**Transmission lines and electromagnetic waves**
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**Lecture – 07**
**AC Signals in Loss-Less Transmission Lines**

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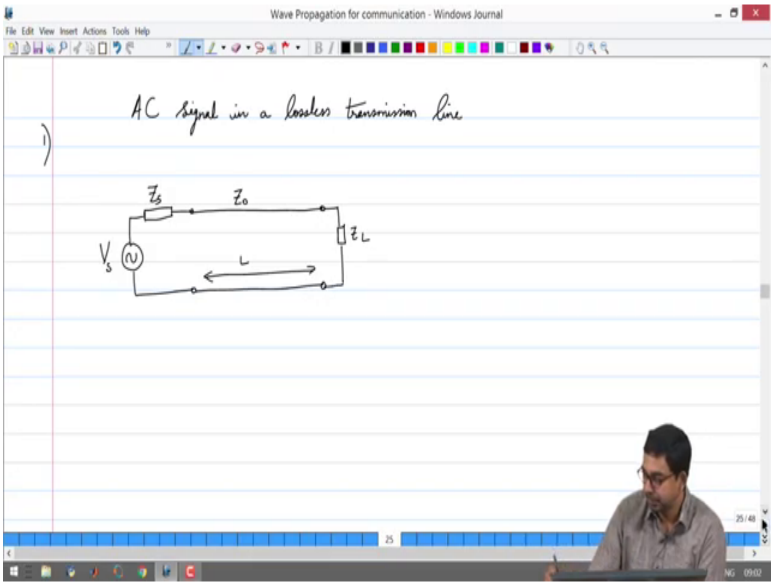


We will get started. We are going to start look at AC signal in a lossless transmission line ok that is why we have seen only the DC cases ok opening and closing switches and we know many things now, but let us go ahead and look at what happens with AC signals and try to see with the higher frequencies what happens, what is the impedance characteristics and impedance matching characteristics ok.

So, let us take the configuration of the transmission line, where I have an alternating source ok in the mark this as $V_S$. It has a series impedance $Z_s$ right and connected to a transmission line ok. The transmission line has a characteristic impedance of $Z_0$ similar to the characteristic resistance we will see more details about it as we go further ok. And the length is L ok and I am going to replace the load resistor, load impedance right ok. So, we want to study this configuration and figure out what the voltages and the currents in this transmission line would look like ok.

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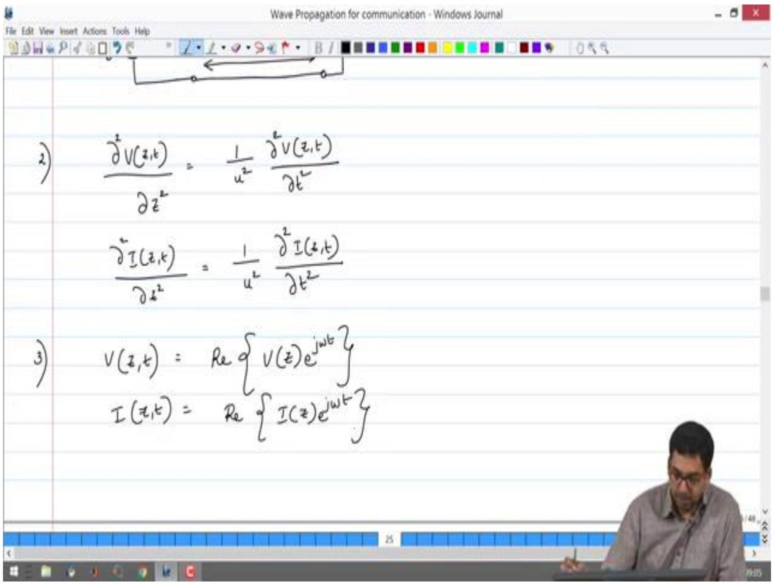


So, we start with the wave equation ok for the voltage ok. So, I have

$$\frac{\partial^2 V(z,t)}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 V(z,t)}{\partial t^2}$$

and the wave equation for the current will have a similar form all right.

$$\frac{\partial^2 I(z,t)}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 I(z,t)}{\partial t^2}$$

The previous time, we had written some general solutions, it's very simple and straightforward to understand. It was of the form

$$V(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-(t + \frac{z}{u})$$

Here we are just going to assume that our source is going to be sinusoidal in nature. So, the general forms will have slightly different format than what we saw before, but they will still be function

$$V(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-(t + \frac{z}{u})$$

So, I will start with the general solution for the voltage right first; we assume that the source is going to be sinusoidal all right. So, we can say that it could be a cosine right and the cosine could simply be written as

$$V(z,t) = Real\{V(z)e^{j\omega t}\}$$

So, we want to write down $VCos\omega t$ all right, but in order to simplify the math we are actually writing this term as

$$V(z,t) = Real\{V(z)e^{j\omega t}\}$$

Reason is because I have to take two partial derivatives with respect to time all right on the right hand side.

So, I know that the derivative of the complex exponential is simpler, I just have to take $j\omega e^{j\omega t}$ and each time I have to take a derivative, I have to multiply the term with $j\omega$ that is it. So, it's easier for me to do this and in the final results one can always take the real part or the imaginary part depending upon the kind of the source ok.

So, it's just a mathematical simplification ok and the I(z,t) can also be written as

$$I(z,t) = Real\{I(z)e^{j\omega t}\}$$

So, the time dependence is clearly periodic ok.

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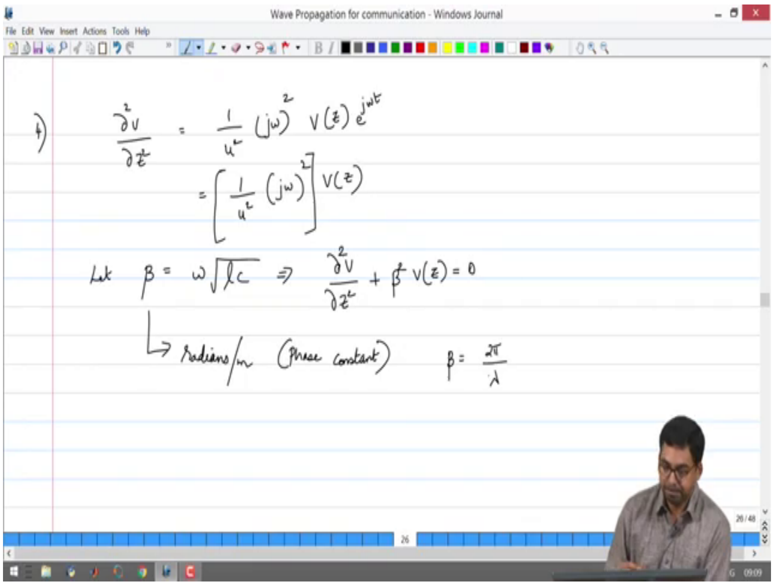


Once we do this all right we can write this down or substitute this in our wave equation right. So, we can write

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}(j\omega)^2 V(z)e^{j\omega t}$$

and then I will say ok.

Now, since the dependence with respect to time is simply periodic right we can try to analyse what happens with respect to space ok. So, we know that it is periodic in time, the signal is periodic in time, we want to know what it is with respect to space ok. We know that the space coordinate and the time coordinate are coupled because there is a term velocity coming into a picture for the transmission line all right. So, we can assume some arbitrary value of time alright that will simplify the amount of maths for us to write down the general solution all right.

So, we could always say that let us pick one time instant, where the phase $e^{j\omega t}$ that is the $\omega t$ part goes to say 0. So, you could always substitute sum t equal to 0 right and then get rid of this term and when you want the time dependency, you can always bring back this term ok. So, this means that you could write this down as $1/u^2$ ok and whenever I want to talk about time dependence, I will multiply everything with the $e^{j\omega t}$ ok. And let us also define a new term for the quantities present over here right.

Let us say that let

$$\beta = \omega\sqrt{lc}$$

This will mean that the wave equation will be $\frac{\partial^2 V}{\partial z^2}$ all right because I have j square. So, I can bring the quantity to the left hand side. So, I will have

$$\frac{\partial^2 V}{\partial z^2} + \beta^2 V(z) = 0$$

We have just rewritten the wave equation for the case, when the signal is time harmonic ok and representing it in the form of exponentials right and we are taking the real part over here to simplify the math ok.

So, the wave equation looks like this and let us quickly do a dimensional analysis while we are add it all right $\omega\sqrt{lc}$ ok. So, this has the unit of radians per meter and it is also known as a phase constant ok. So, a when the solution which is going to be a wave is travelling in space, while it travels the amount of phase it accumulates can be calculated by taking the value of $\beta$ and multiplying it with the distance it has travelled ok.

So, the amount of phase is going to be given by the phase constant. This can also be written in a slightly different wave with respect to $\lambda$ all right. So, $\beta$ can be written as $\frac{2\pi}{\lambda}$, the reason why is it why it is a constant is because the phase has to be accumulated by a number of $2\pi$ radians when the wave travels a distance of $\lambda$, that is the definition of wavelength ok.

So, depending on the value of $\lambda$, the value of $\beta$ can change ok. But for a given wavelength right if it if the wave progresses in one direction with the distance equal to that of the wavelength, it will accumulate a phase of $2\pi$ ok.

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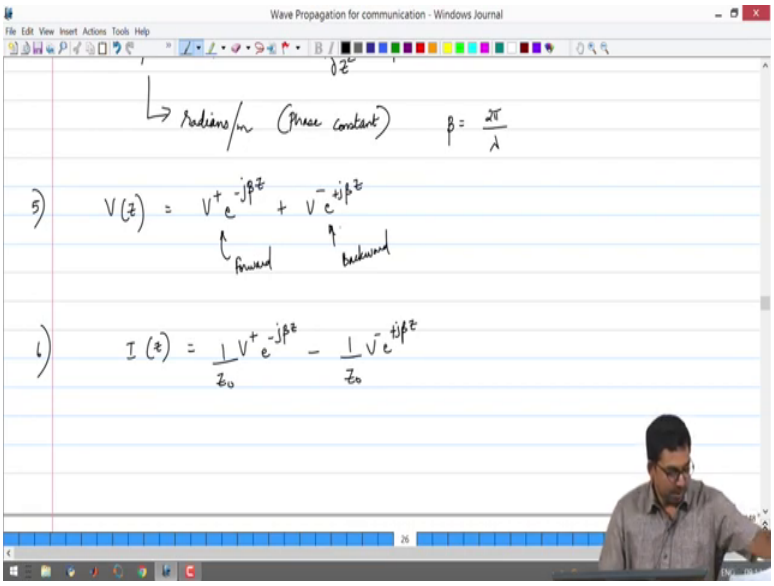


Now, once we have written our wave equation in a slightly simpler form, removing the time harmonic part and writing only the spatial part, we can always write down the general solution to be

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{+j\beta z}$$

It is very similar to the case of forward and backward waves that we saw in the case of previous lectures. All right I will make sure that now it is becoming periodic in time and as a consequence it also seems to become periodic in space.

If you look at the forward voltage wave over here, we see that it has $e^{-j\beta z}$, that means, that as you travel in the special direction, you will encounter some periodicity in the voltage waveform ok.

So, anything that is periodic in time, it’s also going to become periodic in space all right and here we are having the phase constant telling you how fast right the phase will be accumulated by a travelling wave the space right. And just like the transmission line case with the DC excitation, we do have a forward and a backward wave ok ok. One could also write down the currents, how they look ok.

So, if the characteristic impedance of the transmission line is going to be $Z_0$ right then we can just use Ohm's law, and the knowledge that we already have from the previous lectures. Remember that for the backward wave, you will have a minus sign coming into the picture for the current, simply because we have discussed that the reflection coefficient for the current will be negative of the reflection coefficient of the voltage. So, some care has to be taken.

So, it will mean that that the forward voltage divided by the forward current will give you characteristic impedance $Z_0$ and the backward voltage divided by the backward current will give you minus z naught, but one should not worry about -$Z_0$ as negative resistance or negative impedance extra. It just tells you the direction of energy flow is from sink to the source all right not from the source to the sink. So, you should not confuse this with negative resistance, negative impedance and all that ok.

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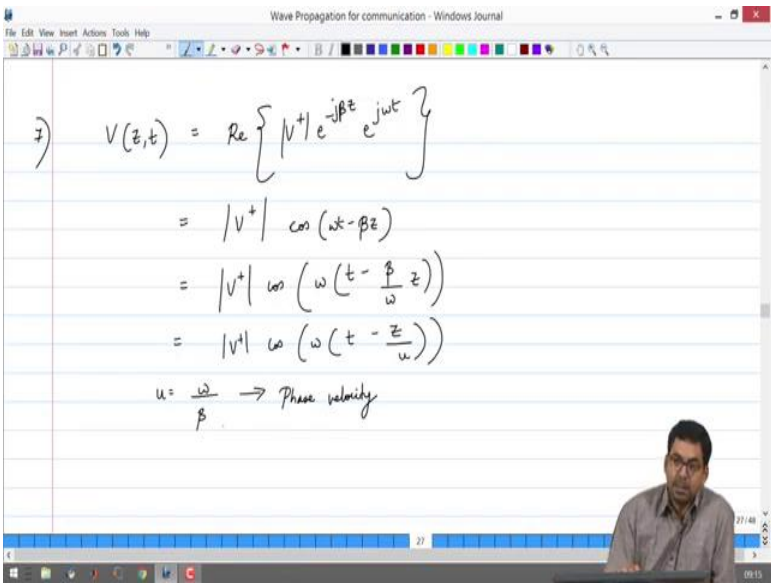


Once we have this, we have to still establish some relationship with the solutions that we have seen in the past all right.

So, we can say that the voltage is a function of space and time all right that is how we started with and the way we broke this down was it say

$$V(z,t) = Real\{|V^+|e^{-j\beta t}e^{j\omega t}\}$$
$$= |V^+|Cos(\omega t - \beta z)$$

$$= |V^+|Cos(\omega\left(t - \frac{\beta}{\omega}z\right))$$

Now we have to establish a clear correlation with the kind of solutions that we had seen before. So, we will just do a little bit of manipulation ok.

The way we have defined $\beta$ all right it is $\omega\sqrt{lc}$ ok. So, we can always go back and try to write down what this $\frac{\beta}{\omega}$ will be right. So, this will look like

$$= |V^+|Cos(\omega\left(t - \frac{z}{u}z\right))$$

Now, once again it is clear that it is a function of t - z/u for the forward wave. So, I think that you will be able to understand that, there is clearly a correlation between the kind of solution that we saw in the past and the solution for steady state AC excitation that we are seeing now. Both are the power cases of function of t - z/u, the function here just happens to be cosine of you know t - z/u multiplied with an omega and for the backward wave one could establish the same it looks like

$$= |V^+|Cos(\omega\left(t - \frac{z}{u}z\right))$$

Now, since this is established we have encountered some new terms alright. $\beta$ alright is known as the phase constant and u has been defined as omega divided by $\beta$ ok. So, you will be having $2\pi$ a radians per second alright for the numerator and radians per meter for the denominator. So, you will end up having meters per second.

This term is known as phase velocity, it just tells you that for a given wave of a specific frequency or a specific wavelength ok how fast it's a phase travelling alright and when somebody defines phase velocity, automatically means that they are talking about AC excitation of a transmission line otherwise in DC you will not talk specifically about phase velocity, we will be talking just about the velocity alright.

So, this omega divided by $\beta$ if it is referred to as phase velocity, they are talking about AC excitation of a transmission line ok.

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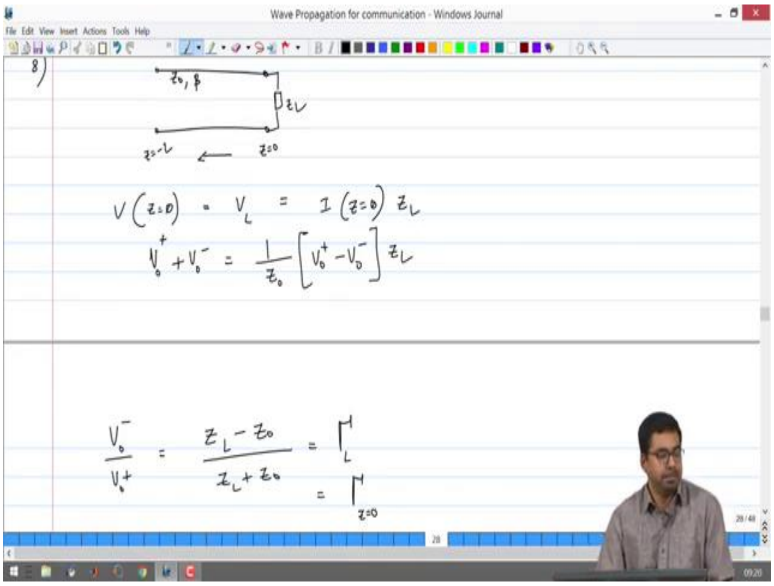


Since we are aware of the general solution to the loss less transmission line for AC excitation right we can always go ahead and try to see what will be the equivalence of what we saw for the DC excitation that is terminations open circuit short circuit conditions, reflection coefficient these are the conditions that we need to see ok.

So, we can start with a setup all right. So, as usual we will forget the source part ok. So, we will start with some terminations, I will have a transmission line ok and let say its characteristic impedance is $Z_0$ right phase constant is $\beta$, I have a load resistor $Z_L$, there are too many $Z_S$, but the z that I am writing below is the special coordinate ok.

So, if you look at the spatial coordinate convention it's slightly weird ok. z equal to 0 corresponds to the load end and z = -l corresponds to the source end you can use any other convenient system for spatial representation. I am doing this simply because I am interested in finding out the reflection coefficient at the load end. So, $\Gamma_L$ what we had seen before is what I am. I want to find out if the position is 0, I can get rid of many terms in my mathematics.

So, I am just doing a simplification over here. So, it is not uncommon to find a slightly different coordinate definition system, usually people will start with z equal to 0 on the left hand side z equal to l on the right hand side. Anything is ok as long as you are consistent with that there are no hard rules regarding this all right. If we do this all right, we can say that the voltage at z equals 0 ok. The voltage at z equal to 0 will correspond to the load voltage all right is equal to the current that is flowing in this branch with the impedance $Z_L$ multiplied by the $Z_L$ itself.

So, you are applying Ohm's law at the load end V=I*z ok. So, it's going to be I at z equal to 0 multiplied with $Z_L$ ok and a at the position z equal to 0, you will be having a forward and a backward wave. If we have a look at the solutions that we have written, we have $V^{+}e^{-j\beta z} = 0$ makes this term equal to 1. So, that makes my math simpler and hence the choice of the coordinate system. $V^{-}e^{j\beta z} = 0$. So, the math becomes simpler over here. So, at z equal to 0 I can write down the voltage to be

$$V_o^{+} + V_o^{-} = \frac{1}{z_0}[V_o^{+} - V_o^{-}]z_L$$

This is the expression for a I mean this is the expression that we are getting from Ohms law on the load end of the transmission line and we are interested in finding out the ratio of $V_o^{-}/V_o^{+}$ ok.

It's a simple rearrangement right and you will end up getting

$$\frac{V_o^{-}}{V_o^{+}} = \frac{z_L - z_0}{z_L + z_0}$$

This is very very similar to the reflection coefficient that you got for the dc excitation, there it was $\frac{R_L - R_C}{R_L + R_C}$ in our lectures here it is simply $\frac{z_L - z_0}{z_L + z_0}$ all right and we will call this as $\Gamma$, and to say that it is $\Gamma$ at the position of the load, we just replace it with the suffix of L we just say that it's a load.

$$\frac{V_o^{-}}{V_o^{+}} = \frac{z_L - z_0}{z_L + z_0} = \Gamma_L$$

Sometimes this convention can become confusing especially when you have the length marked as minus minus l and all that. So, it is ok to be very specific you can always say $\Gamma$ at its ok right ok. Now compared to the previous case where $\Gamma$ was a you know $\frac{R_L - R_C}{R_L + R_C}$, here we are having $\frac{z_L - z_0}{z_L + z_0}$ the most generic case, it means that each of the quantities $Z_L, Z_0$ extra can actually be having some component of resistance, some component of inductance, some component of capacitance extra all right.

However, we are considering some lossless transmission line in this case. So, we are still not attached to our series resistance to the inductor parallel conductance to the capacitor yet ok. So, the solutions are slightly easier ok. And the other thing that we need to start looking at is, what will happen in the transmission line when the termination is extreme.

Say for example, it's a short circuit or an open circuit, these are the two cases that we have seen for the DC excitation, you see the same case and on top of that there is something funny happening with respect to the impedance in the transmission line, we will have to look into that ok.

So, we will start with the case where the transmission line load end is open circuited ok ok.

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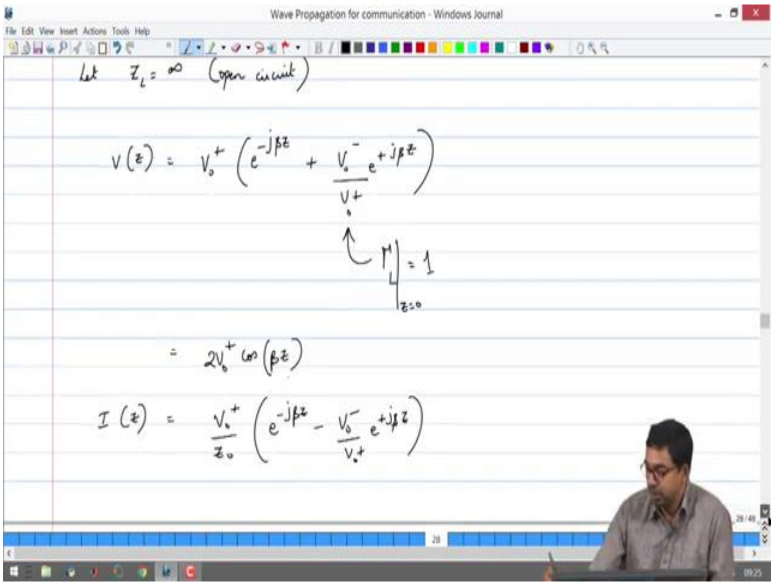


When $Z_L$ is equal to infinity,

$$V(z) = V^{+}(e^{-j\beta z} + \frac{V^{-}}{V^{+}} e^{+j\beta z})$$

So, I have written in this form so, that I can substitute for $\frac{V^{-}}{V^{+}}$ as $\Gamma_L$ ok.

So, when it is open circuited, this is going to be the reflection coefficient, it's going to be equal to 1 ok. So, I can just substitute one for the reflection coefficient and I end up getting

So, we will be having this is going to be

$$V(z) = 2V_0^+ Cos(\beta z)$$

Now, this is a very important thing to look at right. It means that the voltage at any position z in the transmission line ok will vary as $2V_0^+ Cos(\beta z)$, where z is taken from the load towards the source end. But what it also means is that there are values of z, where the voltage in the transmission line is going to be 2 times your forward voltage reaching the load end ok.

So, the maximum rated voltage for a transmission line should always be considered based on the superposition of your **forward** and backward travelling wave. So, the peak voltage at specific positions at the transmission line can be double ok. Similarly, if I wanted to calculate the current in the transmission line at a position z, I can use the expression for the current from the general solution. So, I have

$$V(z) = \frac{V^+}{z_0}(e^{-j\beta z} - \frac{V^-}{V^+} e^{+j\beta z})$$

So, I am having V naught plus divided by $Z_0$ multiplied by some exponentials, but I do have the minus sign here to indicate that it's current reflection coefficient is negative compared to the voltage reflection coefficient.

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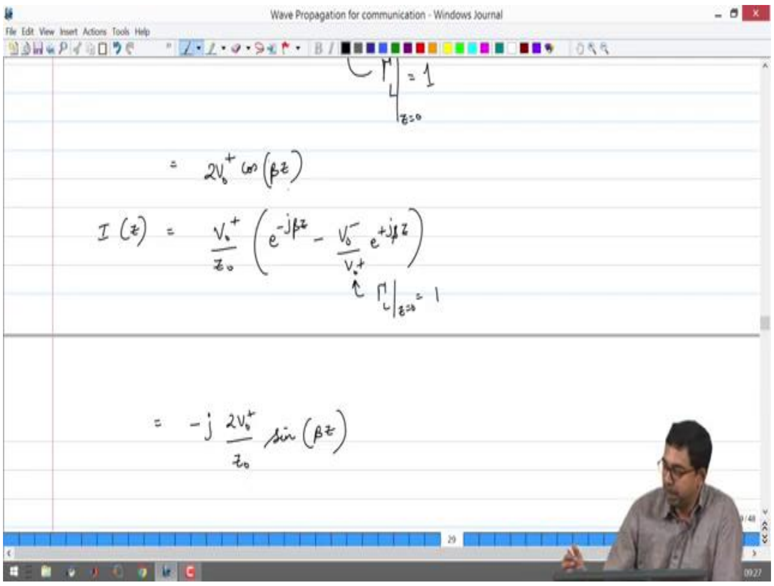


So, here once again we can substitute $\Gamma_L$ and you get a difference between two complex exponentials right and you can write this as some sign right sinusoidal right. So, in this case it will be looking like

$$I(z) = -\frac{j2V_0^+}{z_0} Sin(\beta z)$$

So, when your load is open circuited depending upon the position all right you will have a voltage going with the form $2V_0^+ Cos(\beta z)$ and the current is going to be $-\frac{j2V_0^+}{z_0} Sin(\beta z)$ alright. In this case there are a few things that should be very very clear ok.

First of all, as you go along the distance of the transmission line, there are going to be specific places in your transmission line. For example, z equal to 0 all right and then $\beta$ * z is going to be some multiples of $2\pi$ or $\pi$. You will have repetitions of the voltages occurring in your transmission line so, it will be repeating again and again and again alright.

The peak value of the voltage that can happen in your transmission line is $2V_0^+$, the current is $= -\frac{j2V_0^+}{z_0} Sin(\beta z)$. So, it is also periodic. The only thing that we notice is there is a phase difference between the current and the voltage ok.

This j should signify something ok it should signify something. So, if you take the ratio of voltage to current, we know that we will get an impedance to tell something about the impedance, then the current and the voltage are out of phase extra ok.

So, if we want to have a closer look related to the impedance, we need to write down the expression for V of z divided by I of z and then we need to see how the impedance in this line will change. We already have a very good guess alright cos($\beta$z) divided by sin($\beta$z) alright.

So, there is a good possibility that at position z equal to 0 the denominator will become 0 for the expression for your impedance and it is going to look like it is infinity. But there is going to be another place where the numerator is 0 all right. So, you will be having z equal to I mean the impedance is equal to 0 extra.

So, we need to be clear about what is the distribution of impedance in the transmission line and we need to derive some meaning from it ok. So, in order to make this problem a little bit clearer ok we will draw the entire system with the source ok.

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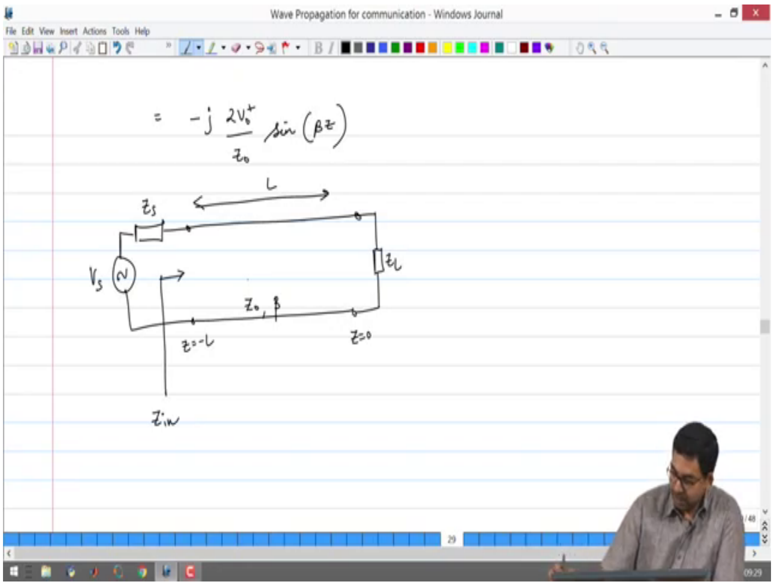


So, I will be having $V_s$, I have a series impedance $Z_s$ I have a transmission line, I have a load impedance $Z_L$ ok. The length of the line is L and I am maintaining the same coordinate system which means that I am going to mark the load end to be position z is equal to 0, position z is equal to minus L ok.

The characteristic impedance of the transmission line is $Z_0$ and the phase constant is $\beta$. I want to understand looking into this transmission line, what is the impedance that I will be seeing right. So, if I look into the transmission line towards this direction to mark the arrow, I want to understand what is the impedance that I would be seeing ok. So, let us take the scenarios.

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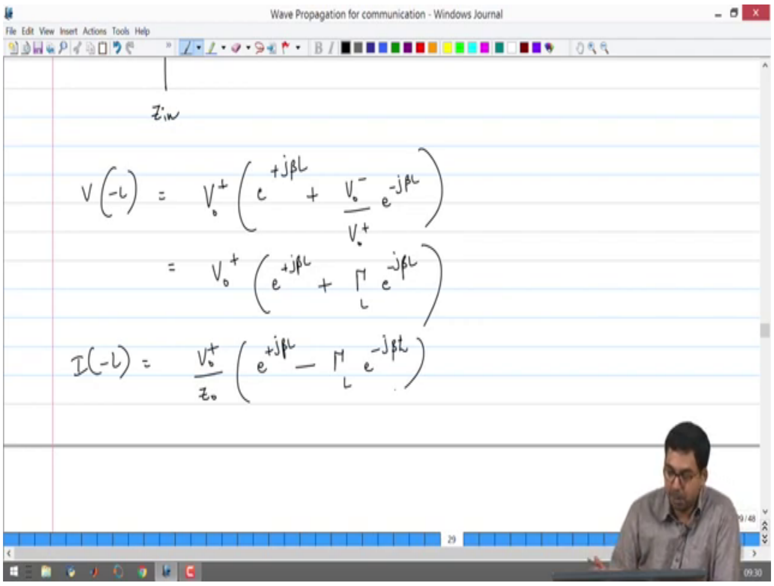


So, I can start with the expression for what will happen at the input. So, it is V at z equal to minus L in our expression alright. So, I am just going to write this term.

$$V(z=-L) = V_0^+(e^{j\beta L} + \frac{V^-}{V^+}e^{-j\beta L})$$
$$= V_0^+(e^{-j\beta z} + \Gamma_L e^{+j\beta z})$$

Where , $\Gamma_L$ is the load reflection coefficient, the current at the input end of the transmission line it's going to look like

$$I(z=-L) = \frac{V_0^+}{z_0}(e^{j\beta L} - \Gamma_L e^{-j\beta L})$$

then all I have done is substitute z is equal to L in the general solution for the voltage and the current is not done any more than that. I just want to take their ratios and I want to understand what the impedance will look like all right.

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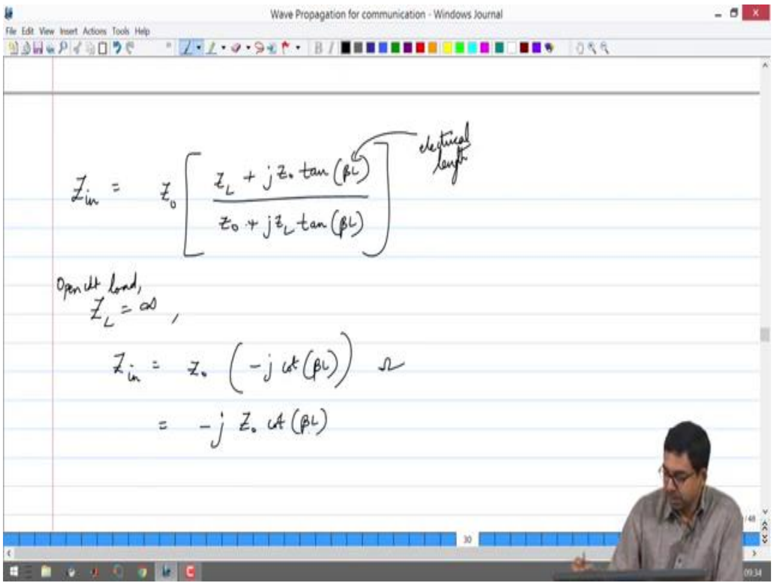


So, I can take the ratio of V at minus L to I at minus L right. So, I am going to be ending up with

$$Z_{in} = Z_o[\frac{Z_L + jZ_0\ tan\ tan\ (\beta L)}{Z_0 + jZ_L\ tan\ tan\ (\beta L)}]$$

This is actually a very very complicated expression ok. It's very complicated expression and one cannot get a feeling of what the z n is actually going to look like when the transmission line is a is of length L. But one thing is clear if the transmission line is of length L alright even within that transmission line, we are going to have some repetitions that are happening.

So, because these are trigonometric functions they do have a period alright all that matters is $\beta$L within your brackets with the trigonometric function it does not matter what your L is. So, it is going to be repeated every once in a while. So, it clearly justifies that if I take a small section of the transmission line and if I understand its properties from say $\beta L$ is equal to 0 to $\beta$L is equal to 2 $\pi$ I would have understood what all the transmission line can do.

So, for this reason the term $\beta$ L is usually known as electrical length ok. The term $\beta$ L is known as electrical length simply because it just tells you the phase between 0 and 2$\pi$ ok and we started our original derivations for the case when the load impedance is going to be infinity or open circuit. So, here we can do the same thing $Z_L$ is equal to infinity, you can as the case we started with open circuit load alright if this is the case right you can express z in.

So, common mistake that people do is a substitute infinity plus something divided by $Z_0$ plus infinity whenever you have infinity coming in, that means that you have to take it to the denominator somehow. So, the you can divide the numerator with $Z_L$, denominator also with $Z_L$ that way you will have $Z_0$ by $Z_L$ and $Z_L$ by $Z_L$ in the denominator will become 1 alright.

So, if you did do that ok you will have

$$Z_{in} = Z_o(-jcot(\beta L))\ \Omega$$

Also one can say that it is $-jcot(\beta L)$ which is once again a very very complicated expression. This means that as you change the electrical length of your transmission line the input impedance for the transmission line with the same characteristic impedance is going to be dramatically different ok. And one could just simply draw a plot for what the cotangent would look like ok as you change the electrical length.

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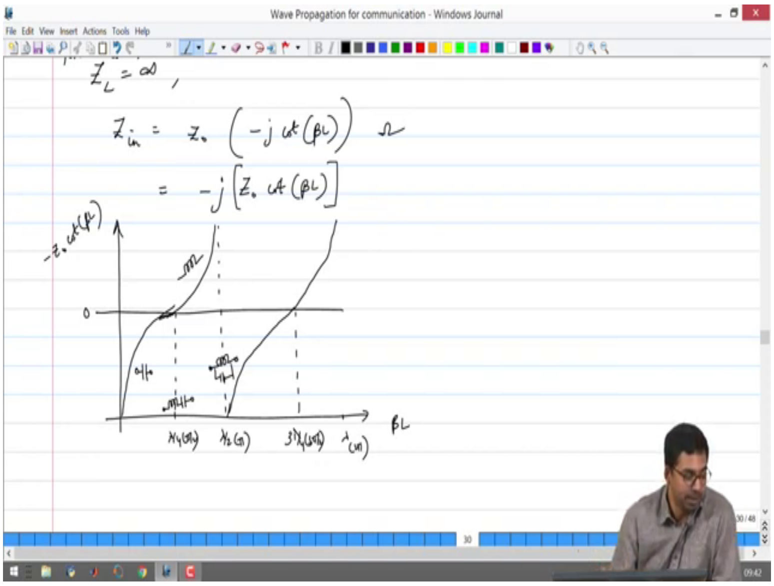

So, you can always draw an axis ok x axis can be your say $\beta$L, your y axis can be minus $Z_0$ cotangent of $\beta$L alright. So, you will have at $\beta$L equal to 0, tangent will be equal to 0, cotangent will become infinity because we have a minus sign in front of it alright we will be having minus infinity alright let us now, forget about this j alright let us just a understand that the $z_{in}$ is purely reactive ok does not have any resistive part, that is what this j is telling us alright.

And $-jcot(\beta L)$if this entire quantity inside that is say this square bracket becomes positive, then z will $z_{in}$ will look like minus j something it is similar to seeing a capacitance alright.

If the term inside the bracket is positive, then you will see capacitance the term inside the bracket is negative $z_{in}$ will look like an inductance alright. It's also possible because it's a cotangent $\beta$L passes through zeros it is quite a possibility that your $z_{in}$ will look like a 0 or a short circuit. So, it can go from minus infinity to plus infinity passing through 0. So, a section of the transmission line can have a variety of impedance depending on how the electrical length of the transmission line is right.

So, in this case you will go from minus infinity to say steadily some $\lambda/4$ alright. So, this will be or $\pi/2$ and then you will have this rising up ok and then there will be a discontinuity ok this will correspond to $\lambda/2$ or $\pi$ alright and then once again 3 $\lambda/4$ all right.

And then there will be a discontinuity corresponding to $\lambda$, $\lambda$ is the wavelength ok $\pi$ by $2\pi$, $3\pi/2$, $2\pi$. I have just marked the x axis to be electrical length going from 0 to $2\pi$, it means that the z in that I will be seeing all right in the transmission line this is going to be your 0 on the y axis all right it will be negative. So, this means that in this region where your z in is going to be negative, the equivalent circuit of your transmission line as seen from the input side is going to look like a capacitor that is all alright.

So, if you try to make inferences about your transmission line from the input end, you will be measuring some negative imaginary part all right. So, you can just write this down to the equivalent circuit to be a capacitor ok in this place it is a capacitor. If you were to a look at the part where the $-jcot(\beta L)$ is a you know a negative or a net input impedance is positive, the equivalent circuit will be like an inductor alright.

So, if you were to make derivations on the transmission line only based on looking at the input impedance you are not talking about R, L, G, C extra you are just looking at a transmission line of some length L with an electrical length of $\beta$L right its equivalent circuit characteristic is a single lumped element.

If you have to place and try to do your analysis for the circuit, it will look either like a capacitor if you measure a you know the value to be a negative imaginary part, it will look like an inductor if it is a positive imaginary part and there are some observed things that are happening.

So, 0 crossing has to be very clear all right. There are some zero crossings in the zero crossing which means that the impedance is 0 or it's a short circuit that is your current is going to peak ok. In those cases, the equivalent circuit where the current is going to peak is going to be a series combination of l and c.

If you have a series combination of l and c and if you have inductive reactance that matches the capacitive reactance alright, then you will end up having a short circuit and your peak voltage is going to be just source voltage divided by source resistance ok that is going to be your peak.

So, the equivalent circuit at the place of 0 crossings is going to be L in series with c. Then we also have to talk about the equivalent circuit at the place of discontinuities that is you are having plus infinity to minus infinity going in your a you know y axis for the input impedance in those cases it will be a parallel combination of l and c ok.

So, it is a very important to know this that a section of a transmission line which is lossless depending upon how long it is could act like an inductor or could act like a capacitor, could acts specifically like a series combination of inductor and capacitor or a parallel combination of inductor and capacitor.

So, that means, the technology for making any of the passive circuits in high frequencies is the same. You need to know how to make the transmission line that is it. The remaining job is where to cut the transmission line, so that you can make any of these passive components.

When we began this course we began by saying that the equivalent circuit of the transmission line is going to be some chopped up versions of l and c, l and c arranged together all right. But it's also true that if you were to replace a transmission line with the lumped element, this is how one could do it alright.

So, it means that the technology for making inductor is not different from the technology for making capacitor, you will be making the same transmission line in your manufacturing company, but depending on the requirement for that particular case you will be you know precisely dicing or cutting here transmission line to where you want alright. So, it means that I can have l, c and parallel combination of l and c. So, you could build a lot of circuits with this alright and all you will need is a transmission line and a cut with different sections.

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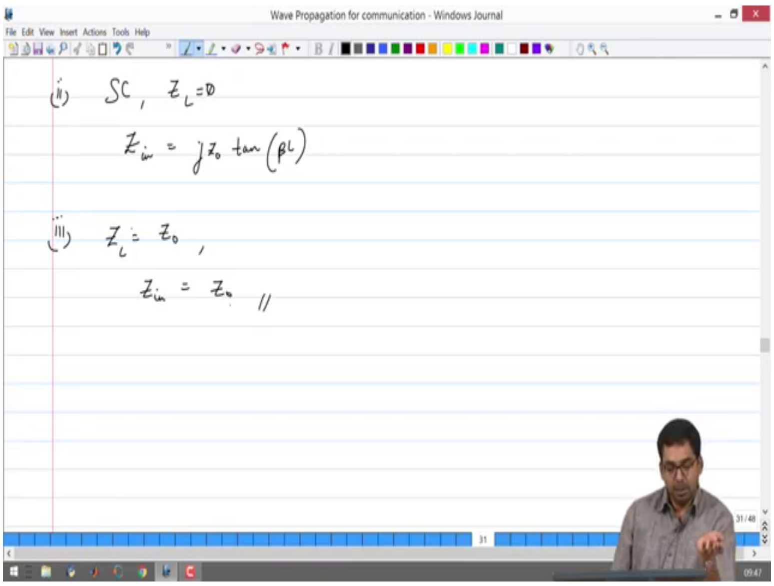

Just to be complete I will also write down the short circuit condition, that means, $Z_L$ is equal to 0. So, far we have seen the case with the open circuit, see the short circuit case alright the short circuit case is going to be

$$z_{in} = jtan(\beta L)$$

This means that in the expression for z in which was $Z_0$*( $Z_L$ + j $Z_0$ tan $\beta L$) is the expression you will substitute $Z_L$ is equal to 0, and you end up with j $Z_0$ tan $\beta$ L. Once again you can draw the variation of $z_{in}$ with respect to the electrical length $\beta$ L it will also have some places where it goes to plus infinity and minus infinity to discontinuity and some 0 crossings alright.

And you will notice that the impedance of the transmission line is repeating every $\lambda$ / 2 ok every $\lambda$ / 2 it will keep repeating and the special case is a where you have $Z_L$ = $Z_0$, $Z_{in}$ is going to be equal to $Z_0$.

This is a special case because if your load impedance is the same as your characteristic impedance, no matter how long or how I mean how short your transmission line is going to be cut no matter what at the input you will see only characteristic impedance $Z_0$ ok. It will look like it is purely resistive ok $Z_L$ will be equal to $Z_0$ alright and the unit of $Z_0$ is simply ohms and its square root of L divided by c norms.

So, it if you have a lossless transmission line alright and excited by a ac source and if $Z_L$ is equal to $Z_0$ no matter what electrical length you take for your transmission line, at the input side you will measure only $Z_0$. So, this means that I know how to create different impedance profiles by making use of this alright.

Suppose I wanted to manufacture a high frequency circuit I could make use of these properties to replace some inductors, some capacitors, some combination of inductors and capacitors and I could also you know target some specific values of z in equal to $Z_0$ to fill up some purely resistive branches of your circuit ok. But this is not very convincing because although we have made l and c very easy, resistance is not something that is coming into the picture all right.

So, the only way to go forward is to actually introduce a different phenomenological model. So far we have been using a lossless transmission line with l and c, we have to be a little bit more practical, include a series resistance in the inductor branch, include a parallel conductor and redo whatever we have done and then we have to see what the consequences would be.

So, thus far the procedure for analysing the circuit is very much the same as in DC you can calculate the load reflection coefficient as ($Z_L$ - $Z_0$) / ($Z_L$ + $Z_0$) and depending upon the termination, you will be able to find out what the reflection coefficient is going to be, but the only difference with respect to DC over here is that depending upon the position you will have places where your transmission line is always having zero voltage or always having zero current extra.

The impedance in the transmission line will be varying from one place to another place depending upon the electrical length ok. These are some of the subtle differences the program that we have written you can always substitute a cosine or a sine and try to see what happens. Here mind you we have considered only the spatial distribution, we have still said that with respect to time it is going to be time harmonic or periodic.

So, it means that even though you will have some places in your transmission line having non zero voltage with respect to time it will go to a maximum and then go to 0 with respect to time it will go to maximum and go to 0, but the nodes where it is 0 its going to be fixed all the time. So, some small things will appear only if you take that simulation that we had already done and plug in a cosine or a sin just to understand what, that means, ok.

So, in these cases even though you have an AC input excitation we do have fixed places on the transmission line where the voltage is going to be 0 or the current is going to be 0 alright and you are going to be having fixed positions where the voltages or currents are going to become maximum.

So, we call these kinds of waves as standing waves ok and soon we have to analyse the meaning of the standing wave, does it tell you anything about the termination can any more information be arrived at what is the consequence of standing waves on the power to be delivered all these things has to be seen. So, we will go forward in that direction and let us stop here.