

$$E_x = Re\{E_{x0} Cos(e^{j\omega t - j\beta z}\}$$

So, in this expression $E_{x0}$ will determine your amplitude or magnitude, all right. It is just magnitude, ok. And $\omega t - \beta z$ will be known as the phase, ok. So, the phase is $\omega t - \beta z$, magnitude is $E_{x0}$, ok. And since we are dealing with Ey in a similar manner we can write down it has some magnitude $E_{y0}$, ok, and you have $e^{j\omega t}$ minus E to the minus j oops, minus E to the j beta Z, ok. I think it is $-j\beta z$ , oops. In fact, I have to multiply, ok. I had to multiply, but I added the term. So, $e^{j(\omega t - \beta z)}$, ok.

So, once again, we can look at the same configuration and decide whether this is the most general case or not, ok. Now, your direction of travel is in one way, the electric field is rotated, all right. It is quite possible that both the components x and y for the electric field may have the same phase or may not have the same phase. This is something to think about, all right.

So, we have written

$$E_y = Re\{E_{y0}e^{j\omega t - j\beta z}\}$$

But it is not necessary. All we are worried about is the resultant electric field, composed of these two components has to point towards your index, but it is not necessary that they need to have the same phase at all, ok. So, you could have Ex and Ey in the most general case out of phase and the resultant would still be pointing in this direction, ok.

So, the better way to write this is actually to say that the most general case you could have a phase difference between Ex and Ey, and it is the most general case, ok, ok. So, you could have Ex and Ey out of phase and the resultant should be having some magnitude pointing in the direction of your index finger that is all, ok.

Now, we can say that similar to your electric circuits you can say that $\phi$ if it is positive. We already know that you know Ey will lead Ex by $\phi$ , ok, if $\phi$ is negative you can say that Ey lags Ex by $\phi$ , ok. Very similar to ac circuits where you will have some a phase between voltage and current. You will say voltage lags current, current lags voltage, extra, similarly they say that Ex and Ey can have a phase between them. So, if we have to write down the expression for the electric field, the right can always write this down as $Cos(\omega t)\hat{x}$ , ok, looks like.

Now, one may notice that a I have not included space over here, it has to be $\omega t - \beta z$, correct. The most technical sense, the real part has to be $\omega t - \beta z$. The way we have defined the word polarization is at a given space, we are trying to find out the temporal evolution. At a given space means that I have to assume a point and then observe that particular point at all instances of time. The simplest point that I can observe is z equal to 0. If I make z equal to 0 I get rid of one term. So, I can focus on the remaining terms easily, all right. So, now I am focusing exclusively on z equal to 0, so I remove some terms, all right. So, it becomes

$$E = E_{x0}Cos(\omega t)\hat{x} + E_{y0}Cos(\omega t + \phi)\hat{y}$$

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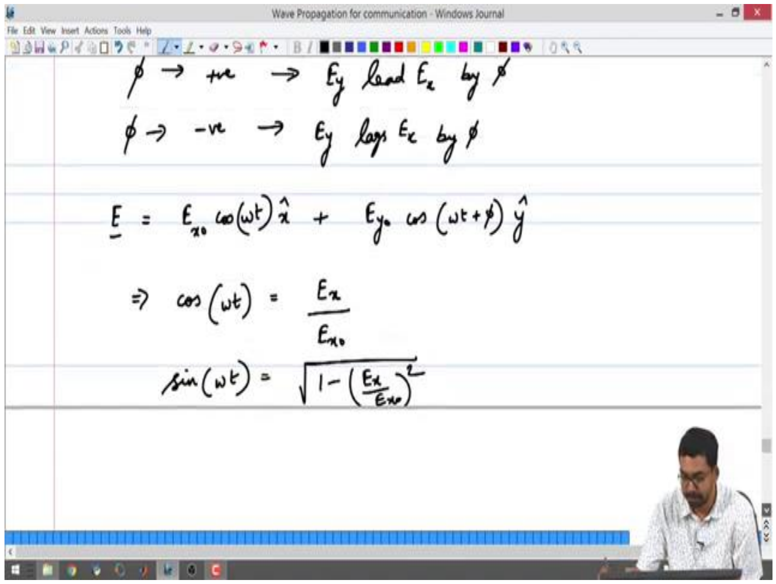


So, at point z equal to 0, I am going to be observing what is happening with respect to you know $E_{x0}$ $E_{y0}$, and what happens to the electric field in general. We can also use some basics of trigonometry and write down that this case cos omega t is simply defined as you know

$$Cos(\omega t) = \frac{E_x}{E_{x0}}$$

And

$$Sin(\omega t) = \sqrt{1 - \left(\frac{E_x}{E_{x0}}\right)^2}$$

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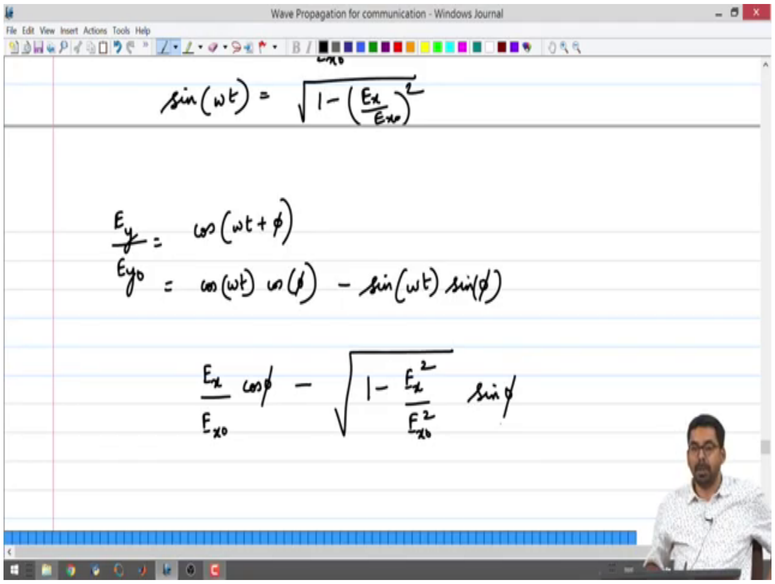


Similarly, you can write down

$$\frac{E_y}{E_{y0}} = Cos(\omega t + \phi)$$

You can use some trigonometric identities, Cos(a+b), you can use the formula $Cos(\omega t + \phi)$ , so you can write this down as

$$\frac{E_y}{E_{y0}} = Cos(\omega t)Cos(\phi) - Sin(\omega t)Sin(\phi)$$

This means that I can write down the expression as

$$\frac{E_x}{E_{x0}} Cos(\phi) - \sqrt{\sqrt{1 - \left(\frac{E_x}{E_{x0}}\right)^2} Sin(\phi)}$$

Now, there are a few things that we can do. One of the things that we can do is a start to look at mathematical manipulations of these terms, and try to see if there is some relationship between Ex, Ey, $E_{x0}$, $E_{y0}$ extra and try to see if we are getting an equation for something that we are able to follow, ok. So, one of the things that we can do is, ok. We can take two sides, all right of an equation we can say that $Cos^2\phi$ if we are able to find out, $Sin^2\phi$ if we are able to find out,

$$Cos^2\phi + Sin^2\phi$$

all right. Can I use this trigonometric identity to establish some relationship between Ex, Ey, $E_{x0}$, $E_{y0}$? This is some question that we can think about, right.

So, I know the phase is $\phi$ , right. So,

$$Cos^2\phi + Sin^2\phi = 1$$

Can I use this equality to establish some relationship between Ex, Ey, a you know $E_{x0}$, $E_{y0}$ extra?

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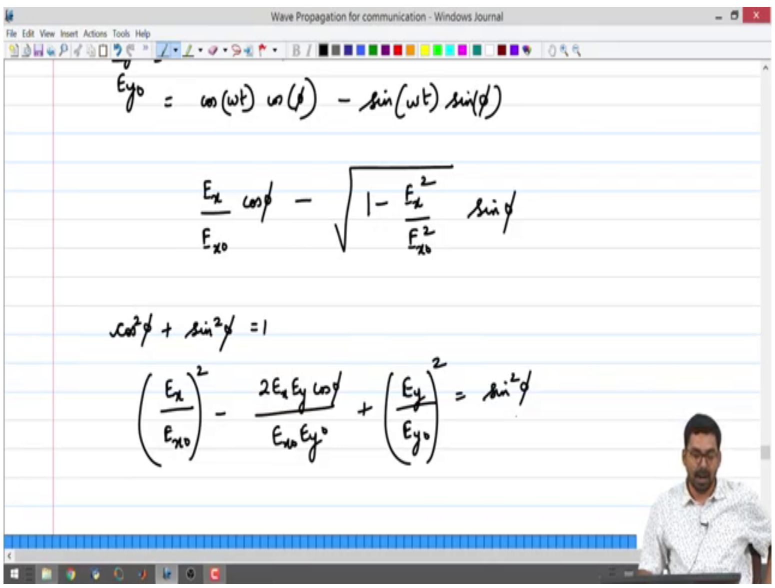


So, you could just say if I am able to find out what is $Cos^2\phi + Sin^2\phi = 1$ and write this down in an equation form. So, I can write down what cos square $\phi$ is going to be like, what sin square $\phi$ is going to be like. Then I can substitute and I can have an equation for Ex, Ey, $E_{x0}$, $E_{y0}$ extra, ok. So, I can do this and you can use the relationships that we have prior to us, all right. So, we can just write down the equation and then spend some time on the interpretation of the equation.

So, I will have

$$\left(\frac{E_x}{E_{x0}}\right)^2 - \frac{2E_xE_yCos\phi}{E_{x0}E_{y0}} + \left(\frac{E_y}{E_{y0}}\right)^2 = Sin^2\phi$$

We are using this to arrive at a relationship between Ex Ey, that is all we are trying to do. And we will spend some time on the interpretation rather than the equation itself, right. So, let us say that I have some equation and I want to find out what it means and I then start to look at different cases, right.

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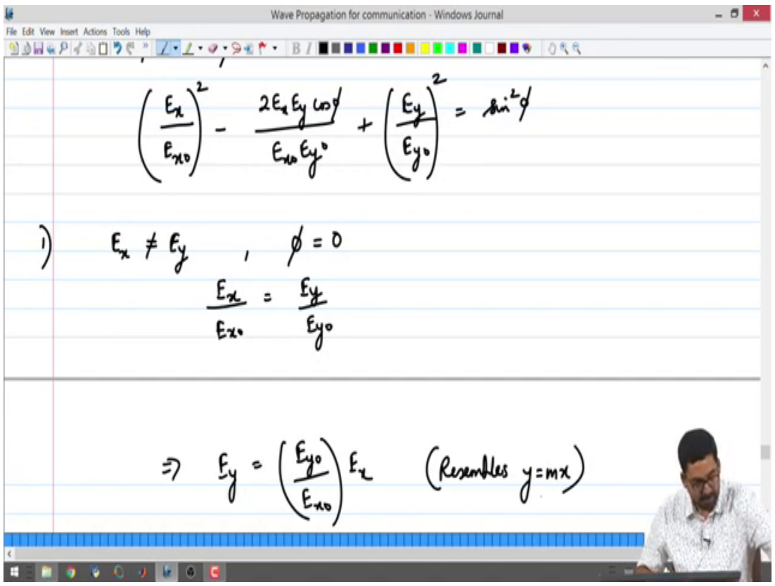


The first case that I look at is Ex is not equal to Ey, ok and say the phase between them is 0, ok. In other words, the way we have written we have Ex, $E_{x0}$ extra. So, the way we have written this means that

$$\frac{E_x}{E_{x0}} = \frac{E_y}{E_{y0}}$$

Let us take this particular case, right or you could also write this down as if I know Ex I could write this down as

$$E_y = \left(\frac{E_y}{E_{y0}}\right) E_x$$

So, this resembles y equal to mx, ok which is the equation of a line that passes through the origin, ok.

What does this mean? This means that Ey is always a constant multiplied by Ex, ok, Ey is always a constant multiplied by Ex. So, instead of looking at all these things you could look at the configuration that you had, right. You are having a net electric field like this, the y component is always a constant multiplied by the x component, ok.

Now, we know that a constant will not change the direction at all, ok a constant will not change the direction, a constant will not change phase, ok. So, it means that if I decompose this electric field to having an x and y component, the phase of the electric field in the x direction is going to be the same as the phase now I direction because I am going to be multiplying by just a constant, only the value will change. If I had 1 volt per meter, maybe I had 3 volts per meter, all right. But the phase does not change their always in phase

So, $E_{x0}$ I mean that the x projection and the y projection will always be in phase. So, even with respect to time it means that, all right, if I have something like this at some instant of time, at another instant of time I will be able to draw the projection. And still I will find that the projection is such that Ey and Ex have a constant multiple between them, it is always a constant, ok.

If that is the case, at any point in time if I know the value of $E_{x0}$ I will be able to figure out what is the value of Ey and the direction of Ey also by just multiplying it with $E_{y0}/E_{x0}$ ok. So, this kind of a scenario, right, where your y component is a constant times x component, all right of the electric field is known as linear polarization, ok.

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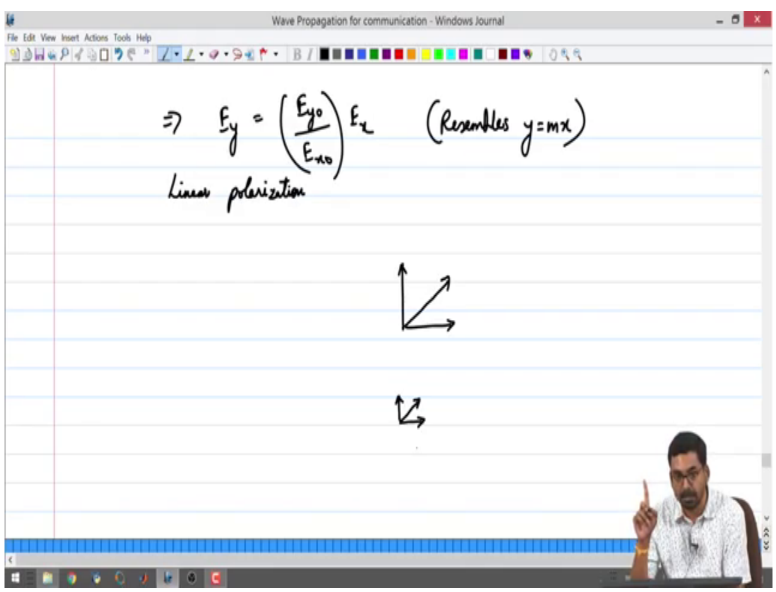


This also means that there is no phase between Ex and Ey, they are oscillating with the same phase. Imagine that your electric field is oscillate bits with a time dependent electric field source that means, if you draw the projections at some instants of time it will go like this, all right it will reach a maximum, then the arrow will string it will go to 0, then it will go to minimum and then it will keep coming to 0, and it will be during this periodically, all right. This is going to be how your x component is doing things.

The y component is just going to increase go to 0, decrease and go to 0 extra. But they are going to be doing this at the same time, that is if your x is increasing, y is also going this and then at the same instant of time they will do this and then it does this. That means, there is no phase difference between them, ok. But the peak values that they can reach could be different, all right.

You can have for example, x going you know this much and at the same time y is going this much, but then they meet here and then they cross extra, ok. So, there is no phase difference between all phases. Ex and Ey will pass through the 0 points at the same instant of time, ok. But with respect to time Ex is oscillating like this, Ey is oscillating like this, all right. They will pass through the 0 at the same time, ok. That is meant by having a no phase difference between your x and the y component of your thing.

What is $E_{x0}$? $E_{x0}$ is since your electric field is doing this the peak value that your x component of the electric field is reaching is $E_{x0}$, and if you multiply that with your cosine omega t then you will get the time dependence of this doing in the horizontal direction that is all, ok. $E_{y0}$ corresponds to the peak of the electric field of the y component, it does this. So, I will just mark this as $E_{y0}$, ok. So, $E_{x0}$, $E_{y0}$ extra is very simple.

If you know with respect to time the electric field is actually oscillating like this. So, the x component is doing this, y component is doing this. But they arrive at the origin at the same instant of time which means that they do not have any phase, but they could have different values, all right, they could have different values.

Now, one of the things that a person can do is try to see, ok at all instances of time what shape describes this kind of polarization, right. That means, that at every instant of time you will take an x component you will take a y component you will draw the resultant, ok. Now, for the resultant in this case it is going to point in this direction, correct. The resultant is going to point in this direction.

What will happen at another instant of time is say the arrow has decreased, this arrow has also decreased in value. The resultant is going to be smaller, but it is going to point in the same direction because Ey is equal to constant times Ex, all right. The resultant has become smaller, but the angle it makes with respect to the x axis is the same, ok. Then what happens? You reach the 0 point, both of the Ex and Ey reach the 0 point at the same time resultant is 0, ok.

For a vector of length 0, you cannot determine the angle, so you forget about it and then you see what happens more at some other instant of time.

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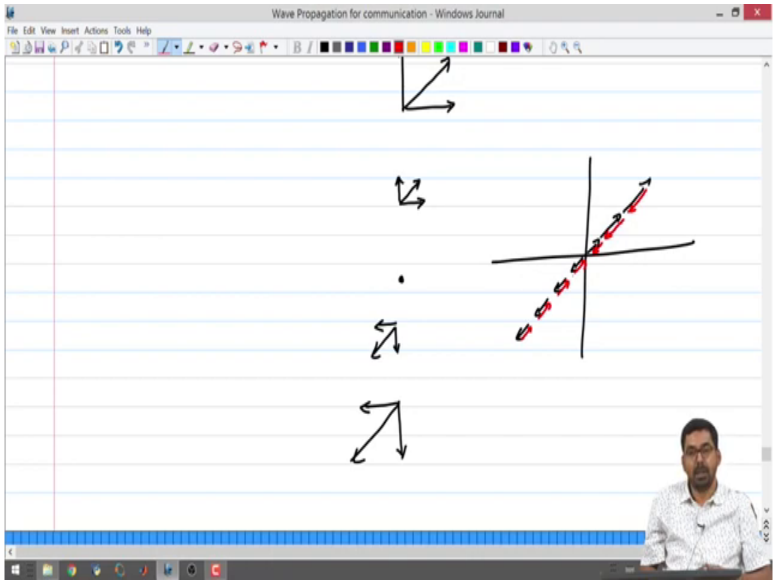


It is only correct that your electric field x will switch direction, all right to a small value, but since your y component is a multiple by a constant it will also switch the value, switch the direction, right. So, it will be like this.

The resultant will then be like this, ok. And then your electric field x component will increase in value, correspondingly your y will increase in value, but this is going to now be like this. Now, if you take this all together and try to draw for several cycles of time, what will happen? All right. If I take an xy axis like this the resultant will go like this all the way and then will trace this path back oops, right. It will pass through the 0, ok. And then what will happen? This thing will keep going like this. Remember that the red and black are collocated just for the sake of clarity I am drawing like this. So, it will go like this and it will come back.

So, the locus of all the points that the resultant will sweep with respect to time is a line, ok. So, linear polarization is actually very tough to imagine from the way we have written the equation extra, ok. But a linear polarization just means that your y component is going to be a constant times x component at all instances of time. So, the resultant, if your x component is 0 y component is also 0, ok. If your x component is maximum y component is maximum, but the values need not be identical because you are multiplying with a constant. If your x component is negative y component is also negative.

So, you will go between the first quadrant and the fourth quadrant, but you will not go to the other two quadrants at all, ok. So, if you are assuming that the Ex is going to be positive then you will be having always you know, first and fourth quadrant only you will not be going to the other quadrants it is like that, right, ok.

So, the resultant with respect to time in space makes a line, ok. So, this is known as linear polarization. This line could be aligned in any way perpendicular to the direction of travel, but with respect to time the resultant is always along a particular line this is known as a linear polarization. You should be able to visualize. What will do, so we will also write a simple program to just make this concept clear because the equation the way in which you are driving is slightly complicated to imagine, ok. So, it will be easier to just see pictorially what is happening by plugging in those equations and then seeing what it sweeps with respect to time.

Now, since we have done this we also realized that there are other possibilities. It is not always necessary that this is the only locus that is possible there are other things which are possible. We will see them in the next class. But what we will do is we will write a program, ok. And then what we will see is, we will vary Ex, Ey with respect to time and then try to plot the resultant and then you know you will realize what is what, ok.

Of course, many of you would have guessed there is something circular polarization, elliptical polarization and all that. But we will arrive at that through the program rather than doing this first, ok because this may become confusing because there is Ex, $E_{x0}$, Ey, $E_{y0}$ extra. So, people will start getting confused as to what is happening, ok. So, but once you see the program things will start to register what is meant by Ex, what is meant by $E_{x0}$, all these things will become very very clear, what is meant by $\phi$ everything will become clear, ok.

So, I will stop here.