Transmission lines and electromagnetic waves
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Lecture – 30
Octave Simulation of Modes of a Rectangular Waveguide

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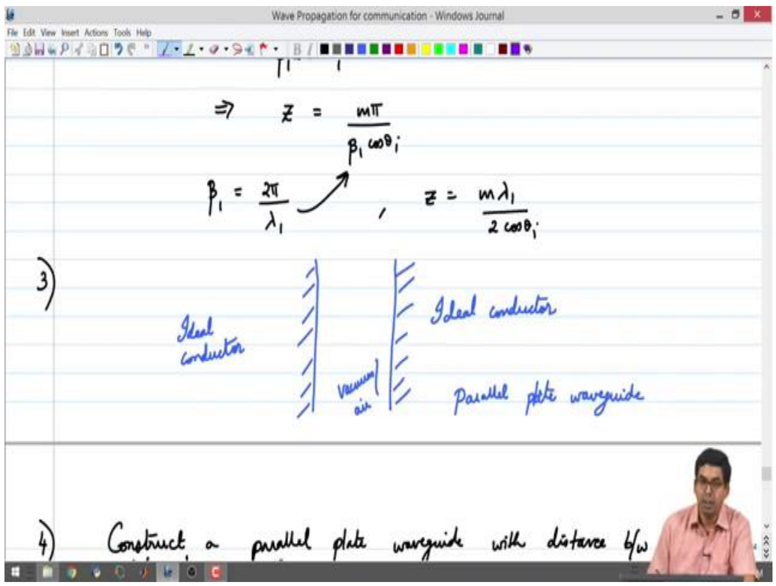

Ok, we will get started quick overview of what we have seen with respect to parallel-plate and rectangular waveguide so far, right. So we tried to derive the transmission and reflection coefficients for dielectric-dielectric interfaces and dielectric-conductor interfaces. Once we were able to get the transmission and reflection coefficients for dielectric-conductor interfaces, alright there was no transmission coefficient, it was 0, alright. Only the reflection coefficient had to be found.

Then we placed the second conductor in such a way that it would not disturb the fields which are produced by this interface with the dielectric- conductor, alright.

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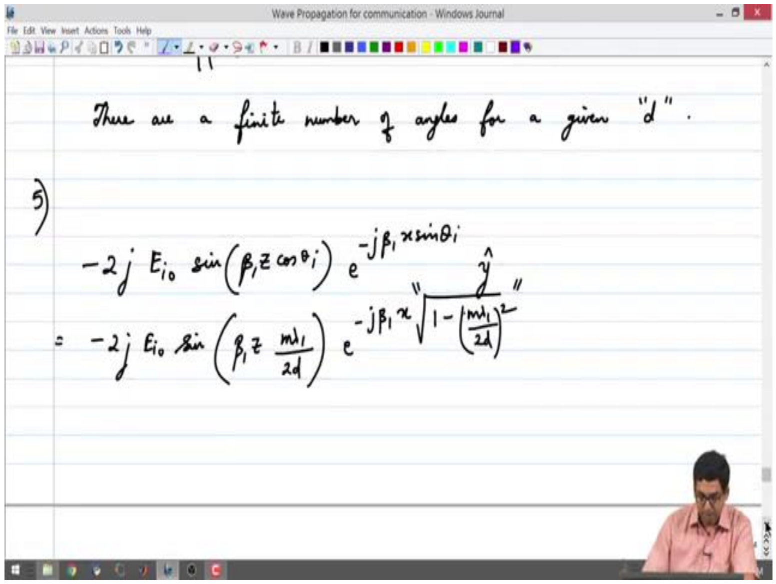

And then we went on to derive a few parameters, alright ah.

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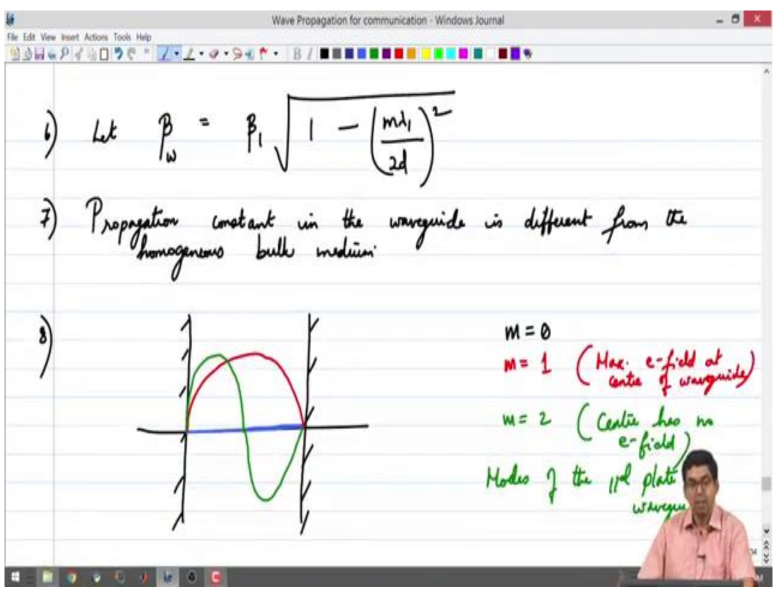

First of all, we found that a the number of angles at which you can launch was discrete right, it was not continuous. The field patterns that will emerge because of this placement of 2 conductors with the dielectric in between or a parallel-plate conductor is also discrete alright. So, for different values of m in your propagation constant, alright, call the mode numbers you will end up with having different kinds of electric field patterns within the 2 parallel plates. This is what we had seen before.

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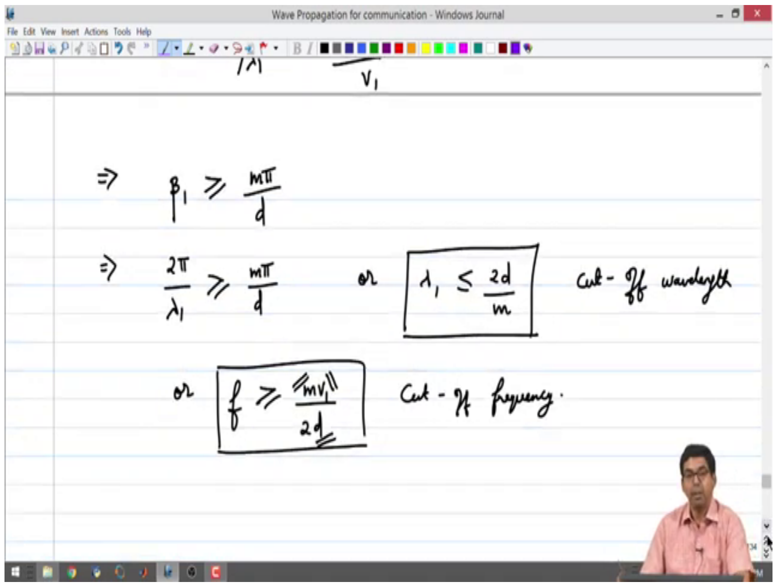

And we also saw that in order for the a waveguide actually to support a travelling wave, you needed to have what is known as signal frequency higher than cut-off frequency and the cut-off frequency is

$$f = \frac{mv_1}{2d}$$

in the case of a parallel-plate waveguide. Where v1 is the velocity in the medium, right. Ok so, we had all these derivations for the parallel-plate and we extended this a idea to the rectangular waveguides alright.

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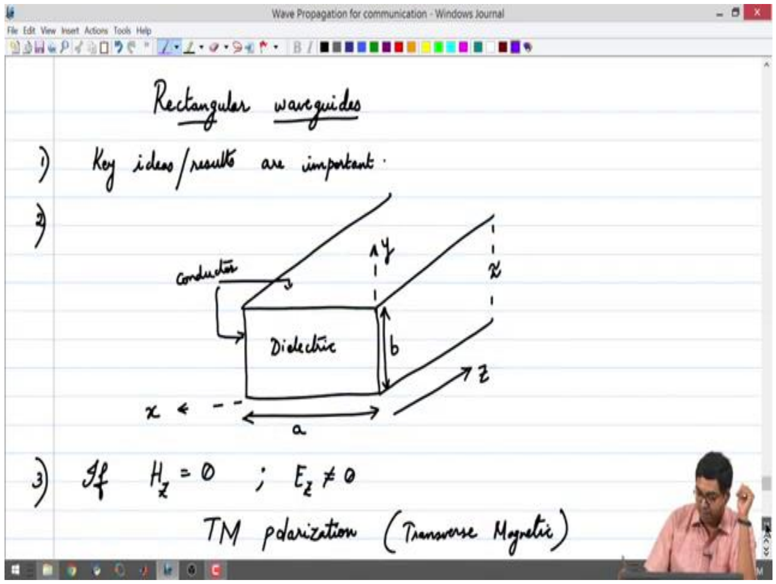

So, we had now a parallel-plate configuration on the horizontal axis and a parallel-plate configuration on the vertical axis.

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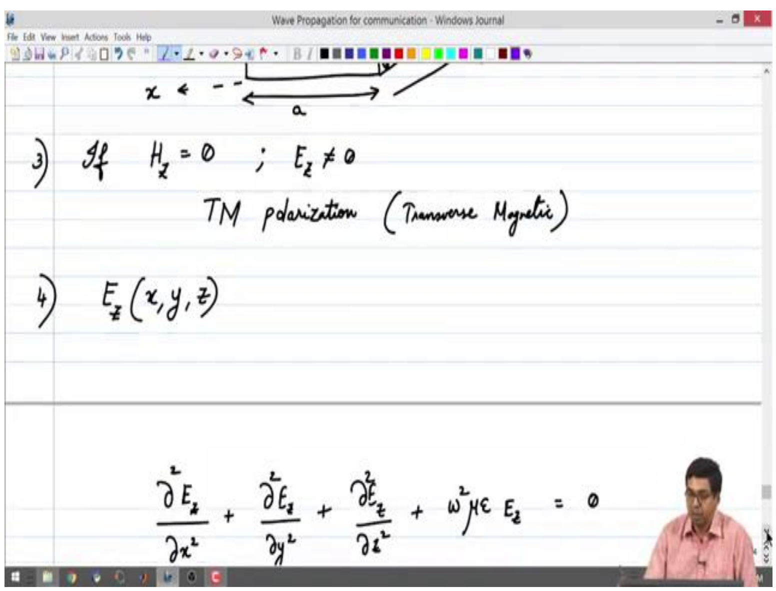

And then what we did was, we used a procedure where we will start with the wave equation alright, we will assume some polarization configuration. In this case, we have used a specific polarization configuration where Ez is not equal to 0 and Hz is equal to 0, alright. Hz equal to 0 means that you may have only Hx and Hy. Ez not equal to 0 does not place any other restriction on the electric field it could have Ex, Ey, it could be anything right.

So, a we had picked up such a configuration and this polarization is known as TM polarization because there is no magnetic field component in the longitudinal direction of the waveguide, ok. So we picked a polarization like this and we wanted to see what the field patterns would look like. You could also pick up and alternate field configuration where Hz is not equal to 0 alright, and Ez is equal to 0, ok.

And that would be known as a TE polarization alright. Consequently, we would rewrite the wave equation for Hz in that case alright and try to see what the field patterns would look like using the appropriate boundary conditions and also using proper general solutions that will satisfy these boundary conditions, alright.

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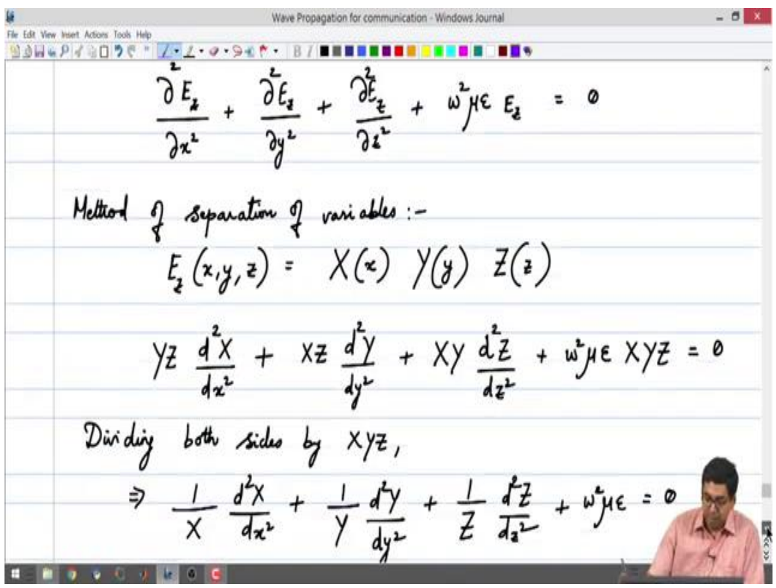

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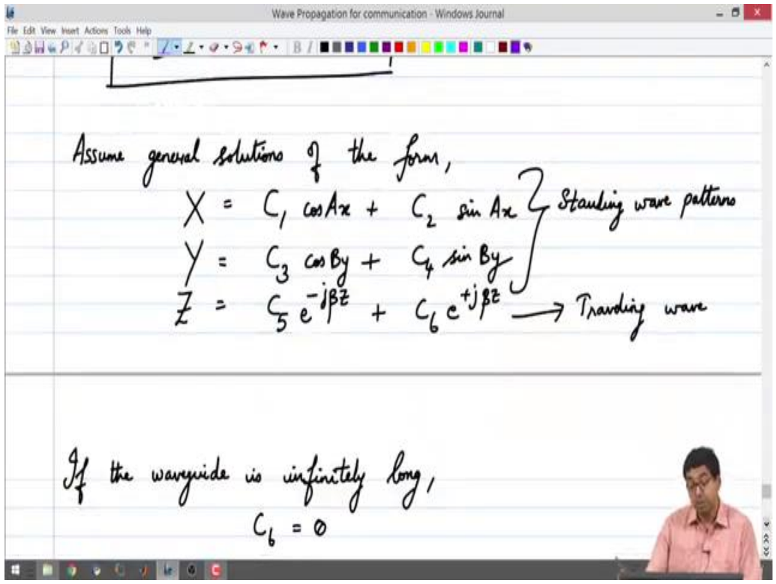

So, one of the things that we already know and we are using from prior knowledge is that we have to choose some boundary conditions which will you know follow the boundary which we have to choose the form of the solutions which will satisfy the boundary conditions. That is what we have to see.

So, in this case we already know from the parallel-plate configuration that there has to be a standing wave pattern in this direction, standing wave pattern in this direction. And along the length of the wave guide, there has to be a travelling wave. Since we already know these things, so we are able to pick up some specific general solutions alright, corresponding to standing waves and we are also able to pick up solutions that satisfy boundary conditions reasonably well.

For example, electric field some component may have to become 0 on an interface extra right. These details are very important, right. So, you can pick up in the case of TM, a general solution that looks like $C_1 Cos Ax + C_2 Sin Ax$ that corresponds to a standing wave in the X direction.

And similarly, for the Y direction its a standing wave you can assume the same form, ok. And in the Z direction we just use a travelling wave forward and backward, so we already are aware of this solution that is we have a forward travelling wave with $e^{-j\beta z}$ and a backward travelling wave with $e^{j\beta z}$.

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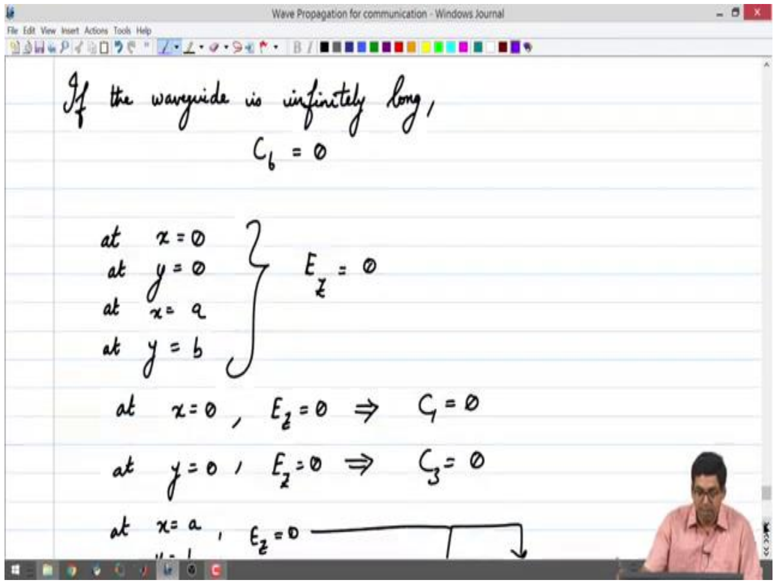

If we take these into consideration, then we may be able to find out the a constants which we are interested in alright, by applying some boundary conditions. There were some constants that become 0 alright, which reduce the number of terms in the expression for the electric field. And there were some constants which we did not solve for, alright.

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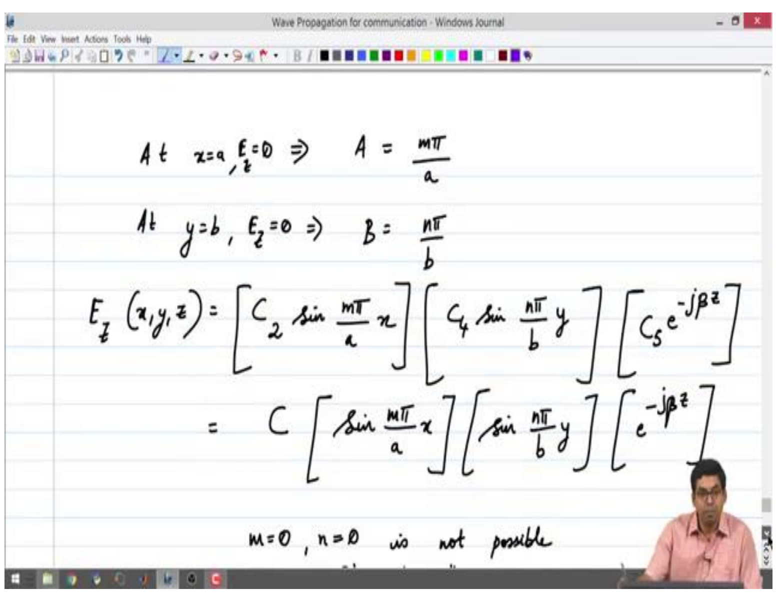

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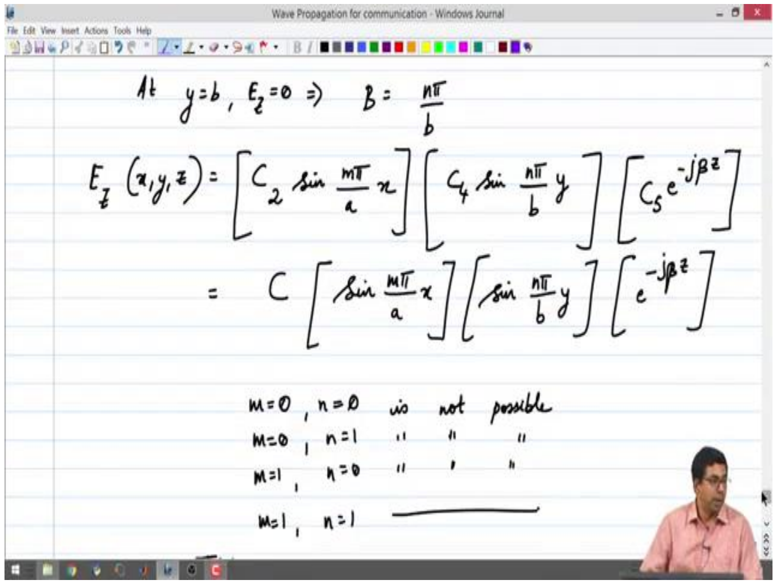

So, we clubbed them together made it into 1 giant constant, alright. So, we said that

$$E_z(x, y, z) = (C_2 SinAx)(C_4 SinBy)(C_5 e^{-j\beta z})$$

alright. And this satisfies the boundary conditions on all the 4 walls, alright. So, x equal to 0 alright your field becomes 0, x equal to a again your field becomes 0. alright. Similarly, y equal to 0 the field becomes 0 y equal to b the field again becomes 0. So, it satisfies the boundary conditions well and it also has a travelling component in the form of $e^{-j\beta z}$, ok.

So, we have seen all these things and then we also saw what possible values of m and n are a going to be with this particular polarization configuration and what they mean with respect to the standing wave patterns. That is where we were a heading towards. So, m equal to 0, n equal to 0 if you merely substitute in the electric field expression, you will end up getting Ez = 0 alright.

So, this is not going to be if there is not there is no electric field z component that is going to be in your wave guide at all, alright. And m equal to 0, n equal to 1, once again, your field is identically 0 m equal to one n equal to 0 is not possible again and so, the fundamental mode or the least value of m or n that is possible for you to give a non-zero value of Ez.

The way that you had started Ez not equal to 0 was the way you had started alright, you cannot end with Ez equal to 0 because that is in contradiction of the assumption you made in order to get to this point, right. So, the solution which will correspond to Ez not equal to 0 requires you to play some minimum criterion on m and n right. So, this m and n are 1 and 1 and the corresponding case will have some spots in your waveguide, alright.

Now, in this class what we are going to do is, we are first going to go ahead write a simple program ok. To visualize this electric field, to get an idea about what these spots are and how they are going to look like. And then we are going to proceed with a few other things for example, how do I find out remaining field components of the waveguide. For example, we did not place any restriction on Ex and Ey, we have not found out anything about Hx and Hy.

Now, if you have found out this form of expression for Ez, can you use this to find out all the other field components directly? Ok. So, we are going to see that and then we are going to make some inferences and get prepared for you know a different polarizations and also a different concepts in waveguides for the future classes.

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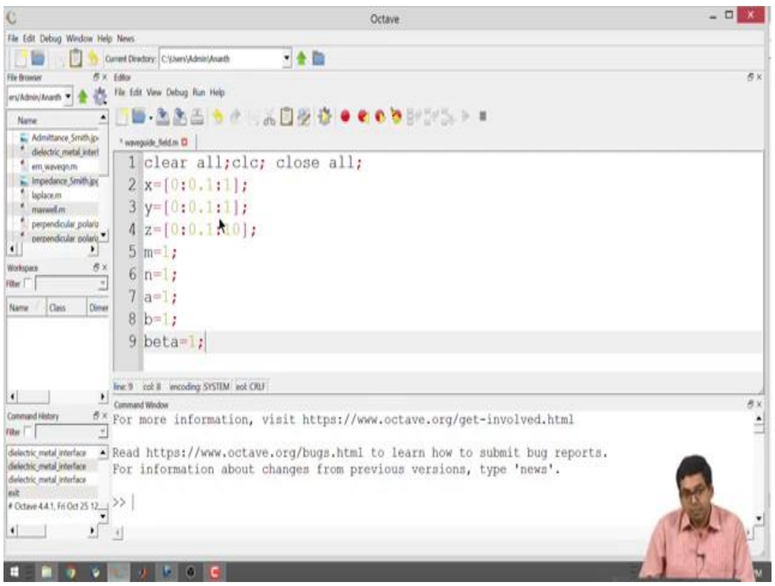

So, I will first fire octave. alright . Ok , hm I am creating a new program the idea is to just take the form of the general expression, we are not going to solve the wave equation and try to find out. All we are trying to do is we had

$$E_z(x,y,z) = C\left[Sin\left(\frac{m\pi}{a}\right)x\right]\left[Sin\left(\frac{n\pi}{b}\right)y\right][e^{-j\beta z}]$$

Because its a very complicated expression, we have to see what it looks like. That is the whole objective right. So, the program is very simple right. So, I am going to need some three-dimensional space, ok. 2 dimensions will represent the cross-section of the waveguide or the transversal section of the waveguide. 1 dimension is going to represent the longitudinal section or the third length of the waveguide right.

So, I am just going to take some x dimensions, I am going to say that is going from 0, that is what we had as boundary condition x equal to 0 to x equal to a, alright. I am going to pick unknown quantities to be equal to 1, to make my calculations very easy. So, x equal to 0 was already fixed from my derivation and the other extreme was x equal to a. And I do not know what is a so, i am going to make it 1, alright.

So, the length is 1 unit, ok. So, I am going to say that x is equal to 0 in steps of 0.1 going all the way to 1, that means, I am taking discrete points in this direction separated by 0.01 units going all the way to 1, right. And at each of those points I want to be able to calculate the value of Ez, ok. Now, I am going to also, so, what I will do is I will just begin with smaller number and then increase it slowly. So, I will go with 0 to zero point I mean 2 1 in steps of 0.1, I will do the same thing with the y, alright.

This represents the cross-sectional space right, ok. So, y is going from 0 to b. Once again, I do not know what is b. So, I am just choosing it as 1.

So, in this case it is you you need not call it as a rectangular waveguides, its a special case. Its a square cross-section waveguide, that is all right. So, I have z ok. To signify that, this is the longitudinal direction, what we had done in our derivation was we had considered the z direction to be very large, alright. Now very large means very large compared to something, alright. So, very large compared to the maybe the cross-sectional lengths, alright.

So here, what I am going to do is to just signify that z direction is much larger than your cross-sections. I am just going to take 10 times alright, the same unit. So, if I took 1 by 1 millimeter to be the cross-section, I am just choosing 10 millimeters to be the length. So, just signify that the longitudinal direction is large right, ok. And m and n were also present in your expression for electric field z component. Now I am going to start with something minimal m equal to 1, n equal to 1.

So, all unknown quantities I am just taking it to be 1, right. Again, my formula had some value for a and b, so I am going to you know a is equal to 1, according to the way I have taken x, b is also equal to 1 right. This is also present in my formula I had $n\pi/a$ alright. I am just going to take the formula and plug it in over here and I am going to make all the other quantities equal to 1 that is all, ok.

And so, I had $\left[Sin\left(\frac{m\pi}{a}\right)x\right]\left[Sin\left(\frac{n\pi}{b}\right)y\right]$, And then I had a travelling part it was $e^{-j\beta z}$. So again, I have to tell what is β, right. I do not know β so, I am just going to make β equal to 1, alright.

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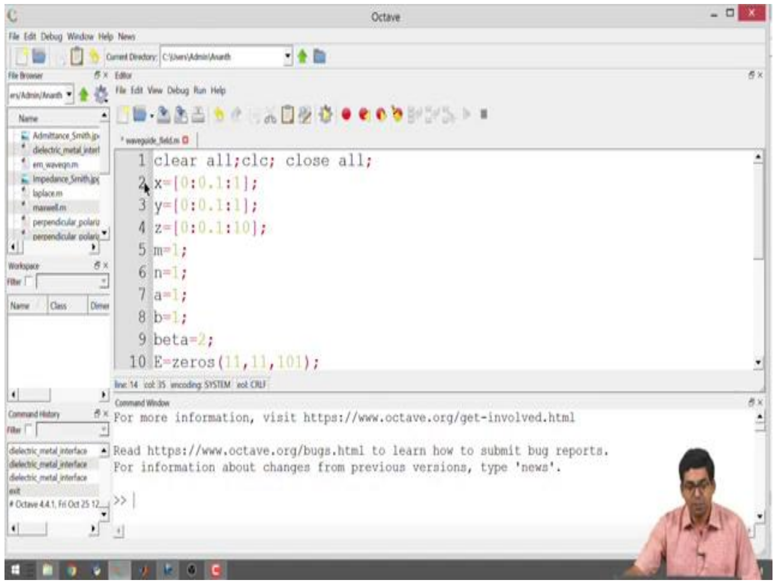

Or just to give it a twist, we can make it β equal to 2 to just signify then that is the travelling direction component of some kinds, alright. So, its a constant relating to some travelling. So, making its slightly different from the other things. You can make it whatever you want, it does not matter, ok.

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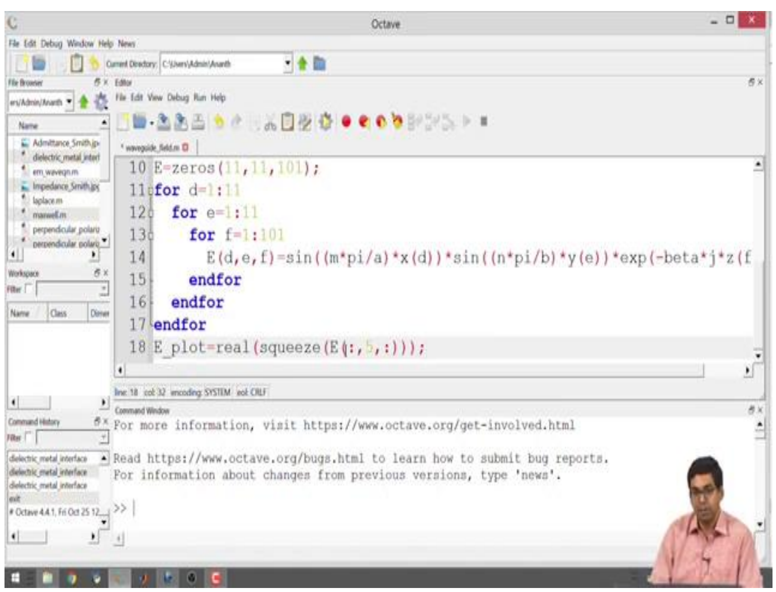

Now, I need to calculate the electric field z component, but I will just make it like you know, I will begin with zeros, alright ah. So, all I need to do is I need to create a matrix alright, the matrix will have three-dimensions x, y and z alright. So, I need to figure out Ez at given x, y and z. What is the Ez looking like? That is all I want to find out, right.

So here, I am just going to use these things as my indicator for the dimensions of the matrix. So, I am having x going from 0 to 1 in steps of 0.1, that means, I will be having an array whose length is 11, alright whose size is 11, right. 0 to 1 in steps of 0.1. So, that will be 10 plus 1. So, 11 will be the size of that array. Similarly, in y direction I will be having 11 and in z direction I will be having 101, right.

So, I will be making, alright. Now I will go, I am not going to write a very efficient code or anything like that, I am going to write as simple code as possible to do this, alright. So, I am just going to use since a and b are already used for some constants alright, i and j the thing is j comes in $e^{-j\beta z}$, alright. So, I do not want to use those constants in far, so I am just using d, e, f extra ok, because I am just running out of constants. So, I am just using d e and f. So, I am just going to make it d equal to 1 to.

So, this is going to be 11, this is for x right for maybe e equal to 1 colon 11 is this for the y, for f equal to 1 colon 101. So, I have 3 loops, one in the X dimension, one in the Y dimension, one in the Z dimension. And all I need to do is plug in the formula for the electric field that whatever we have derived or whatever we have got after evaluating all the constants, right.

So, we had a an expression so, e of d comma e comma f. So, E at a position x,y,z right. And we had a constant c multiplied with these things. I do not know the constants, I am going to take it as 1 because, if I took it as 0, the field becomes 0. So, I understand that that constant is going to tell me something about the amplitude of that field. So, I am going to take it as 1, alright. So, I am going to remove that constants, I am just going to write the remaining terms.

I had $Sin\left(\frac{m\pi}{a}\right)x$ alright. So alright, see my x goes from 0 to 1, ok. I have to now it is a vector, I have to use a particular point d will tell me which point in x I am going to be evaluating this, alright.

So, I am having x of d and then this was multiplied with $Sin\left(\frac{n\pi}{b}\right)y$ multiplied with $e^{-j\beta z}$. So, that is exponential. So, the way to write $-j\beta z$, so I will just do it as $-j\beta z$ of f, right . I just changed the order instead of $-j\beta z$, I put $-j\beta z$ because that is easier to do in octave, otherwise it will throw some error that j is not found and all that right.

So, j by default it will define it as square root of minus 1 over here, if I do not specify anything, right. Ok, just put a and there is nothing more, I can calculate the field using the formula, ok. Now all I want to do is, I want to visualize. The unfortunate thing is now, its a three-dimensional matrix alright, and I can display only two-dimensional images, alright.

So, I have to pick up some slice alright. So, we will pick up a slice like this, alright. What we will do is right.

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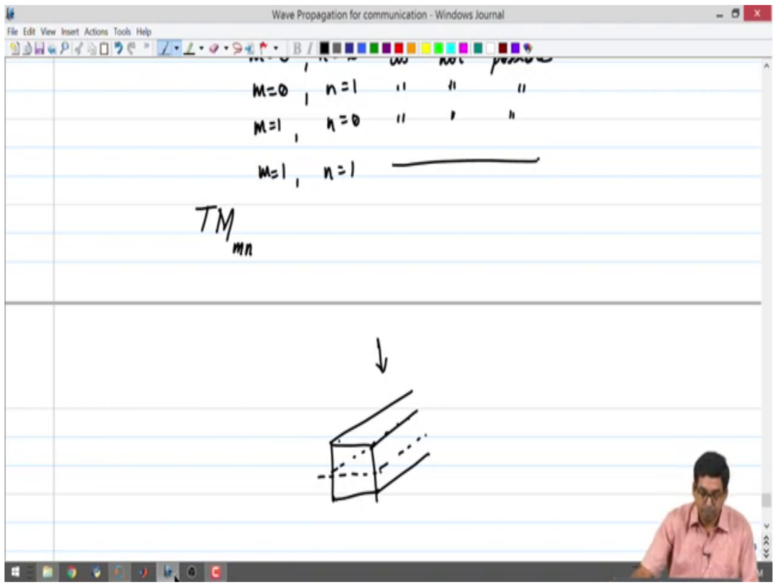

This is the schematic of the waveguide that we are considering. Its approximately a square in the transversal section and a long you know waveguide in the longitudinal section a. What I will do is, I will take a slice right at the middle over here, and plot. That means, I am looking at the top of the waveguide, I am looking at it from here, alright.

Left side will correspond to say, input side and the right side, when I am looking from the top we will look at the other edge longitudinal edge of the waveguide, I am going to be looking from the top, but I am looking at it at the center plane first, alright. Then I will go ahead and change a few things for you to understand more. But, first I will begin with right in the middle of the waveguide, I will take a slice along the length and I will see from the top, ok.

So, I am going to go ahead and do this. So, what I want to do is, I want to create a field to plot. So, I am creating a new variable e of d comma e comma f will have a the value of fields at all positions. But for the sake of plot, I am just picking up a slice, ok. So, I am just going to say that there is a command which allows me to do this in octave right, right ok.

So, what it means is at all x at all z for y equal to 5. So, y equal to 5 in this case means, y is going from you know, 0 to 11 or 1 to 11 alright, its going from 1 to 11. So, I am picking up the fifth column alright. So that means along the center of the waveguide I am drawing a slice, ok. Remember that this is not the exact value of the position coordinate,it is the value of the index having that coordinate.

So, we have a 11 elements, center is approximately 5 alright, ok. So, I am just taking the fifth y-index and all x and all z. So, this will allow me to create a two-dimensional slice ok, squeeze command. a now a there are also a few other things that I want to make note of over here. If you see the expression for electric field alright, of course, I have $Sin\left(\frac{m\pi}{a}\right)x$ alright, this is a real number. $Sin\left(\frac{n\pi}{b}\right)y$ that is again a real number, alright.

And multiplied with exponential -jβz well, that is not going to be a real number there, right. So that is going to give you the phase, right. So that means that your electric field that you are calculating will have a real and an imaginary part, ok. So, when I want to plot the image, I have to tell whether I want to plot the real part, imaginary part or I want to plot the magnitude, i want to plot the phase, extra, right.

Now, I will begin by plotting the real part ok, then later on I will show you different things that can be plotted. Mind you, we are not writing this program to calculate the electric field, we just want to see the visualize the calculated electric field or the fields form that we have already assumed, alright. This is to just give you another confidence that sometimes if you have an analytical expression, you should be able to go back, write a simple program quickly see what it looks like, alright.

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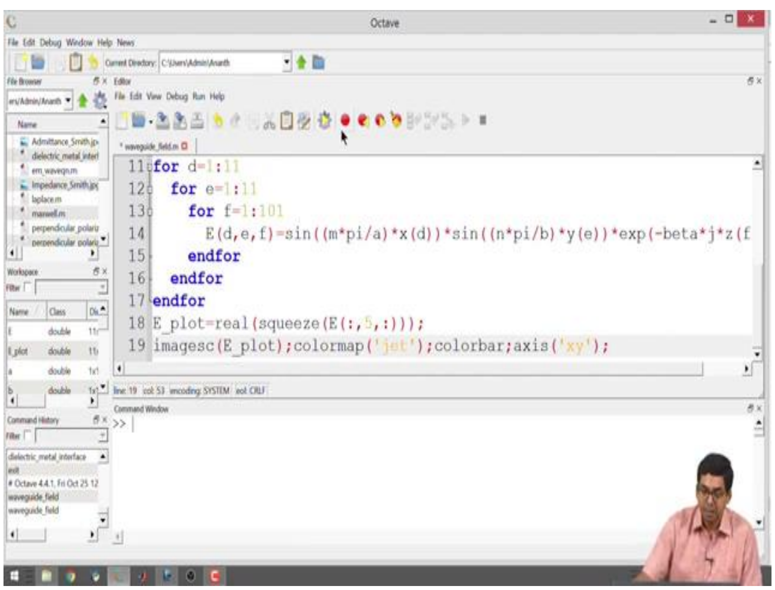

So, I will just use the commands that we have been using before.

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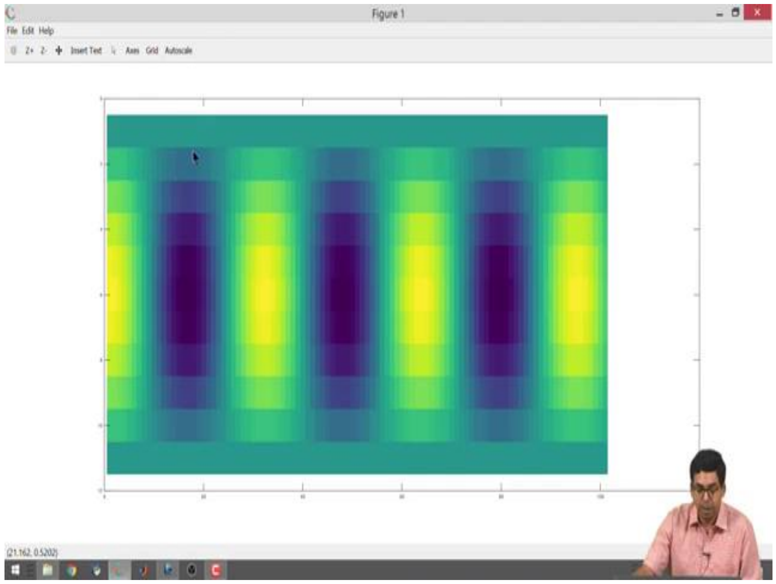

We may just see ok, that is it. So, I am going to run the program and then we will see what happens, ok. There are a few things that we have to look at, first of all those colours, I am not even able to see the colour from here. So, I will just change the color bar and the color map.

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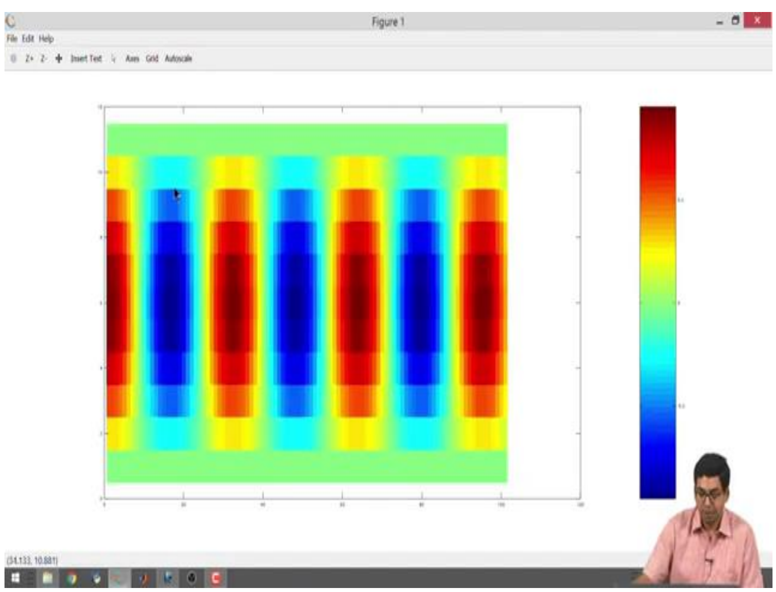

The second thing that I am noticing is image is c. By default, we will put the image coordinates 0 comes from the top to bottom and from here to here, I do not like it so, I will just a its difficult for me to interpret that. So, I will just make it conventional axis, axis x y right. So that, alright. So, now, this is your z-direction alright, this is your z-direction alright, and we are plotting for slice.

So, this is your x-direction x-direction going here, right. We have picked up m equal to 1, ok it is saying that if you pick up m equal to 1, and n equal to 1 is what we have picked up and we have taken a slice right in the middle of the waveguide, m equal to 1 is telling you that you are going to be having a spot in the middle of the waveguide alright. And along the z-direction, it is a travelling wave. So, it keeps switching between plus 1 and minus 1 depending upon the β value alright, that you have given.

So if you count the number of spots, you will be able to figure out with the length what is the value of β, ok. So, you should be able to do all that.

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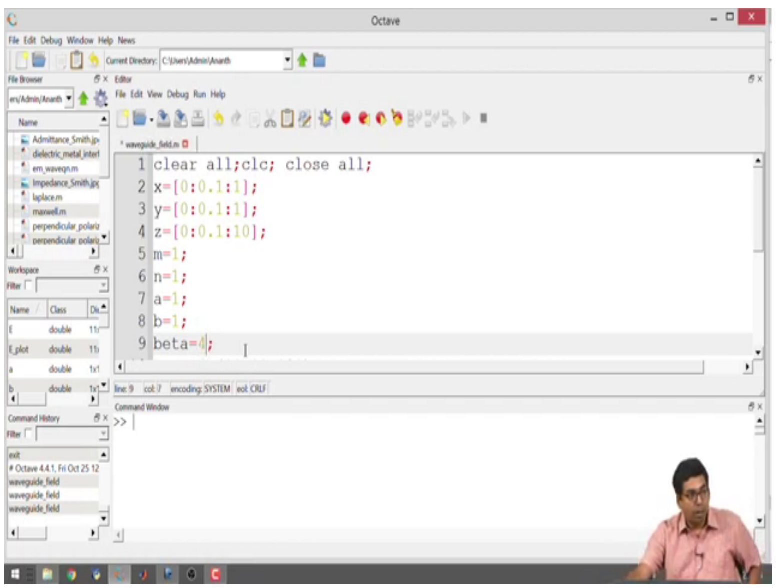

So for example, if I go back and if I make the β as 4 instead of 2 right, ok.

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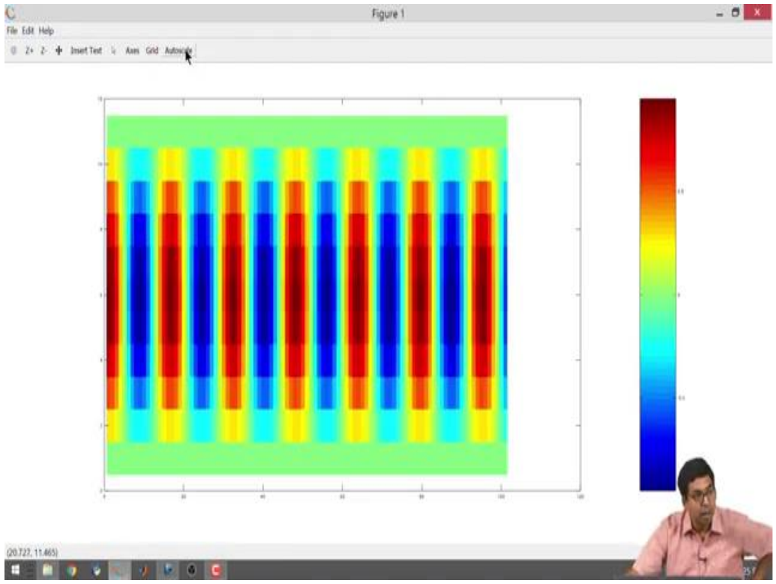

The phase changes faster along the z-direction, alright. So I will I will be able to see which side is the travelling side, alright. So, it does not matter what β is alright. In the transversal direction, I have one spot for m equal to 1, that is what we are seeing over here right so, you could play.

One of the things that I notice here is there is a too much square you know, things which are looking like this because of the way we have set the points. If you were to choose a going from 0 to 1, in steps of say 0.01, alright then you will be able to calculate using much better you know visualization, but this is good enough right.

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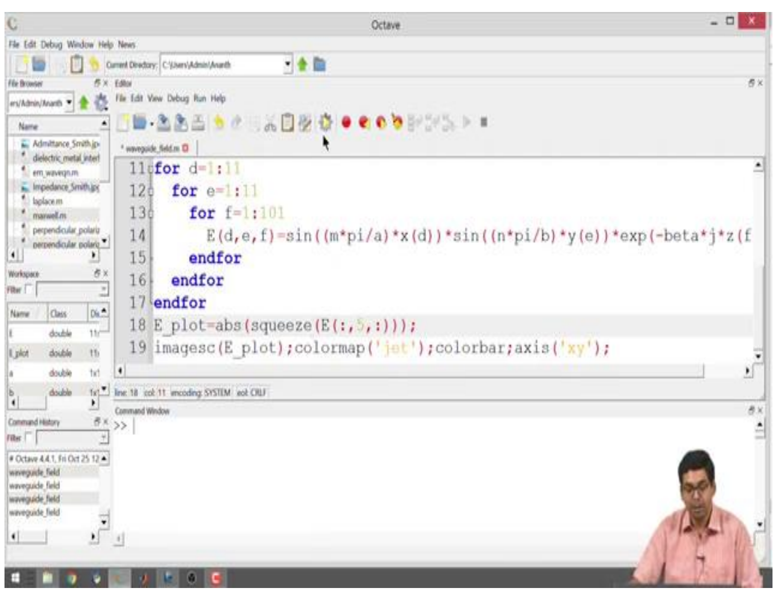

Now, I had taken the real part right, we can always, when we are not aware of this we usually take absolute, that is magnitude right.

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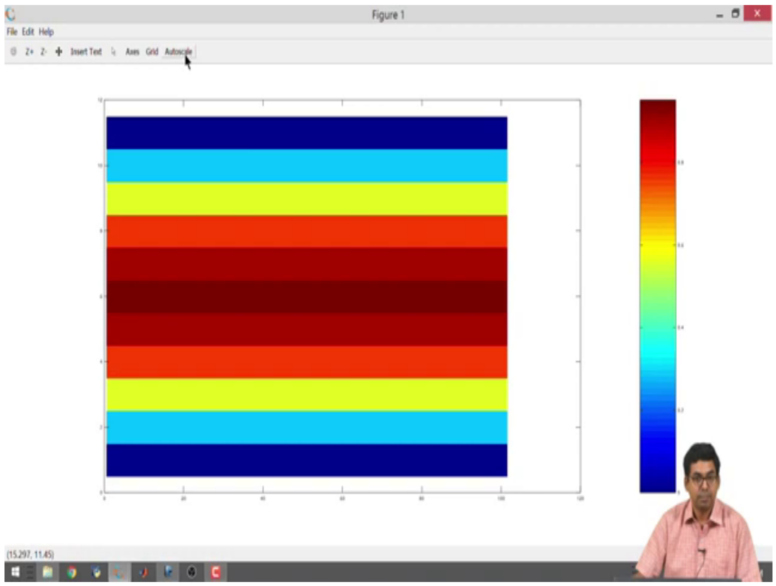

If we were to look at the magnitude of the electric field, and if you were to plot it, it would look like this, alright. One of the things that we notice here is with the magnitude alright, I am not able to figure out what is happening along this axis, alright. I cannot see that β parameter influencing in the a longitudinal direction.

So that is one of the reasons while I why I started the real value. Just I just wanted to see going from plus 1 to minus 1 extra, so that I can figure out the the effect of beta. But if I pick up absolute value, it just looks uniform everywhere, right. So you could do this, but the effect of β is not clear. But in the transversal direction, so again clear that the center of the waveguide carries the maximum electric field.

As you go away from the center, your value of the field decays and on the edge, 0 ok. This is how you will see something, right. Now, I will just go back to the real, ok.

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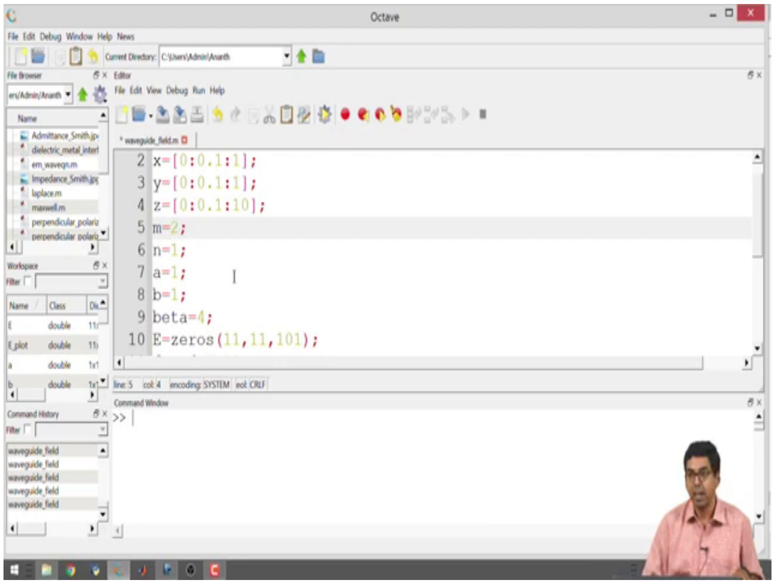

What I will do is, I will take my m to be equal to 2 ok which would be making this $TM_{2,1}$, right. Instead of $TM_{1,1}$, it is become T $M_{2,1}$.

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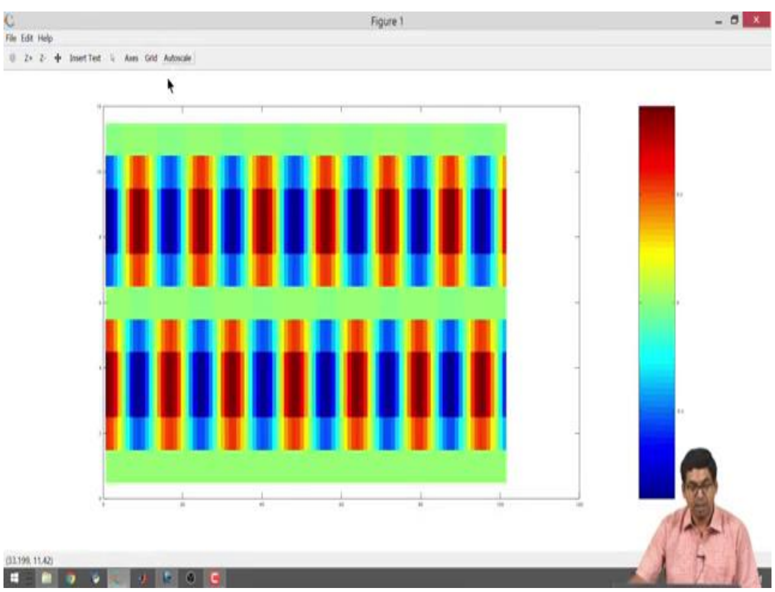

And I am plotting a slice only in the middle. That means, I do not know what is happening with the y-axis alright. I am going to draw this alright, ok. Now it means that in your waveguide, in the center of the waveguide, your electric field value has become 0 and there are 2 spots alright, and there are some distinctions that we can we can also see.

For example, one spot is having a value of say plus 1 electric field. At the same a you know, the same waveguide the other spot is having minus 1, right. So, it just tells you that your electric field is having 2 spots along this direction, but 1 spot is having 180 degrees phase shift with respect to the other spot.

So, we have to now remember that when we were talking about this half cycles of sinusoids coming into the picture, we noticed that one spot was going into the positive direction and the other one was going in the negative direction. So, its telling you that the relative phase shift between the 2 spots is a 180 degrees, ok. But they are travelling with the same β, ok. So wherever, you have a red on the top, you will have a blue in the bottom and so on alright.

So, this is ok. So, you can also keep increasing the number of value for m.

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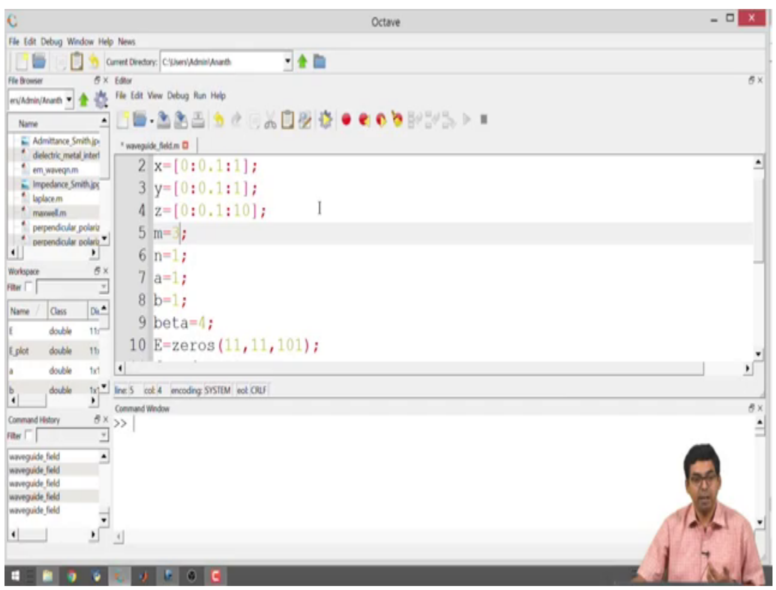

So, this is now TM $_{3,1}$ alright. And I am plotting the value of the electric field z component, alright .

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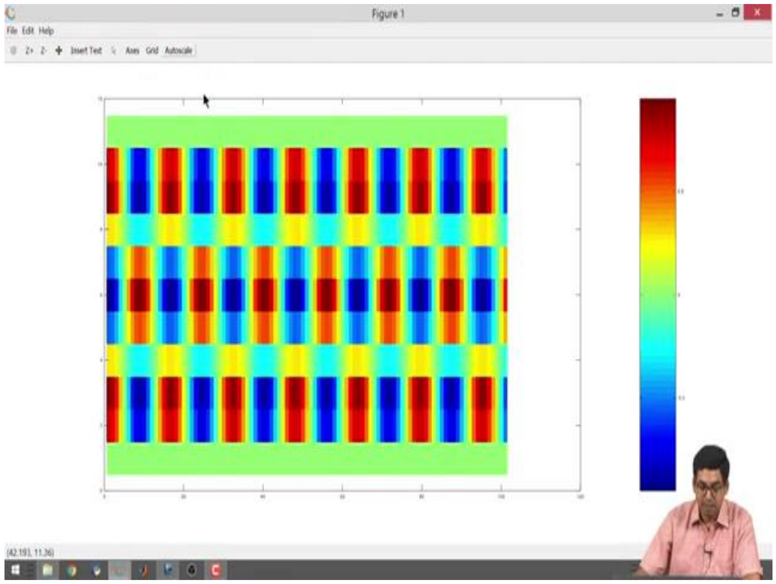

I start getting 3 spots starts to look very square and a little bit ugly, that is because of the way you have discretized. If you wanted to fix that and we are looking in the x-direction.

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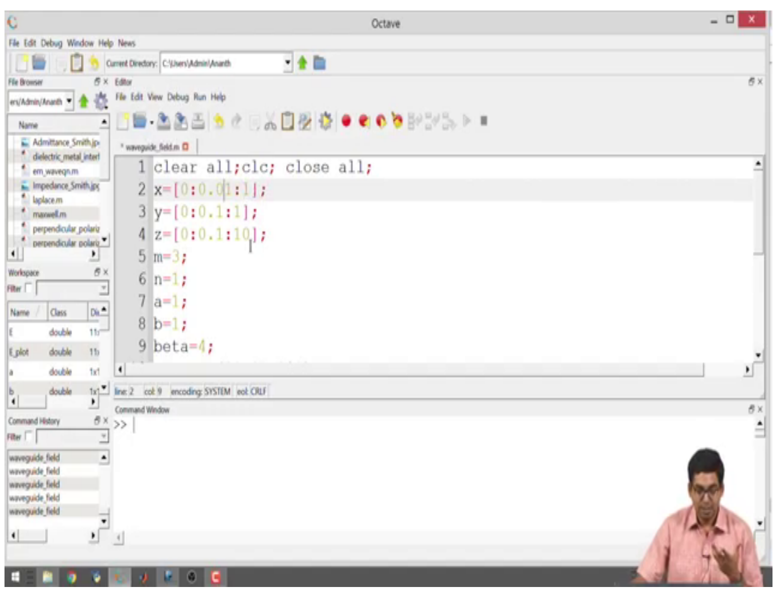

So, all I can do is I can just make it 0.01 alright, just to get a little bit more resolution and change the sizes of the arrays.

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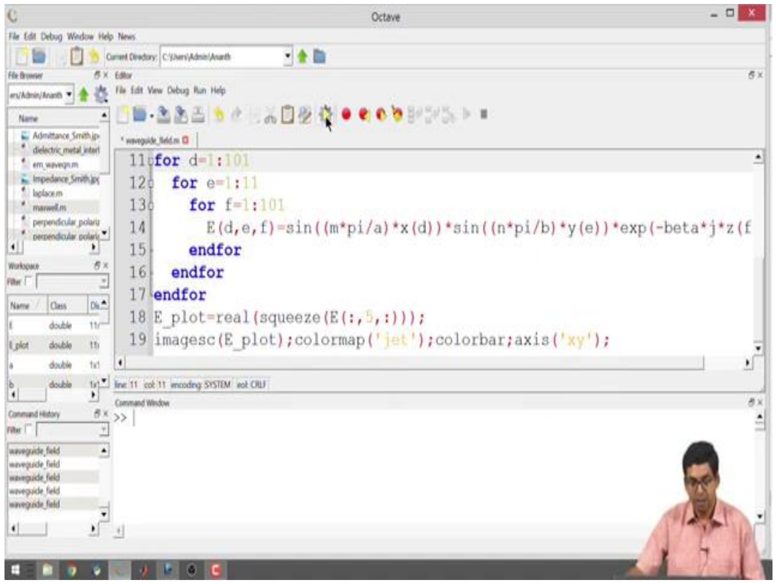

For example, I need 101 101 here, that is fine. And now the consequences it will take some time for it to calculate, right.

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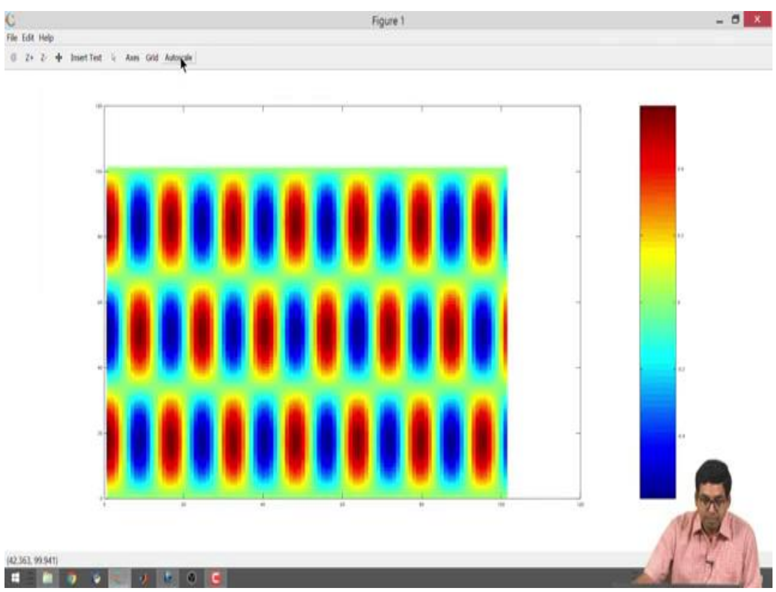

Now the spots are much clearer, alright. It looks like you know sinusoid half cycles very clearly. So, you could improve the resolutions and you could keep calculating in with better accuracies right. So now, this is corresponding to increasing values of m alright, and there is no end to it.

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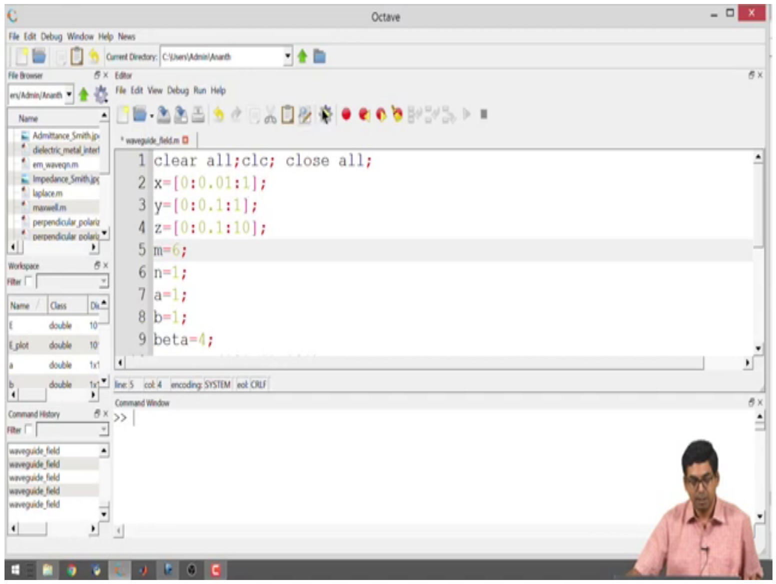

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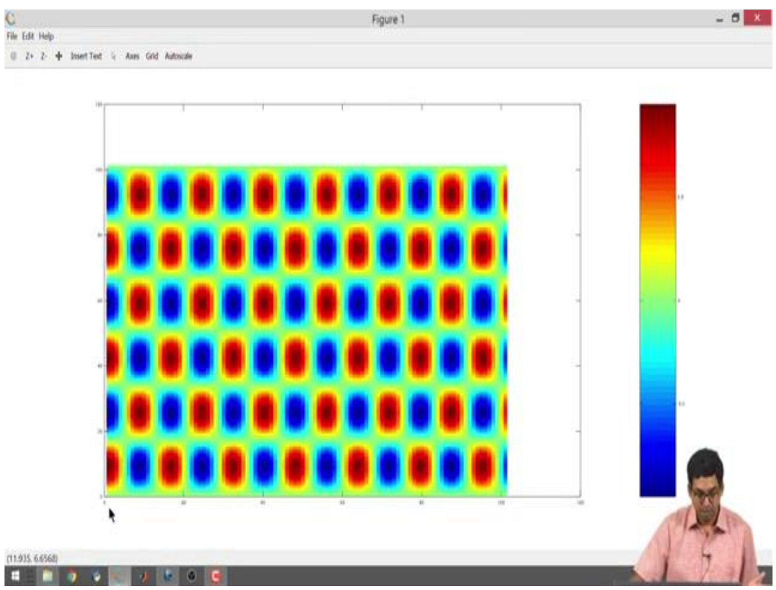

For example, if you want to make m equal to 6 this is $TM_{6,1}$, you will end up having, you know 6 spots. Each of them is having 180 degrees phase shift with respect to the previous spot, travelling with the same value of β along the length of the waveguide alright.

So, there is no end to a visualizing anything over here right. So, one of the things that we want to now do is, we will go back to say you know m equal to 3, ok. I want to play with n, alright. Now n equal to 1 corresponded to a single spot in the vertical direction and you have taken only the slice in the middle, alright. Suppose, I make n equal to 2, I should be able to tell because its a square wave it should have the similar effect.

If I make n equal to 2, you should have 2 spots: one at the bottom, one at the top, right in the middle there should be no field for n equal to 2. So, I want to verify this.

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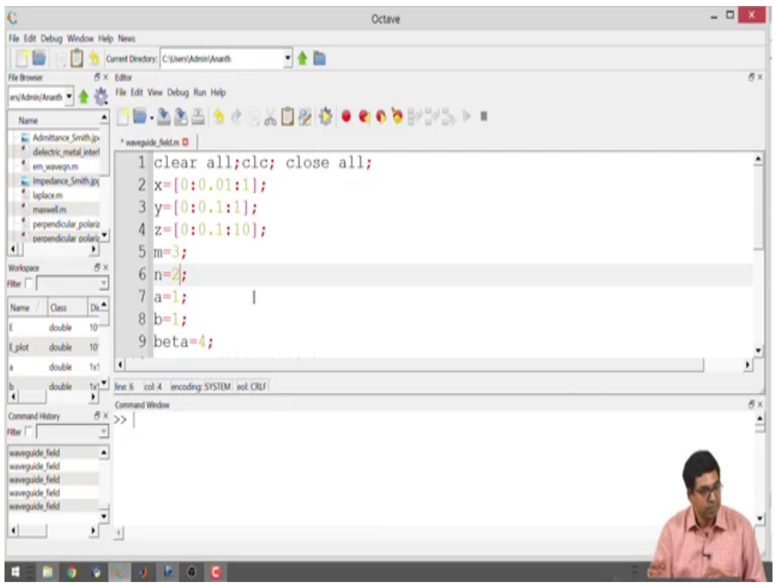

So, I am just going to make n equal to 2 and I am going to run it, ok. Now what is happening alright, we will have to sort this out, alright. Now n equal to 2, right in the middle alright, its not right in the middle, but its slightly away from the center, its having some value right, its not supposed to have that.

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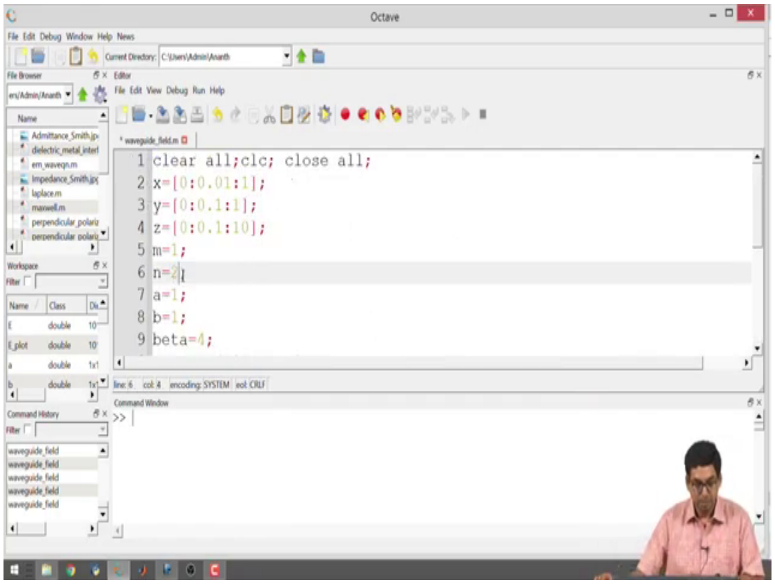

So, I want to go back and m equal to 1 and I want to sort this out, alright.

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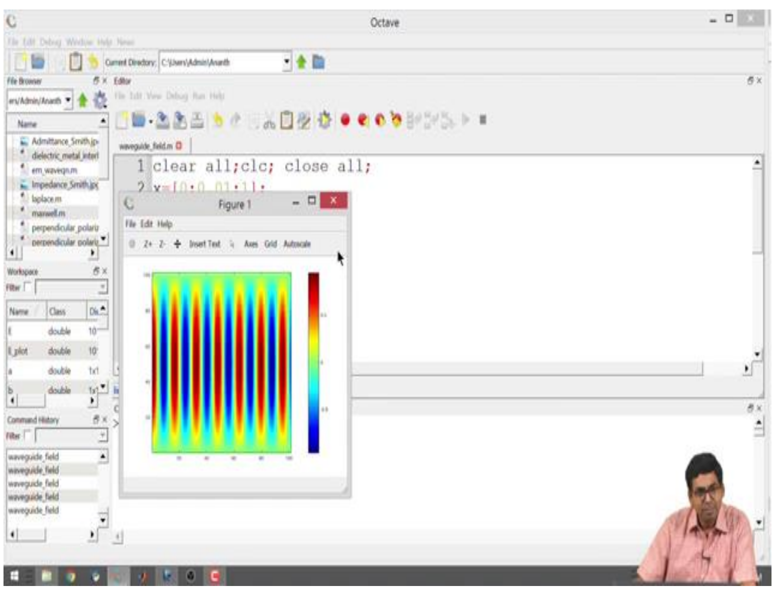

I am first of all, I am noticing that the value of the peak is lesser compared to the previous case.

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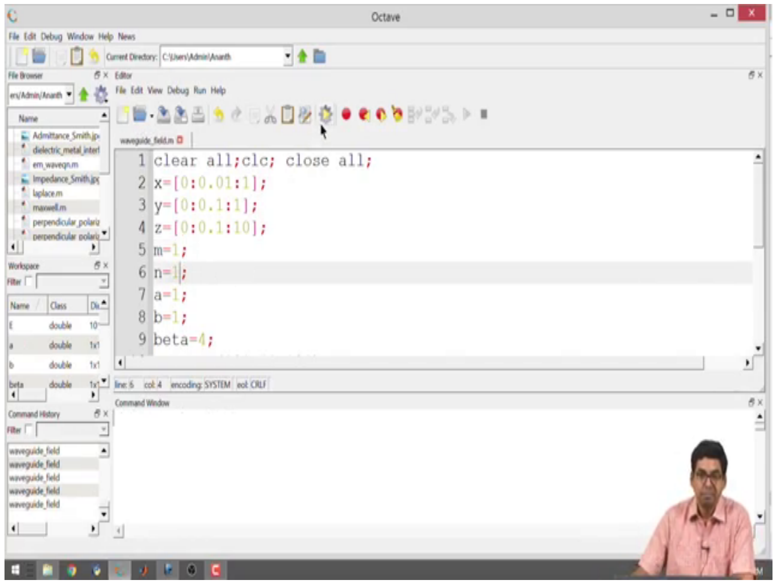

For example, in the previous case when I had m equal to 1, n equal to 1, right, the value is higher ok, and the value is becoming lower, that means, I am not hitting the center maybe, alright.

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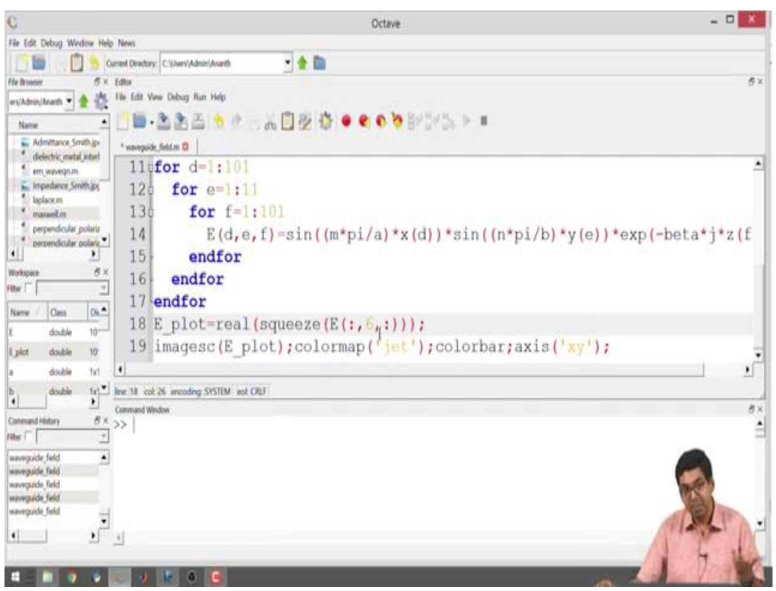

So, I can try to plot instead of 5 maybe 6 and see what is happening, right. So now, I am having I am going for n equal to 2, right.

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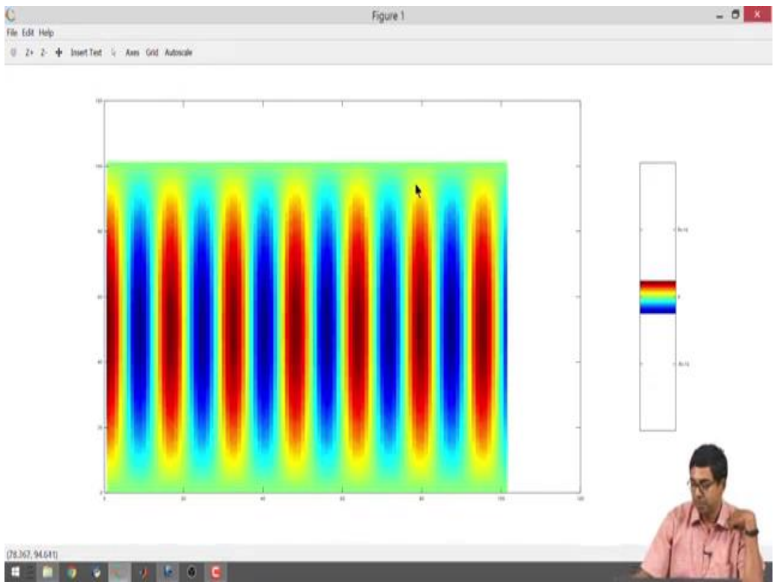

Alright, now I am able to see clearly that I was not plotting right at the center because, I had 11 elements, 5 is not exactly the center, alright. Now the little bit away from the center so it was getting some field from the extension of one of the spots but now, 6 seems to be like you know, closer to the center in this particular case. Even though, its an odd number, does not matter.

But if you see the y-axis , it clearly tell you that you have 0, the maximum is 5 times a e to the minus 16 which is again, you know, you can approximate it to 0, alright. And minus 5 e to the minus 16 over here. So, all the red to blue is focused around 0 alright. So even though it gives you colours, that means, that it is able to represent very tiny numbers accurately and it is giving you.

But technically, if you pick the a magnitude of this compare it to a n equal to 1 alright, here we have taken n equal 2, n equal to 1, n equal to 1 was having like 0.5 excess of 0.5, alright peak value. But here its only less than 1 times e to the minus 16 ok. So, you are having almost nothing in the middle alright. So, just to be just to confirm this again.

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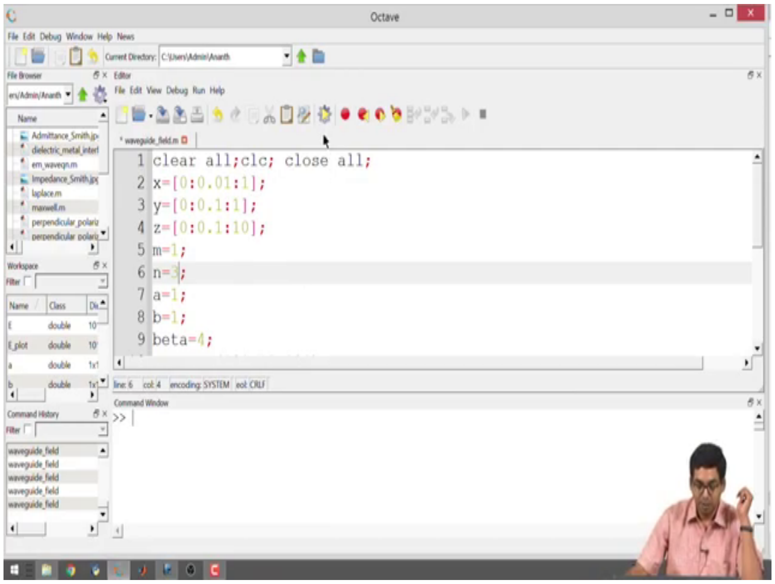

I am going to do n equal to 3 and see whether I get back the higher value of field closer to the center.

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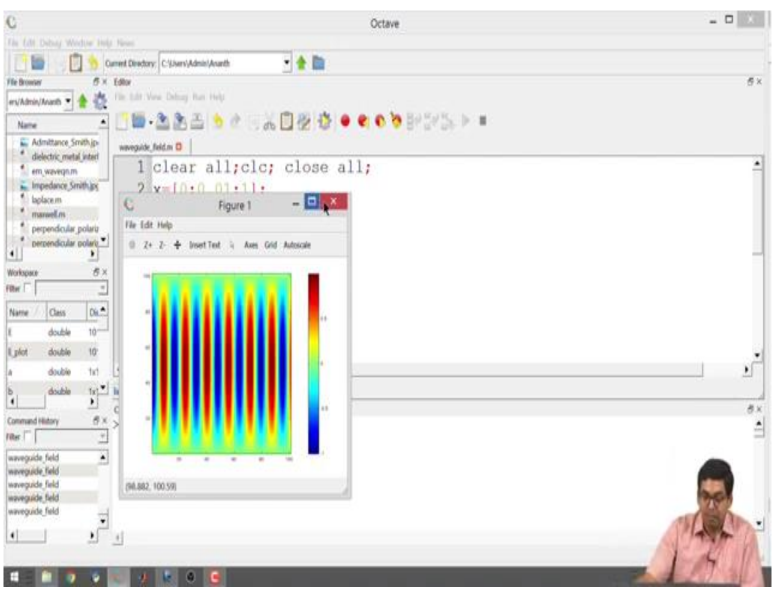

So, here I am able to see my field goes from minus 1 to all the way to plus 1, alright. So, you can run some experiments on this and you can always see what is happening and instead of taking a slice in this direction you could take a slice in the other, but I am not doing that because its a square cross-section. All its going to do is show the same effect in the 2 directions and its its not going to give me any significant difference, right.

So, this is a simpler way to visualize when you do not have to solve from the wave equation, but actually you know the general solution and you just want to make a plot of what is happening. This is a simple thing you could do, ok. Now we go back to our analysis alright, and look at a few points alright, few more points alright ok .

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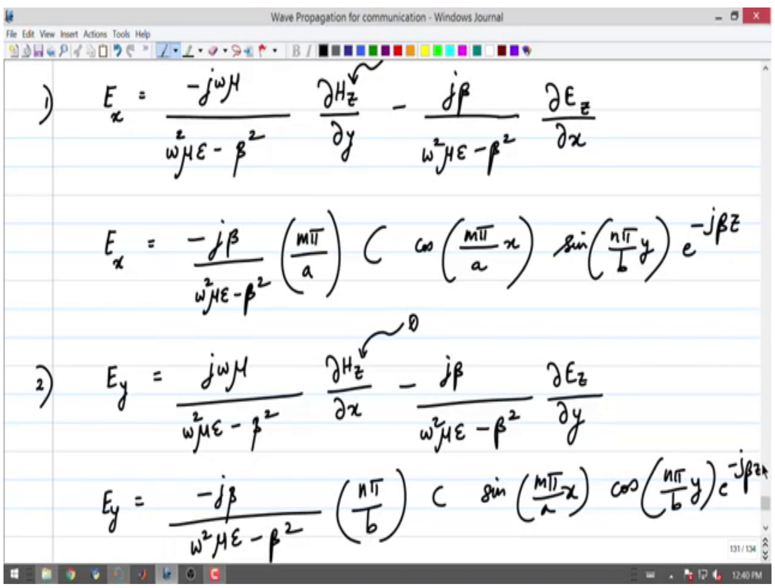

So, the first thing one could ask is, I want to be able to find out other field components and I want to be able to plot them, alright. Since you have a program that is plotting electric field at x, y, z you could also plot for example, Hx, Hy alright Ex, Ey extra. If you did know the general form that satisfies all these criteria.

Now, you have imposed particular boundary conditions, you have picked some general solutions which satisfy your boundary conditions, you have applied those boundary conditions and you have simplified the expression and you have got something for Ez, alright. But generally, you may want to find out what the other components look like, ok.

Now, that is very simple because now you are aware of the curl equations and in this stage in the course, all you need to do is go to the curl equations. Try to plug in this form of Ez into the curl equations alright, and you have to apply one more condition which is Hz is equal to 0, alright.

So, Ez is equal to this form, Hz is equal to 0 in your 2 curl equations and then you will be able to find out all the other components ok.

So, for example, I will just I will not derive it but I will just write down the forms and what it looks like and then you will be able to see that you will be able to get the a you know, form of Ey, Ex, Hx, Hy extra, right. So, there are 2 things over here, right . It would look like

$$E_x = -\frac{j\omega\mu}{\omega^2\mu\epsilon - \beta^2}\frac{\partial H_z}{\partial y} - \frac{j\beta}{\omega^2\mu\epsilon - \beta^2}\frac{\partial E_z}{\partial x}$$

Can it be is what Ex looks like, ok. And one of the things that I know is I know the form of Ez, alright. So, I could take a partial derivative with respect to x for Ez alright, and I can get rid of that term alright and Hz, the way we have done this is 0. So, first term goes off and there is only another term. So using this, I will be able to find out the form of Ex, alright.

If I did do this, I will just write down what I have got you can do the simplification when you have free time, alright . So because you take a derivative with respect to x, you get $m\pi/a$ because you had sin $\left(\frac{m\pi}{a}\right)x$. So, $m\pi/a$ and then you will have some cosine coming into the picture. So, I am having constant $(m\pi/a)x$.

With respect to y, there is no term that changes anything. So, you will have the same $(n\pi/b)y$ and it will also be travelling in the y direction. So you could in theory, find out all the other components using the curl equations it is a bit of an effort alright, alright.

$$E_x = -\frac{j\beta}{\omega^2\mu\epsilon-\beta^2}\left(\frac{m\pi}{a}\right)C\left[Cos\left(\frac{m\pi}{a}\right)x\right]\left[Sin\left(\frac{n\pi}{b}\right)y\right]\left[e^{-j\beta z}\right]$$

But its not impossible you can go back to the wave equation and you can look at you know how you can use the existing components to find out the remaining components. It is an effort, but it is not impossible, but I do not want to spend time on deriving each and every one of these, you will be able to do this or there will be references for this in many many I mean many sources. You could also have a look at them, but its its there is no magic or anything over here its just a laborious steps, right.

Just for the sake of completeness, I will just write it for Ey also. So I have

$$E_y = -\frac{j\omega\mu}{\omega^2\mu\epsilon - \beta^2}\frac{\partial H_z}{\partial y} - \frac{j\beta}{\omega^2\mu\epsilon - \beta^2}\frac{\partial E_z}{\partial x}$$

Once again Hz will be made 0, the way we have assumed, so you can get rid of this term and then you will have to substitute for Ez. I have to take a derivative with respect to y. So, I can pretty much write down what Ey would look like.

So, it will look like

$$E_x = -\frac{j\beta}{\omega^2\mu\epsilon-\beta^2}\left(\frac{n\pi}{b}\right)C\left[Sin\left(\frac{m\pi}{a}\right)x\right]\left[Cos\left(\frac{n\pi}{b}\right)y\right]\left[e^{-j\beta z}\right]$$

Similarly, you can also try to find out Hx, Hy, Hz, the way we have started all these things is by assuming that Hz is 0. So, there is no need to find.

Now, one of the things I want to highlight over here is the is in the in the configuration that we have seen so far ok, one of the questions that was arising in the previous class is what would happen if Ez is equal to 0, does it mean that everything else is 0? Or you could have a look at the expressions now. Hz is 0 alright, in the way we have formulated. Suppose, Ez becomes equal to 0 that could happen if m equal to 0 and n equal to 0, alright.

If that is the case, if you see the expression for Ex, it becomes 0, Ey will become 0, Ez, Ex, Ey will become 0. Should mean that your magnetic fields will also start to become 0, Hz is not there alright, and then your Hx and Hy everything will become 0. And then there will be no electromagnetic field components at all inside of the waveguide.

That is why m equal to 0, n equal to 0, is not possible scenario for the fields to exist inside of a waveguide ok. So, you will get all the field components to become identically 0, ok. So, when some things are identically 0, then you you say that there are no fields existing inside, ok. So, so, something to think about, ok.

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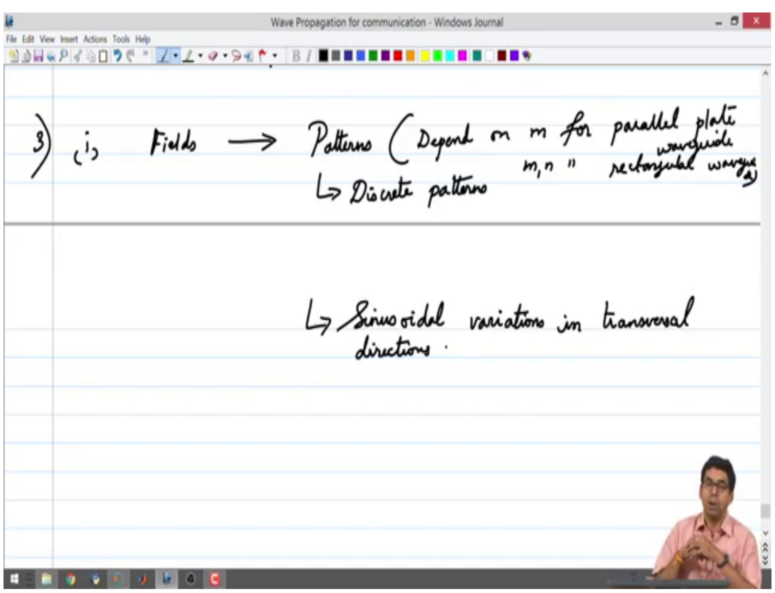

So, let us try to draw some more inferences. You can also do this for Hx, Hy, I think references are available, you can go back have a look at them, you can also plot them in the same program, alright. You can just add components and plot them there are no issues at all.

Now, a few things that we are seeing commonalities between this rectangular waveguide and parallel-plate waveguide is that the fields that we are talking about they form patterns. In the case of standing wave patterns were formed, in the case of parallel-plate waveguide, some standing wave patterns are formed in the case of rectangular waveguides also. So, the fields are existing in the form of patterns alright, and the patterns depend upon the value of m in the case of your parallel-plate waveguide, m comma n in the case of your rectangular waveguide, ok.

So, we can write that it depend. Ok, now since they depend on m and m comma n alright and we have to satisfy boundary conditions and m and n are integers, ok. Its not continuous numbers, m and n are integers. So for m equal to 1 you get one pattern, m equal to 2 you get a completely different pattern, m equal to 3 you again get a different pattern extra.

So, these patterns are actually some discrete patterns, ok. Here some discrete patterns alright, so, they go from being a single spot to suddenly 2 spots. They do not, you do not have one and half spots or something like that, alright. So, you have 1 spot or you have 2 spots or you have 3 spots, but you do not have something in the middle so, these are these are discrete patterns.

And the patterns consists of both in the case of rectangular and parallel-plate waveguide, they have sinusoidal variations, sinusoidal variations in transversal directions ok, that is if you go from one edge of the waveguide to the other side, you will be able to see for example, half a sinusoid or a full sinusoid or one and half sinusoids and so on and so forth.

So, you can see that the variation along the temporal or mean along the transversal direction is going to be sinusoid. Irrespective of how many dimensions you have, in the case of parallel plate you had only one direction going from one plate to another plate, in the case of rectangle you can go from one plate to another plate in the horizontal direction or you could go from one plate to another plate in the vertical direction. In both of these cases you had sinusoidal variations right.

So, these are some commonalities alright, and there are a few more commonalities that we have to talk about right. First of all, on the walls you will not have any fields, electric fields right, and you will also notice that in these cases, if you try to go back and look at what the magnetic fields would be like and try to plot them you will find out that the magnetic fields will be maximum on the walls.

That is because we have done the interface between a dielectric and a conductor alright, and we will we have already noticed that electric field will be a 0 on the interface alright, but at the same place the magnetic field will be high, right.

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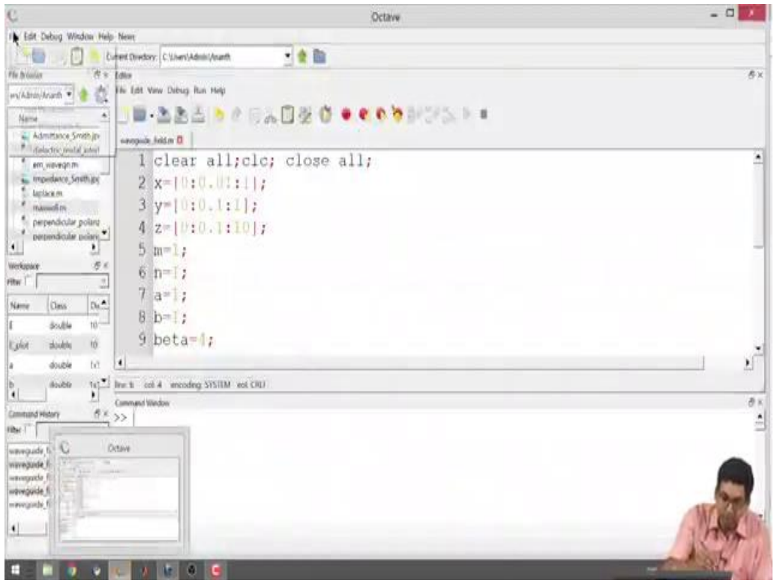

So, we also did some a calculations for a the you know, a program. We wrote a program a simple program in octave, I think it should be here for the perpendicular polarization with an interface, alright.

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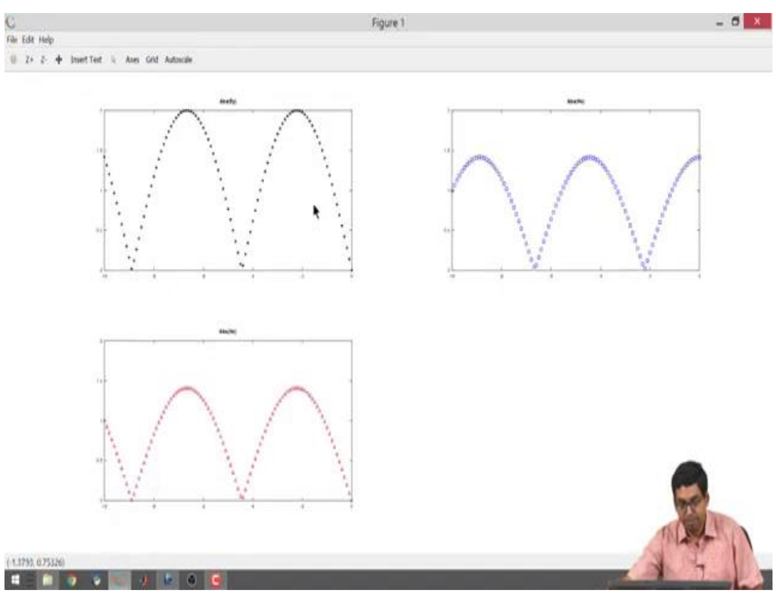

So, we also saw that on the interface, electric field is 0. Hz we have assumed it to be 0 in our case so, it does not matter. So, its going to be 0, but we saw that Hx which is a component that can be present in our case, is actually maximum alright. So, you will also notice that the magnetic field alright, is maximum on the interface electric field becomes 0 on the interface right.

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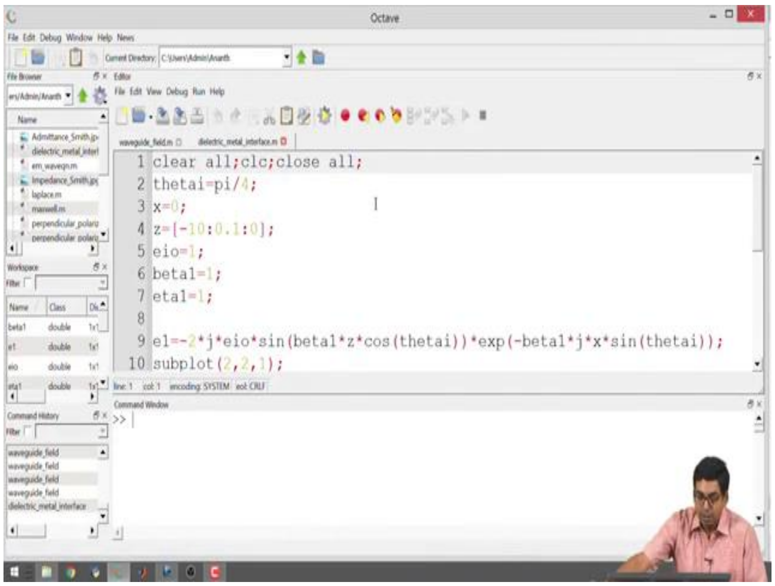

So, this is going to be common for both the parallel-plate waveguide and the rectangular waveguide, its one more commonality, alright. And more thing is that in the case of parallel-plate waveguide, we had seen that there exists a condition which is known as a your frequency of the signal has to be higher than the cut-off frequency for the fields to exist. Is there a commonality like that in this case also?

Because technically, it is also made up of 2 sets of parallel plates, this could also have some cut-off frequencies, ok. So, this is this also a high-pass filter right, and one of the ways to verify whether it is a high pass filter or not or whether there is a requirement on m, n extra, alright, is to take the solution to Ez what you have got.

So, this is the solution that we have got from our analysis, from the assumed general solution to applying the boundary conditions for this case, this is the solution you have got, right. What one can do is.

Student: 0.5.

Take this solution and actually plug it into the wave equation that we began with a, right. So, we had

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu \epsilon E_z = 0$$

This is the original equation that we started this derivation with.

So, you can go back and substitute right. One of the things that you could say is, I am verifying if Ez is a valid solution, whether it is satisfying the equality, whether the right hand side equal to 0 means that left hand side should become equal to 0 correct. So, you could go ahead and substitute it will be a lengthy process, alright. But one of the things you will notice is left-hand side has got lot of terms the sum of all these terms should become equal to 0 right, that is how this we started this.

So, for the solution to be valid and we know it is, it has to satisfy this equation and all the sum of all the terms on the left-hand side should become equal to 0. Now, you will end up with a condition for that also alright. You will notice that when you do the substitution, you had omega square mu epsilon to be the last term alright, and you will also end up getting some other terms alright, you will end up getting because of the way we have written Ez expression.

Ez has β alright, you go back to your original wave equation that you had started with, it had only

$\frac{\partial^2 E_z}{\partial x^2}, \frac{\partial^2 E_z}{\partial y^2}, \frac{\partial^2 E_z}{\partial z^2}$, it had $\omega^2 \mu \epsilon E_z$ it did not have β, it did not have m, it did not have n, it did not have a, it did not have b extra. So, I know that by substituting for Ez, I am introducing m, n, a, b alright, and I am also introducing β, alright. So, I need to account for all that alright, then I have to see what is the condition that is needed.

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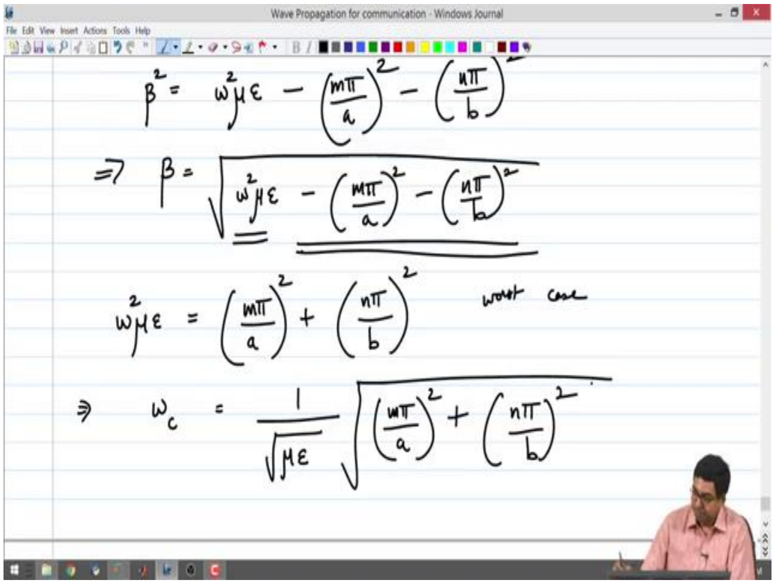

So, while introducing this right, you will notice that only way this equality is going to be satisfied is this

$$\beta^2 = \omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

This is just a rearrangement of the same expression ok, and I can write that this implies, I can take square root on both sides. So β will be

$$\beta = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Ok, just rearrangement and taking a square root and I can also make a say similar analysis as to what I did with my parallel-plate waveguide.

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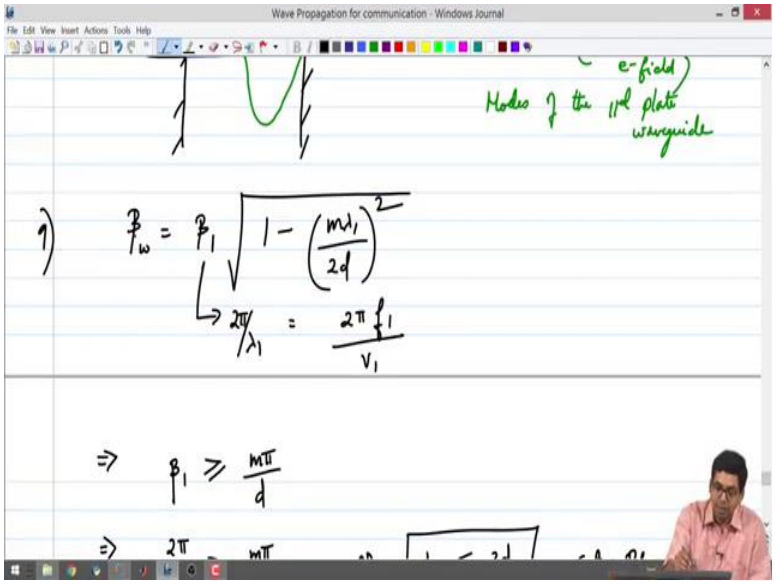

So, if you go back and have a look at what we did with the parallel-plate waveguide so, I had a condition for β of the waveguide is equal to something on the right-hand side.

So, I said that I had a square root term coming in between β corresponds to propagation constant for a travelling wave. So, you would have written this to be $e^{-j\beta z}$ which means, that β has to be a real number. β cannot be an imaginary number right β has to be a real number, alright. That means, the term inside the square root has to be positive that was our condition, ok.

So similarly, I am having a square root coming into the picture here also, and I am having the general solution for Ez to be having $e^{-j\beta z}$ which means β again has to be real which means that this square root should be for a positive number. I cannot have a negative number over there.

So, this gives me some idea as to what can happen, right.

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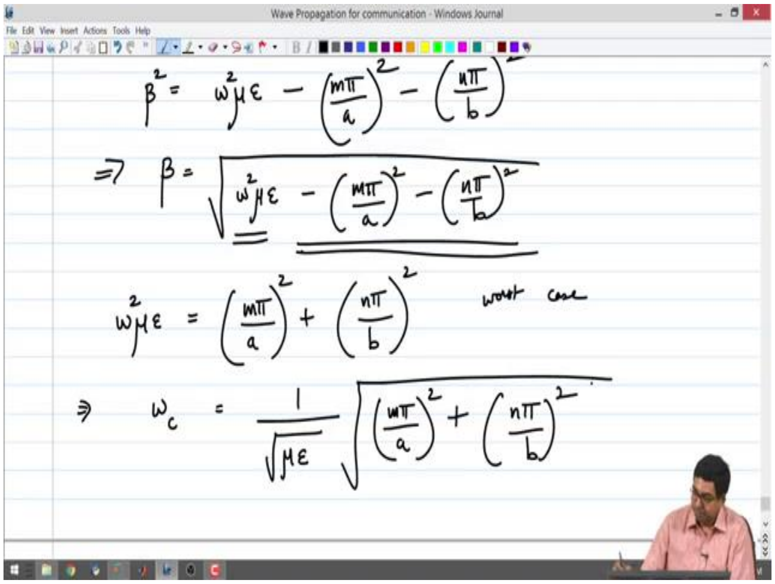

So this means that the worst condition that I can have the worst condition, alright is a the positive numbers being equal to the negative numbers. So I will become, I will be having β equal to 0 means, i do not have a travelling wave and anything above that.

When β is real, I am having a travelling wave, alright. So this means that I can write down

$$\omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Now this gives me something related to a you know, ω^2 alright. So, what I can do is I can take $\mu\epsilon$ on the other side and I can take a square root. So, I will just get up for the worst case alright, so I will just mark this as c right.

Its going to be

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

This is the worst case where β becomes equal to 0. β becomes equal to 0, you do not have a travelling wave alright this forms the ultimate lower bound and this is the frequency at which β is equal to 0.

That means, at frequencies higher than this alright, β is going to be a real number. So, you have a travelling wave. So, you can say that any frequency of your signal higher than this frequency has a real value of propagation constant alright, beta. So, you will end up having a travelling wave. So you can say that your signal alright, whatever you are giving has to be higher than this cut-off.

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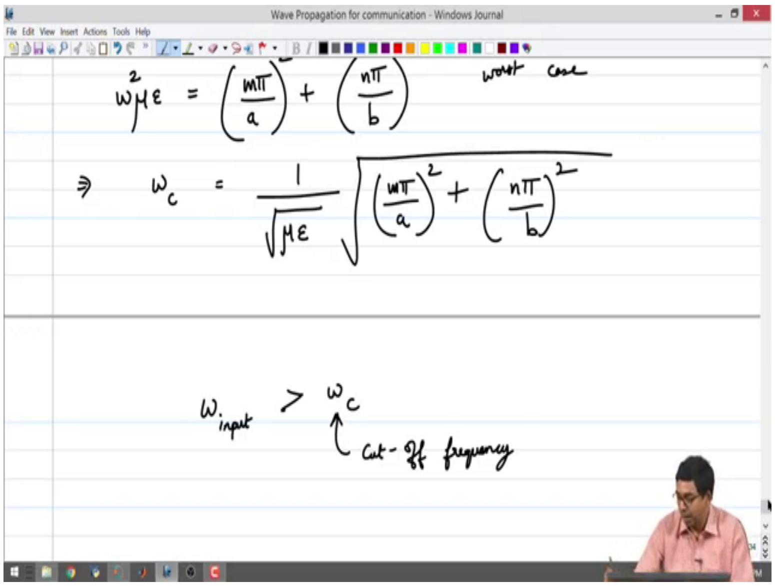

Also, we can say that omega input should be higher than omega c for you to have any travelling wave inside of this waveguide. So this condition is just saying that your input frequency has to be higher than a specific frequency which means that it is again a high-pass filter just like the parallel-plate waveguide, alright and you can call this absolute minimum case to be a cut-off frequency, right.

So, that does exist a restriction on omega input for it to function as a waveguide, ok. So, these 2 cases are very similar, parallel-plate and rectangular just that the parallel-plate was very easy to analyse because it had only you know 2 interfaces, but now you have many interfaces coming into the picture.

So we had used a different approach, we started with the general solution and then we tried to see a based on the standing wave patterns on the travel direction what kind of approximations you can make and come up with the solution that actually seems to be valid. Travelling wave in the z direction, standing wave pattern in the cross-section Because we had all this prior knowledge we had a different way of getting a valid solution, alright.

So, but even though we did that for the case of parallel-plate waveguide we still did it with the original method that is where we will write the incident wave, reflected wave, extra and then we will write down the expressions for the incident and the reflected, take some of the 2 to form the total field, all that.

So, these are 2 approaches to find out you know the expressions for the electric fields and also to find out cut-off frequencies, ok. There are also other ways for example, a you could use the start with the wave equation basically, and try to use the computer to do a few of the solutions to the MODS, ok.

So in other words, when you take the wave equation and you have a look at what the wave equation that we had started with, immediately, you will realize that while we were beginning, we did not know Ez we did not know what form it would have, what variation with respect to space it would have and all that, right. And we did not know much about omega square mu epsilon Ez as a result, because we did not know about Ez, alright.

And omega square mu epsilon alright, the way we have a written further alright. So, in the last line of our derivation if you see omega square mu epsilon is m pi divided by a square plus n pi divided by b square. That means, if I choose some value of m and n, I get some omega square mu epsilon. If I choose different value of m m and n, I get a different value of omega square mu epsilon extra.

So, omega square mu epsilon also is a discrete number that we were not aware of, right. Depends upon m and n. So, we were not aware of m, we were not aware of n and we are not aware of the pattern a given m and n would form in Ez, ok. So, we were we were having 2 unknowns. One is m comma n which we did not know and the second one was Ez, right.

So, what computers can do is try to find out eigen values of this equation and eigen vectors this equation. Eigen values will tell you about the omega square mu epsilon, right because these are discrete values and eigen vectors will tell you about the distribution of the field itself, the standing wave pattern of the field Ez, right. So, there are ways to do it.

So, those are all treated in your higher level courses in computational electromagnetics, where they will be talking about mode solvers and things like that. But essentially, it just means finding eigen values to the wave equation. Eigen values will tell you about m comma n and $\omega^2\mu\epsilon$ alright, and eigen vectors will give you the Ez, ok. And its not a very complicated a program or anything, but it is already covered in a higher level follow up course on computational electromagnetics or in waveguides.

So, you should be in a position to grasp that when that material comes in your course, that is the only thing. So, there are lot of commonalities and there are some differences also, right. These differences there are there are a tiny number of differences we will go over them the next class, right. For example, the fundamental mode supported here is TM_{11}, TEM mode alright, transverse electromagnetic mode. Is it supported in one case, not supported in the other case?

There are some differences also, even though both of them are constructed with parallel-plates, there is some differences. So, we will just highlight those differences briefly in the next class and go for a few more a concepts. That is we will then talk about what is known as phase velocities and we will talk about group velocities we will also talk about dispersion, right.

Its a very important thing because, if the waveguides have 2 plates and you have vacuum or air filling it, we already know from our prior classes that vacuum or air is a non-dispersive medium right. But if you make a waveguide out of it, suddenly it becomes dispersive. That is it will have different values of velocities for different you know, a different frequencies right. So, $\frac{\omega}{\beta d}$ all these things we will be seeing in the next class ok.

So I will stop here, we will meet in the next class hm..