Nonlinear System Analysis
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Lecture - 19
Limit Cycles- Examples
Part 02

Hello everyone, I am Shriram Jugade and I welcome you all to the lecture number 11 of non-linear dynamical systems.

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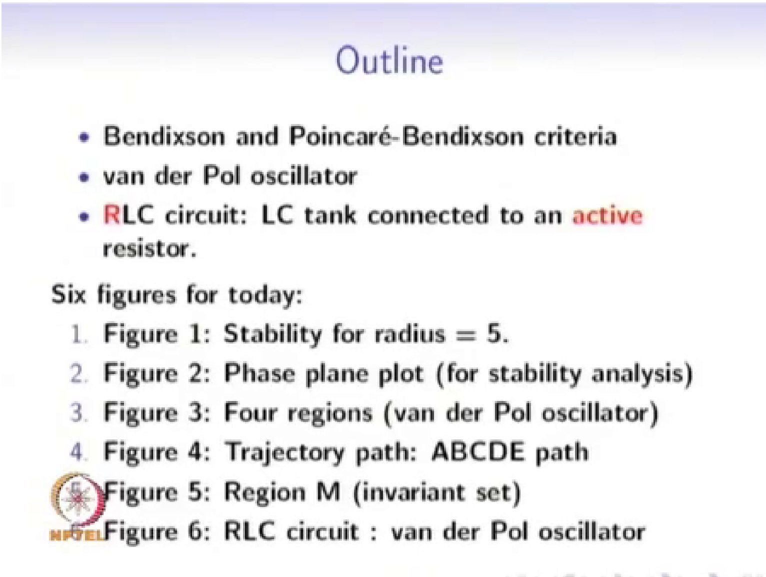

In today's lecture we will look into the Bendixson and Poincare-Bendixson criteria. The application of it we will consider one 2 examples of that. Next, we will consider van der Pol oscillator we will study van der Pol oscillator which is a non-linear oscillator. Then we will take the example of a van der Pol oscillator which is a RLC circuit LC tank connected to a active circuit. During the lecture we will refer to the following figures six figures.

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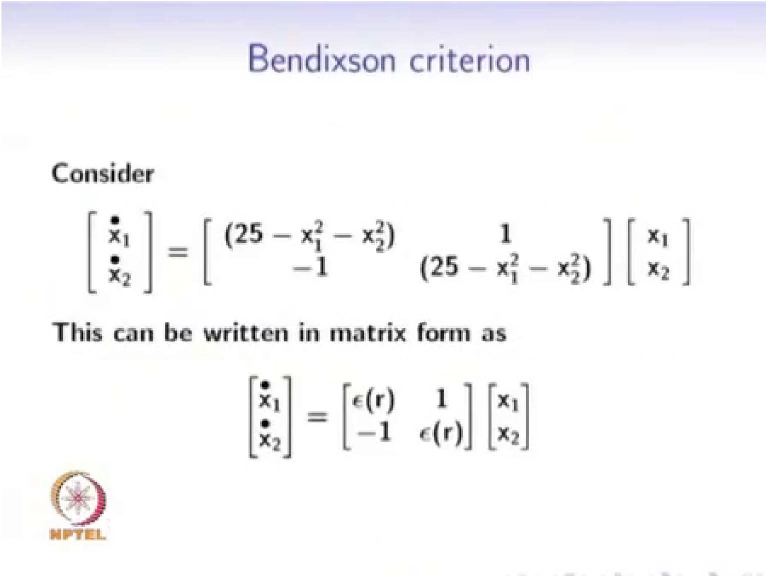

First, we will consider the Bendixson criteria. Here, are the state equations of an examples. Here we can see in the matrix the function 25 minus x 1 square minus x 2 square. This function is dependent on both x 1 and x 2. Let us say the radius is r, then we can say that x 1 square plus x 2 square is equal to r square. So, the function 25 minus x 1 square minus x 2 square is can be written as 25 minus r square. So, the function is dependent on r let us represent that function as epsilon r a function dependent on r. So, the state equations reduced to the following form which are shown in this slide.

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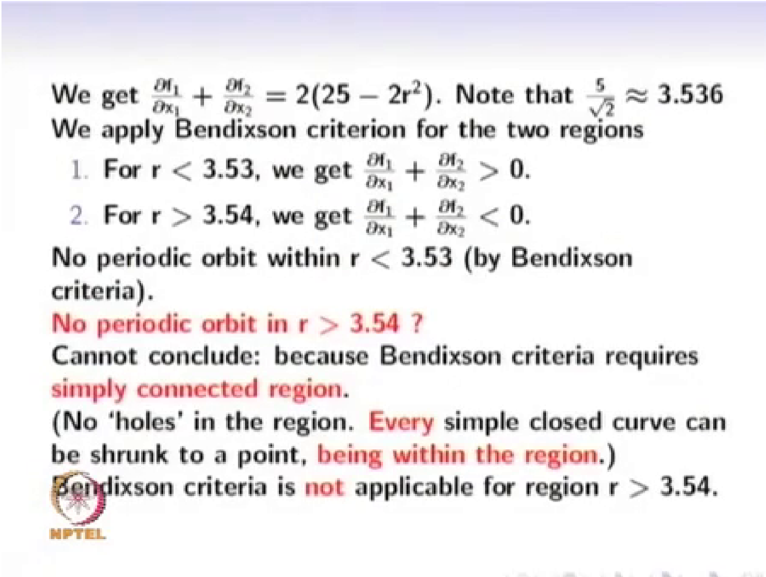

Next let us consider the expression dou f 1 upon dou x 1 plus dou f 2 upon dou x 2. This results out to be 2 into 25 minus 2 r square. The root of this equation is r equal to 5 by root 2 which is approximately equal to 3.536. So, at r equal to 3.536 we will have this expression value to be 0.

Now, let us try to apply Bendixson criteria to the following example. For this we will consider 2 regions; the first region we will consider bounded by r; r which is bounded by 3.53. So, r is strictly less than 3.53. Now, for this region we will get the expression dou f 1 upon dou x 1 plus dou x dou f 2 upon dou x 2 as strictly greater than 0. Now, the second region we will consider for r strictly greater than 3.54.

For this region we have the value of the expression dou f 1 upon dou x 1 plus dou f 2 upon dou x 2 strictly less than 0. So, we can see in both the regions the sign of the expression does not change. In the first region, the sign of the expression remains positive and in the second it remains negative. So, let us try to apply a Bendixson criteria. Let us consider the first region where r is strictly less than 3.53. So, there is no sign change, we can say that by Bendixson criteria no periodic orbit exist in the region.

Now, consider the second region where r is strictly greater than 3.53. Now, the question arises whether we can apply Bendixson criteria to this? The answer is no, the we cannot conclude in this case because Bendixson criteria requires simply connected region. Now, what is simply connected region? A simply connected region is a region which has no holes or we can define it in other sense.

If we take that region and if we take a simply closed curve in that region and shrunk it to the point, then it should also remain within the region. During the shrinking the every curvature and up to the point it should be remain in the region. So, that is how we define simply connected region. So, for r greater than 3.54 we cannot conclude whether there are periodic orbits or not or we cannot apply Bendixson criteria since this region is not a simply connected region. So, we conclude here that Bendixson criteria is not applicable for region r greater than 3.54.

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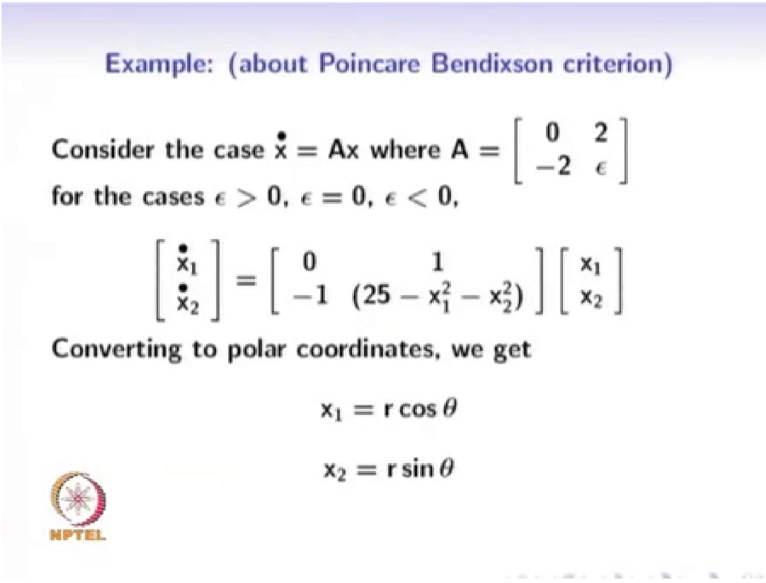

Next we will consider an example of Poincare-Bendixson criteria. Here we take a system of the form x dot equal to a x where a is equal to 0 2 minus 2 epsilon. The diagonal elements are 0 and epsilon. Now, in this case as epsilon will vary the behavior of the system will change. So, we will consider 3 cases in the first case we will consider epsilon as greater than 0. In the second case we will consider epsilon equal to 0 and in the third case we will consider epsilon less than 0. Now, let us take that example previous one.

In the previous one we had both the diagonal elements as 25 minus x 1 square minus x 2 square. Now, in this example we have one diagonal element as 0 and the second as 25 minus x 1 square minus x 2 square. So, we will try to analyze the behavior of the system and for that we will convert the coordinates into polar coordinates. So, we will have a clearer picture. So, we can convert a polar coordinate into the following form. So, x 1 will become r cos theta and x 2 will become r sin theta where r is the radius and theta is the angle.

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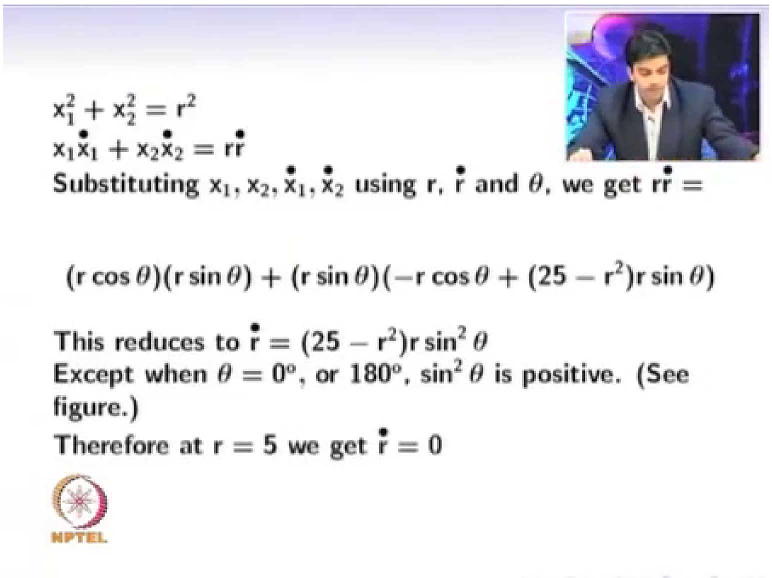

So, x 1 square plus x 2 square we get as equal to r square. Differentiating it with respect to time we will get x 1 into x 1 dot plus x 2 into x 2 dot equal to r into r dot. Now, we have the expressions for x 1 and x 2 and x 1 dot and x 2 dot from the state equations. So, we can substitute in this expression and we will get the following expression at r dot equal to expression is shown on the slide.

So, we will cancel out the common factors and rearrange it and the equation will reduce to the final form r dot equal to 25 minus r square into r into sin square theta. So, except theta equal to 0 or theta equal to 180 degrees theta is sin square theta is always positive. So, let us consider one figure.

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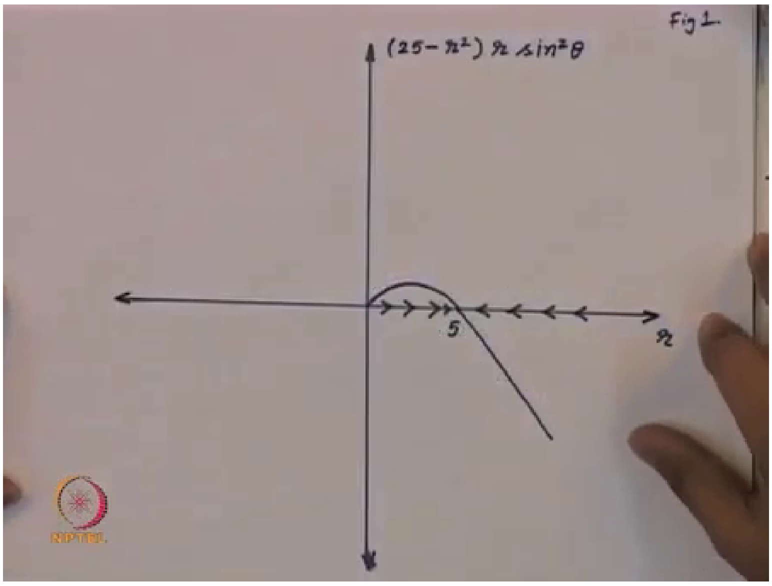

In this figure we have plotted 25 minus r square into r into sin square theta versus r. In the first case we have considered r greater than 0. So, we can find it for r equal to 5 the expression reduces to 0. So, we can see that for r equal to 5 r dot is equal to 0. So, when r dot is equal to 0 we can say that the circle with radius 5 is a periodic orbit, since the radius is not changing.

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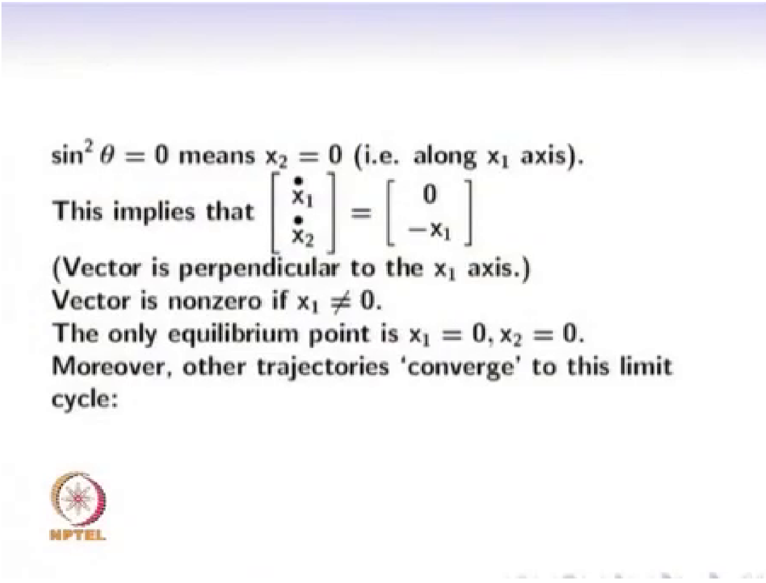

Now, we will consider the second case where sin square theta equal to 0 means theta is equal to either what 0 or 180 degrees. So, in that case we will get x 2 equal to 0. So, the x 2 equal to 0 is say is along the x 1 axis. So, thus if we substitute this values in the state equation and our state equation reduced to the following form where x 1 dot equal to 0 and x 2 dot equal to minus x.

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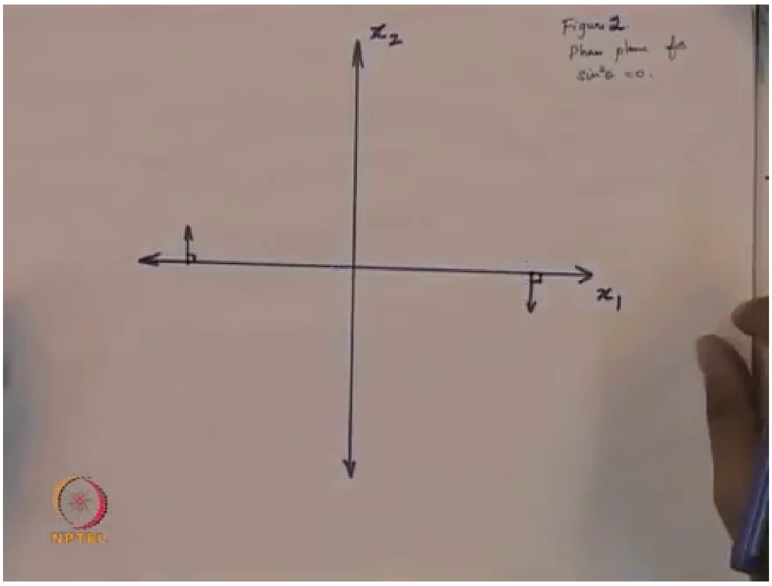

In this case we have a point on the x 1 axis which is a initial condition, then according to the state equations we will have x 1 dot equal to 0 and x 2 dot equal to minus x 1. So, for x 1 to be x 1; x 1 is positive we will have a vector pointing in downward direction and it will be perpendicular to the x 1 axis.

Similarly, when x 1 is negative direction vector of the vector will be pointing outwards and it will be perpendicular to the x 1 axis. So, the magnitude of this vector will depend on the value of x 1. So, we can see that for x 1 not equal to 0 the vector is always nonzero. So, from the state equation we can see that for the equilibrium point we need x 1 dot to be 0 and x 2 dot to be 0. So, in this case the only equilibrium point we can see is when x 1 dot is equal to 0. So, the only equilibrium point is x 1 equal to 0 and x 2 equal to 0. So, we will go back to the previous figure where we are drawn plotted the r dot versus r.

In this case we can see that for r equal to 5 if there is a disturbance or a perturbation, then the trajectories are approaching towards r equal to 5 when r is greater than 5 or r is less than 5. So, the point at r equal to 5 or the periodic orbit, we will conclude that it is stable. We can also say that the limit cycle is a isolated. The meaning of isolated is if we consider a small region around r equal to 5, then we will have no periodic orbits. So, in that region r equal to 5 is the only periodic orbit existing. We already proved that how it is stable since for disturbance all the trajectories are pointing towards r equal to 5. So, it is a stable limit cycle.

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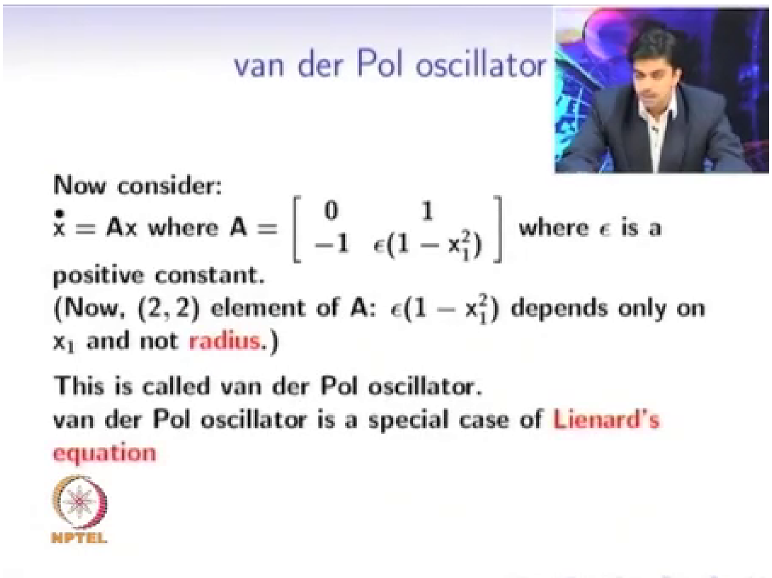

Next we will consider van der Pol oscillator. Van der Pol oscillator is a non-linear oscillator. If we consider the same for system form as x dot equal to a x where a is equal to 0 1 minus 1 and epsilon into 1 minus x 1 square. So, epsilon here is a positive constant. Here we can see that 1 diagonal element is dependent on x 1 square. So, it is not the diagonal element is dependent only on x 1 square it is not dependent on x 2 square.

So, we cannot conclude that it depends on radius. It does not depend on radius, it depends only on x 1 square. So, this system we will call it as a van der Pol oscillator and van der Pol oscillator is a special case of Lienards equation.

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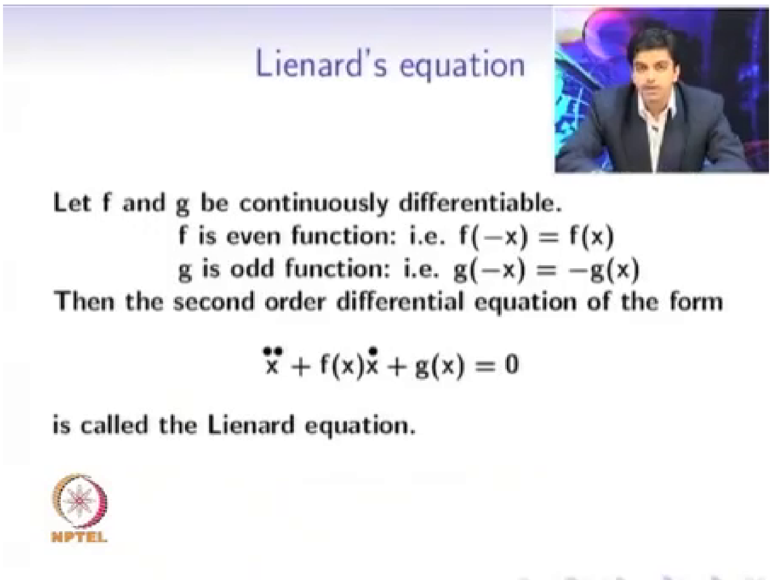

Next we will consider Lienards equation. As we previously mentioned that van der Pol oscillator is a special case where, Lienards equation defines the generalized case for the non-linear oscillators p. And let us consider 2 functions f and g which are continuously differentiable. Yeah and let us consider that f is a even function that is f of minus x is equal to f of x and g is a odd function.

So, that g of minus x is equal to minus g x. So, this the second order differential equation of the form x double dot plus f of x into x dot plus g of x is equal to 0. This equation is called Lienards equation this is a generalized form of the equation for non-linear oscillators. In general non-linear oscillators are considered for the modeling of the physical oscillator.

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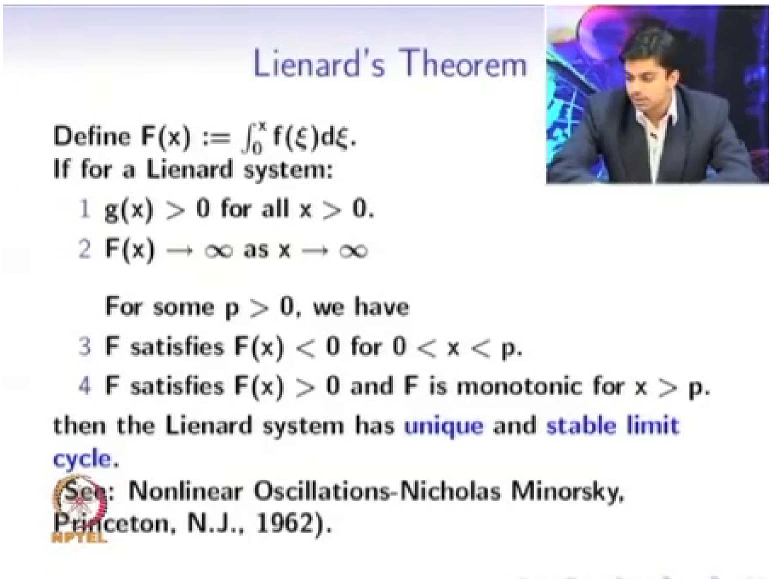

We have defined a generalized Lienards equation for the non-linear oscillators. We will have to next look into the stability of the oscillations for the non-linear oscillator. For that we will first define a function capital F of x which is equal to integral of small f of x. Then for a Lienard system if we consider the following conditions like g of x is greater than 0 for all x greater than 0, then capital F of x tends to infinity as x tends to infinity.

And for some p capital F of x satisfies that it is negative for the range when x is between 0 to p and F of x also satisfies that it is positive and monotonic for x greater than p that is when x is greater than p the F of x is monotonically increasing. If these conditions are satisfied then we can say that the Lienard system is having a unique and a stable limit cycle and this is what is called Lienards theorem.

Lienards theorem gives the conditions for the stability of oscillations for non-linear oscillators. For the reference you can see the book title non-linear oscillations by Nicholas Minorsky with the respective edition.

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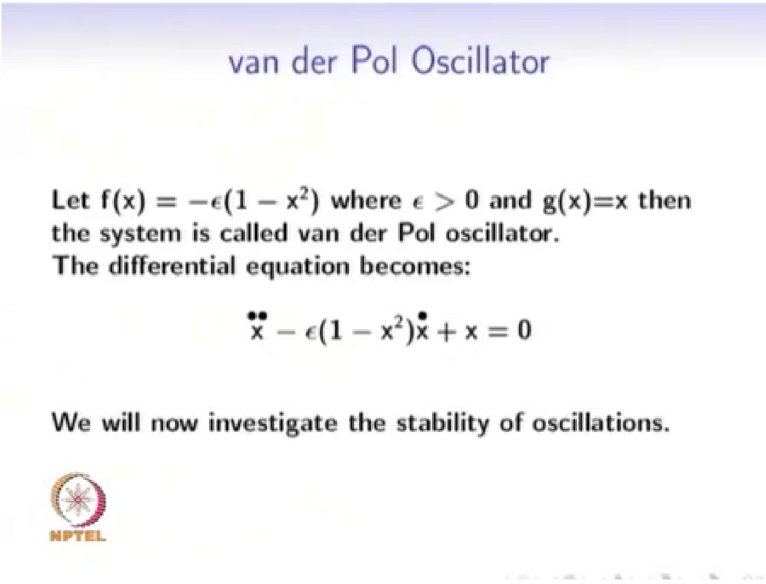

Next let us consider f of x is equal to minus epsilon into 1 minus x square where epsilon is a scalar and it is strictly greater than 0 and g of x equal to x, then the system is called van der Pol oscillator. Previously, we consider a Lienards equation where it was a generalized case. Now, we are defining van der Pol oscillator in that equation where f of x and g of x are defined as I said before.

So, the differential equation the Lienards equation gets transformed to the form x double dot minus epsilon into 1 minus x square into x dot plus x equal to 0. Let us now investigate the stability of oscillation. We had Lienards theorem which gives us the condition for the stability of oscillation. For van der Pol oscillator we can specifically investigate for it is stability like if epsilon is much greater than 0 then our oscillation for the van der Pol oscillator are very stable.

Now, as epsilon goes on decreasing the relative stability of the oscillations goes on decreasing. When epsilon is equal to 0 we can see that the equation is transformed to x double dot plus x equal to 0. So, the oscillator no longer remains non-linear. It turns into a linear oscillator and for the third case where epsilon is less than 0 we will have unstable oscillations. So, we can see that the van der Pol oscillator have will have stable oscillation only if epsilon is greater than 0. So, we can also conclude that for epsilon greater than 0 f of x and g of x will satisfy the Lienards condition given for the stability.

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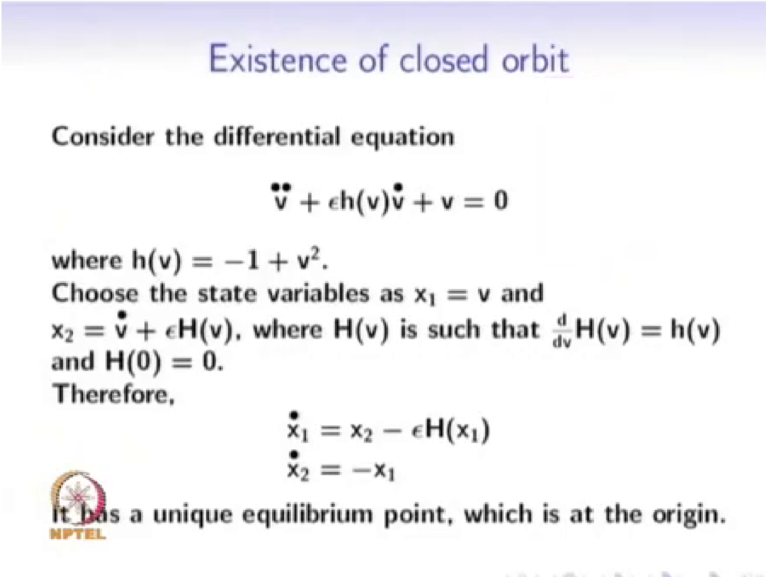

Next let us look into the existence of a closed orbit for the van der Pol oscillator. So, we will consider the same equation where v double dot plus epsilon hv h of v into v dot plus v is equal to 0 where v can be a voltage across a element in the given circuit. Now, h of v here is equal to minus 1 plus v square. Now, for analyzing the behavior we let us choose state variables as x 1 equal to v and x 2 equal to v dot plus epsilon into capital H of v.

Now, here we will define capital H of v as a d by dv of capital H of v is equal to small v and capital H of v at v equal to 0 is equal to 0. So, therefore, if we differentiate the equations of x 1 and x 2 we will get the following state equations where x 1 dot is equal to x 2 minus epsilon H of x 1 and x 2 dot equal to minus x 1. So, we can see here if we put a x 1 equal to 0 and x 2 equal to 0 it has a unique equilibrium point and since H capital H is equal to 0 only at v equal to 0 it is the only equilibrium point So, origin is the only equilibrium point for this van der Pol oscillator.

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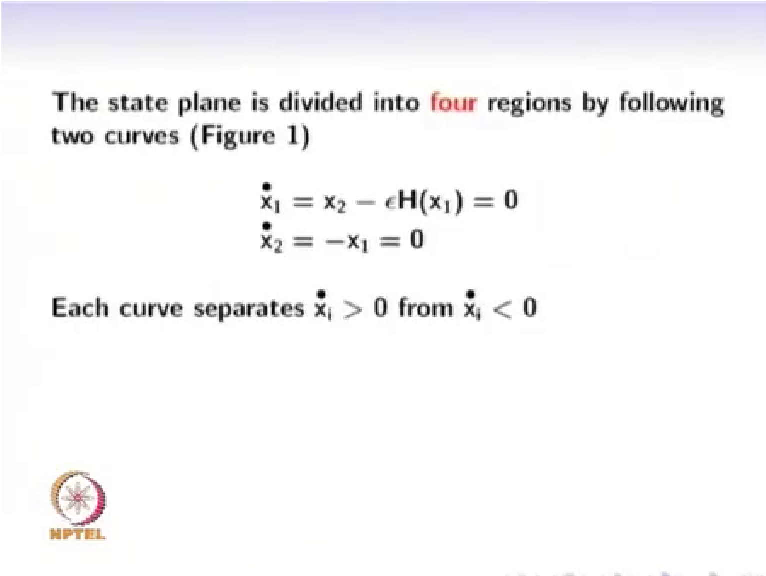

Next we will consider one figure.

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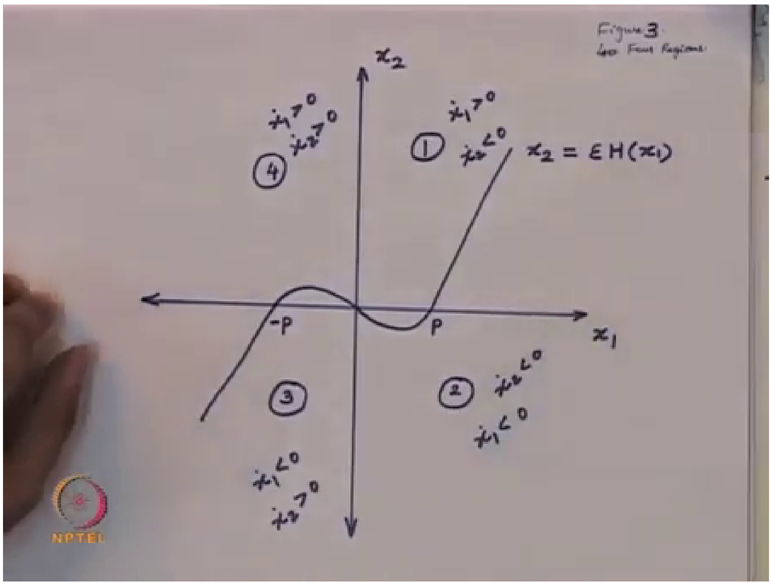

Let us look at the state plane where x 1 and x 2 are the axis. So, we will divide this plane into 4 regions with the help of the curves given as follows, x 1 dot is equal to x 2 minus epsilon H of capital H of x 1. So, it is the this curve where x 2 is equal to epsilon into capital H of x 1 and the second curve is x 2 dot equal to minus x 1 equal to 0 which is the x 2 axis.

So, we will next look into how this curve divides the plane into 4 regions. We can say that each curve divides or separates x 1 x i dot greater than 0 from x i dot less than 0 like for example, let us consider the first curve where x 2 is equal to epsilon capital H of x 1. So, above this curve in this region we can have x 1 dot greater than 0. Since x 2 is greater than epsilon capital H of x 1.

So, x 1 dot is greater than 0 in this whole region and below this region we have x 1 dot which is less than 0. Now, let us consider the second curve which is the x 2 axis. Now, to the right side of the x 2 axis we have x 2 dot less than 0. Since x 2 dot is equal to minus x 1. So, to the right side of x 2 axis the x 1 is positive. So, x 2 dot will be negative. So, x 2 dot is less than 0 to the right half of the plane and to the left half of the plane x 2 dot is positive.

So, now we will consider the 4 regions. Now, in the first region we have x 1 dot greater than 0 and x 2 dot less than 0. In the second region we have x 1 dot less than 0 and x 2 dot less than 0. In the third region we have x 1 dot less than 0 and x 2 dot greater than 0 and the fourth region is x 1 dot greater than 0 and x 2 dot are greater than 0. So, as we have seen here the 2 curves are dividing the state plane in the 4 regions. Now, we will see how these 4 regions will be helpful to us in finding the existence of the periodic orbit.

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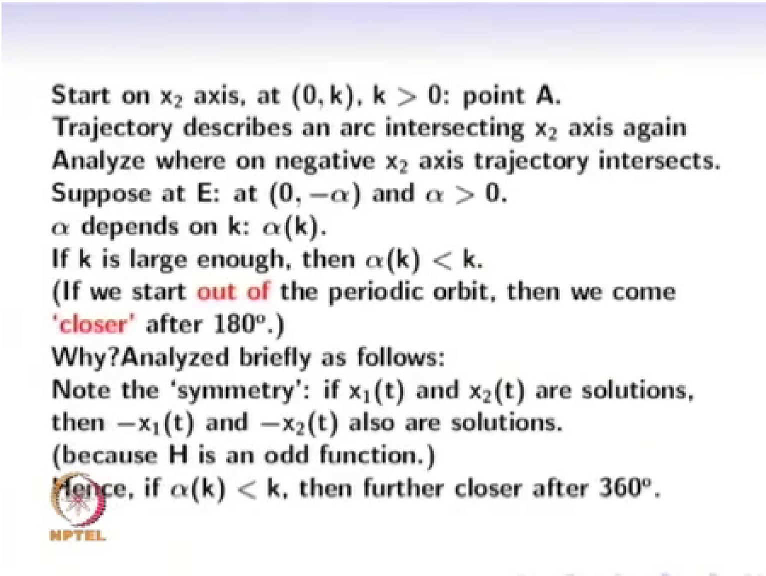

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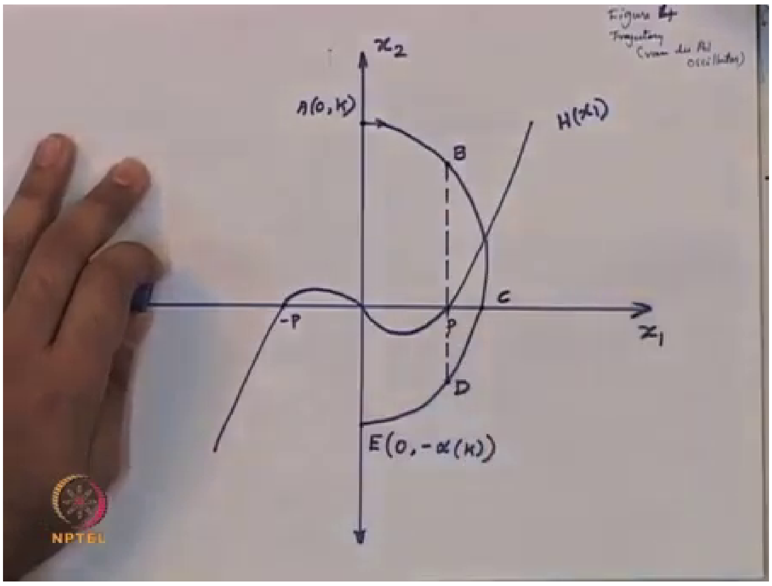

So, we will consider another figure. So, let us take the initial condition on the x 2 axis. So, that x 1 is equal to 0 and x 2 is equal to minus K. These are the initial conditions we have taken and here K is greater than 0 let us name the point as A. If we draw the trajectory according to the directions given. So, here is the trajectory which will be intersecting x 1 axis to at point C and x 2 axis again at point E. Let us say that the coordinates for the E point is 0 and minus alpha of K where alpha is positive.

So, alpha is greater than here. The reason are taken alpha as a function of K because alpha depends on K. Now, if we change the initial condition or if we change the value of K, then we will get a different alpha. So, the value of alpha is actually dependent on K. So, the intersection at point E or at the x 2 axis again, so it is dependent on the value of K. So, we can say that alpha is a function of K. Now, if we take K as large enough then we can prove that alpha K is less than K.

So, that it is same as saying if we start out with the initial condition and around the orbit, if we consider 180 degrees curvature. So, trajectory will come closer to the periodic orbit since K is greater than alpha of K. Now, let us look why it is like that in the first slide we will we consider that x 1 dot and x 2 dot the state equations in that we can see they are the function of capital H of x 1 and x. So, both these capital H of x 1 and x are odd functions. So, we can say that if x 1 and x 2 are the solutions to the van der Pol oscillator then minus of x 1 t and minus of x 1 x 2 t are also the solutions.

Now, as stated before the reason for this is H is an odd function. Now, let us consider that if the trajectory completes 360 degrees, then if alpha K is less than K the trajectory will come more closer to the periodic orbit. Let us consider the function v of x equal to x 1 square plus x 2 square upon 2.

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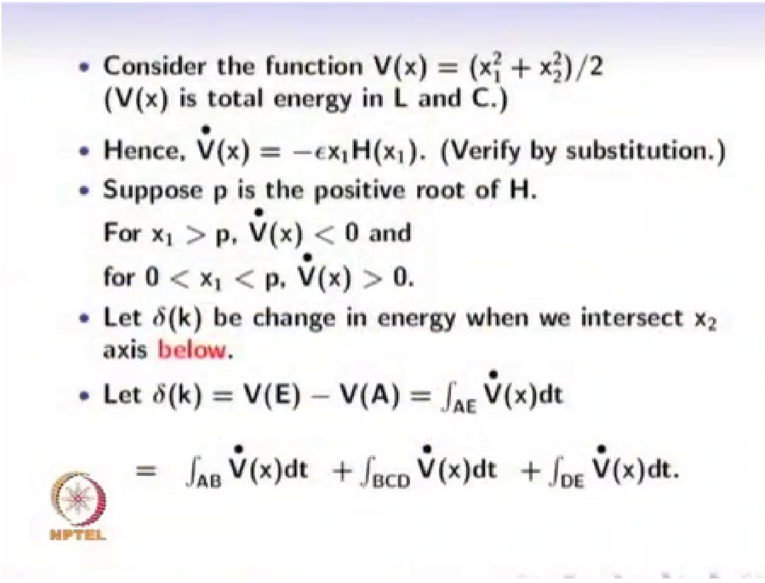

Now, V of x is equivalent to the total energy in a LC circuit in a LC tank circuit. So, if we differentiate V V dot V of x with respect to time then we will get is equal to minus epsilon into x 1 into capital H of x 1. We can verify this by substituting the values of x 1 dot and x 2 dot from the state equations. Now, suppose we consider the curvature of H. So, this is the curvature of H of x 1. So, it has a positive root P when x is greater than 0 and a negative root x 1 is less than 0. So, when x 1 x 1 is greater than P or v dot x is less than 0. Since for x 1 greater than p H of x 1 is positive and x 1 is also positive.

So, from the expression we can see that v dot x is negative. If we consider the region for x 1 to be between 0 and P, then we will get that v dot x is 0 greater than 0. So, we can conclude that the V dot x is changing along the x 1 axis. Now, let us take delta k as a change in energy when we intersect the curvature to the x 2 axis. So, delta k is the change of energy from point A to point E. So, we can define delta K as V E minus V A where V E is the energy at E point

E and V A is the energy at point A. So, it is equal to the integral along the curvature A E of V dot x with respect to time.

We can divide the following curve into 3 curves AB where B is a point just above the x 1 equal to P. So, at point B along the curvature x 1 is equal to P. Point C is the intersection of the trajectory with the x 1 axis, point D is another point where x 1 is equal to P on the curvature and DE is the remaining curvature. So, the whole curvature from A to E is divided into 3 curves AB, BCD and DE. So, delta K can be represent in the as a change of energy from A to B, B to D and D to E.

Let us represent it in the form of delta 1 K, delta 2 K and delta 3 K where, delta 1 K is the change of energy from A to B delta 2 K is the change of energy from B to D and delta 3 K is the change of energy from D to E along the curve. Now, let us take the K where delta 1 K is greater than 0. We can say that delta 1 K is greater than 0 because x 1 here is greater than 0 and H of x 1 as we can see is negative. So, delta 1 K is greater than 0. So, we can say that the change of energy from A to B is positive.

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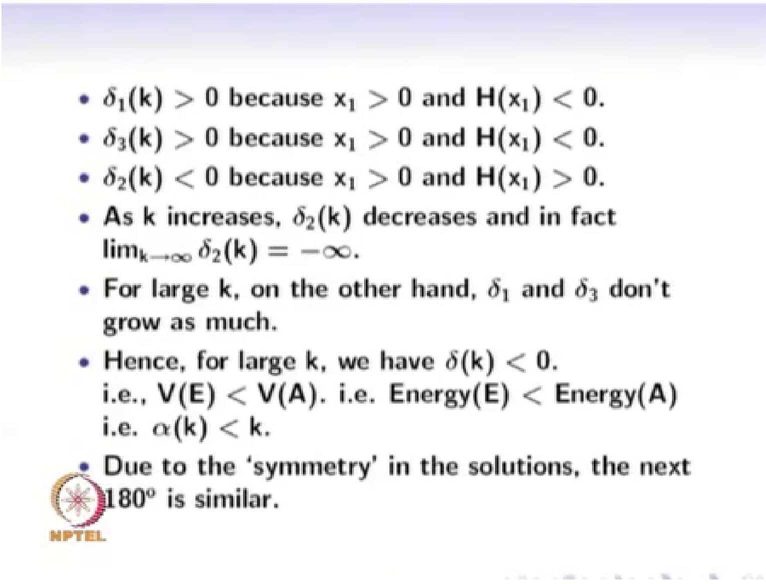

In the second case, we can say that delta 3 K which is a change of energy along the curve from D to E is positive. It is similar to delta 1 K. In this case also x 1 is greater than 0 and H of x 1 is less than 0; H of x 1 is less than 0 because x 1 is restricted to the point P. As we can see B and D are the points along the x x 1 equal to P axis, so we will have delta 1 K greater than 0 and delta 3 K always greater than 0.

Now, let us consider the change of energy along the curve BCD which is denoted by delta 2 K. Here we can say the change of energy along the BCD is less than 0. We can give the reason because x 1 is greater than 0 in along the curve and H of x 1 as we can see x 1 is greater than P. So, capital H of x 1 will be always greater than 0.

So, delta 2 K along the curve BCD will be less than 0. Now, as we see as I increase the initial conditions as I increase the K the curvature will expand and delta 2 K that is the change of energy along BCD will go on decreasing. And we can also say that as limit x tends to minus infinity the change of energy along BCD that is delta 2 K will go to minus infinity. In the other context, if we look at the delta 1 K and delta 2 K expressions that is the change of energy along AB and DE, then for large K they will not go as much as delta 2 K.

So, as K will go on increasing the delta 2 K will grow much faster than delta 1 K and delta 3 K. So, since delta 2 K is negative. So, the net summation of delta 1 K, delta 2 K and delta 3 K will be negative. Hence for a large value of K we will have delta K less than 0. Since delta K is less than 0 we can say that along the curvature from A to E the energy at point E is less than energy at point A that is the energy along the curve is decreasing. Since the energy is decreasing along the curve we can say that alpha K is less than K.

So, the trajectory will move closer to the periodic orbit or it will approach the periodic orbit. Now, due to the symmetry of the in the solutions as we stated before if x 1 t is a solution and x 2 t is a solution when minus x 1 t and minus x 2 t are also solutions. So, the next 180 degrees will be similar to that.

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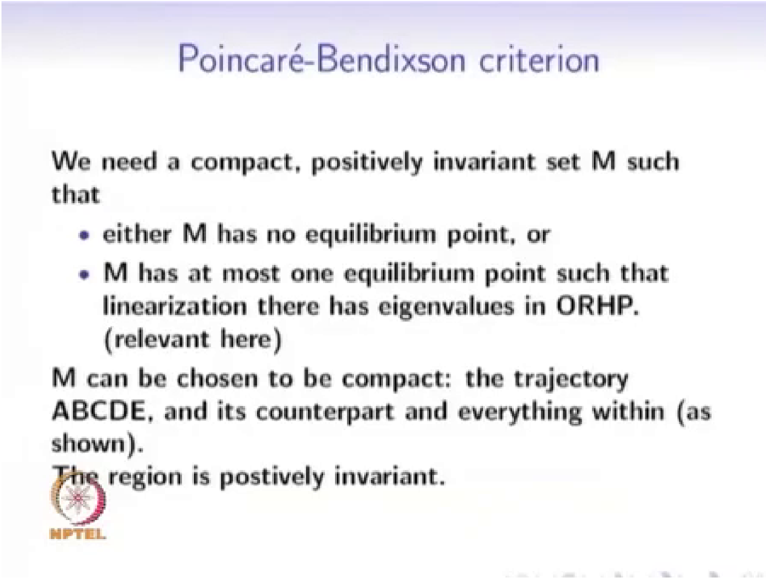

Let us consider Poincare-Bendixson criteria again. For applying Poincare-Bendixson criteria we need a compact positively invariant set M such that either M has a no equilibrium point or it can have at most one equilibrium point such that after linearization, if we consider the eigenvalues they will be in open right half plane. So, the equilibrium point if it is there in the M region it will be unstable.

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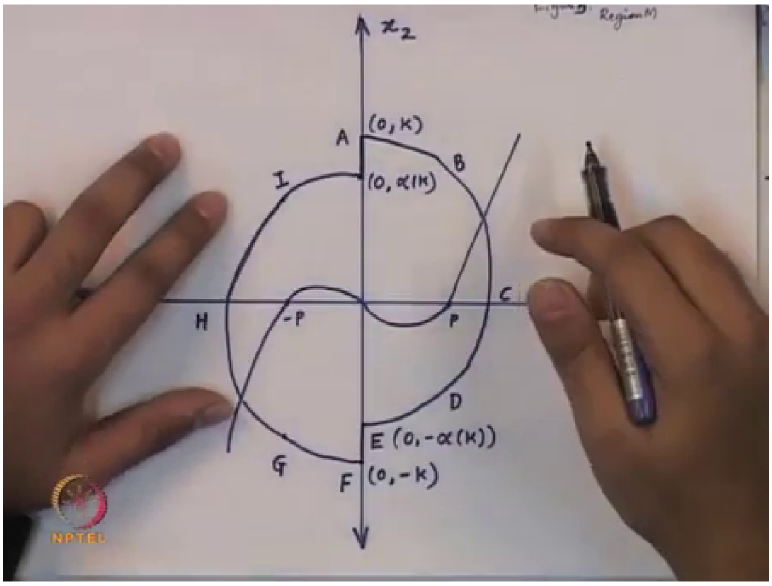

So, here we can choose an region, we consider the curve A B C D E before. Now, let us consider another curve where the initial condition is 0 minus K. So, the curvature will be F G H I and back to A. So, if we consider the whole region if we connect close this region then we can say that this region is a positively invariant set. Now, this curve is also contained in the van der Pol oscillation or it is also solutions.

But due to symmetry as stated before that if a portion x 1 t and x 2 t is in the solution then a portion minus x 1 t and minus x 2 t will also in the solution. So, if we consider the whole region it will be a positively invariant region that is for initial condition in this region the trajectory will remain in the region and it will approach to a periodic.

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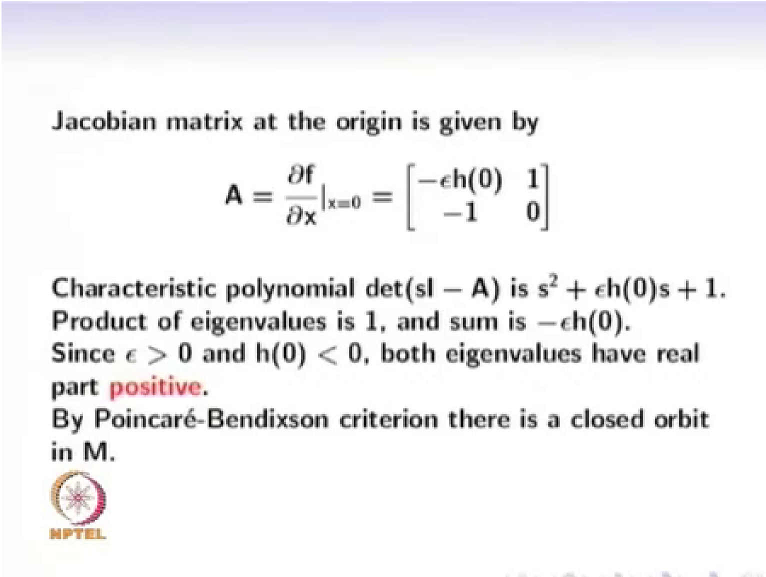

So, next we will consider a Jacobian matrix which will be formed after linearizing the system. So, the A is the Jacobian matrix which is defined as dou f by dou x at x equal to 0. Since we have the equilibrium point that x equal to 0. So, we will get the matrix as shown on the slide which is the minus epsilon into small h of 0 1 minus 1 and 0. S

o, the characteristic polynomial of A will be given as s square plus epsilon into h of 0 s plus 1. Now, from this equation we can say that the product of eigenvalues is 1 and sum is equal to minus epsilon h of 0 because the roots of this characteristic equation are the eigenvalues of the system.

Now, since we defined before epsilon is greater than 0 and h of 0 as negative, then both of the eigenvalues we will have as having positive real part. So, we can say since the eigenvalues are having positive real part the equilibrium point is unstable. So, now, the conditions we concluded for this system a van der Pol oscillator are that we have a invariant set M, then we have a equilibrium point inside that which are unstable. So, we can apply the Poincare-Bendixson criteria here and by Poincare-Bendixson criteria we can say that there is a closed orbit in M.

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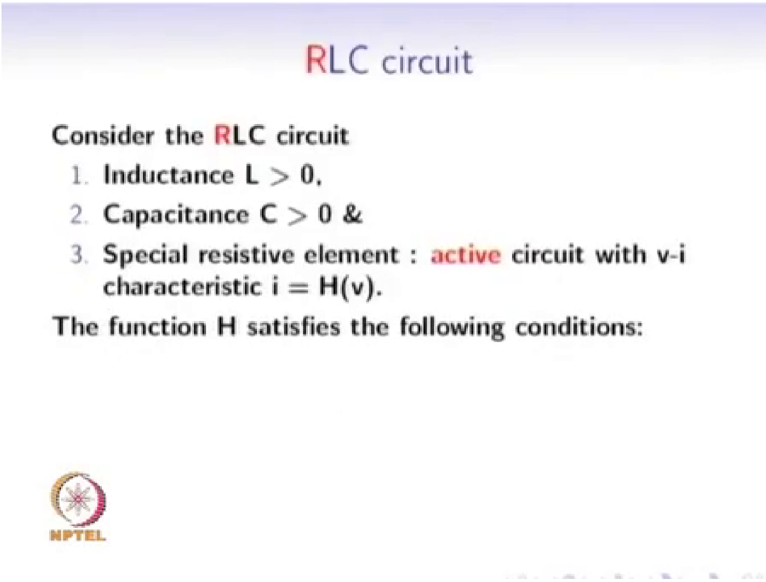

Now, for the example of the van der Pol oscillator we will consider an RLC circuit where the R is an active resistive element.

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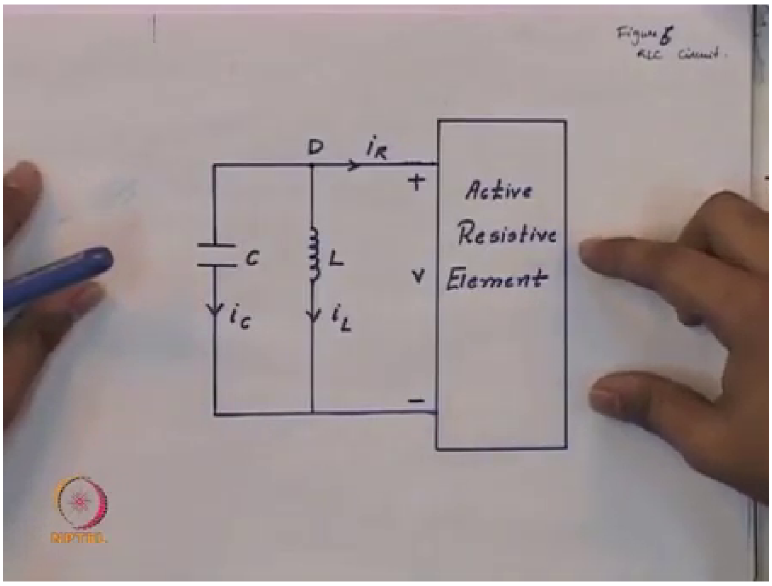

Now, let us consider this figure. In this figure this is an RLC parallel circuit, this is the capacitance C which is greater than 0 L inductance which is greater than 0 and v have connected in parallel an active resistance element. This is having v i characteristics as I equal to capital H of v where we had defined capital H of v before.

Now, so we can say that the capital H of v satisfies the following conditions, capital H of 0 is equal to 0 then capital H dash that is derivative of capital H with respect to v satisfies capital H dash of 0 is less than 0 and capital H of v tends to infinity as v tend to infinity.

Now, here we have a point saying that capital H of v is similar to capital H of a f in Lienards equation. We can see that the conditions that are satisfied by capital H of v capital F of x are same. So, we can say that H and F are both odd functions. Now, let us go back to the circuit again. Here we consider a point D and we will apply KCL here. i C is the current to the capacitor, i L is the current to the inductor and i R is the current for the active resistance element. So, at D we will apply KCL so that we will get the summation of all the 3 currents is equal to 0.

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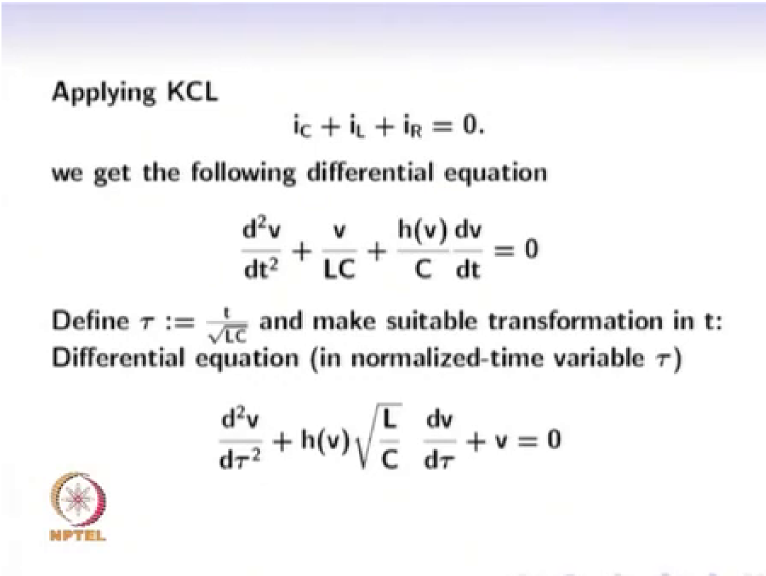

Then in the differential form we will get the following expression where we have h of v coming into the picture. Now, we will use a transformation we will define a element tau equal to t upon root LC and substitute in the following equation. So, we will get a normalized time variable equation. It is a second order differential equation and we can see that it is in the form of a van der Pol oscillator where we have epsilon equal to root of LC.

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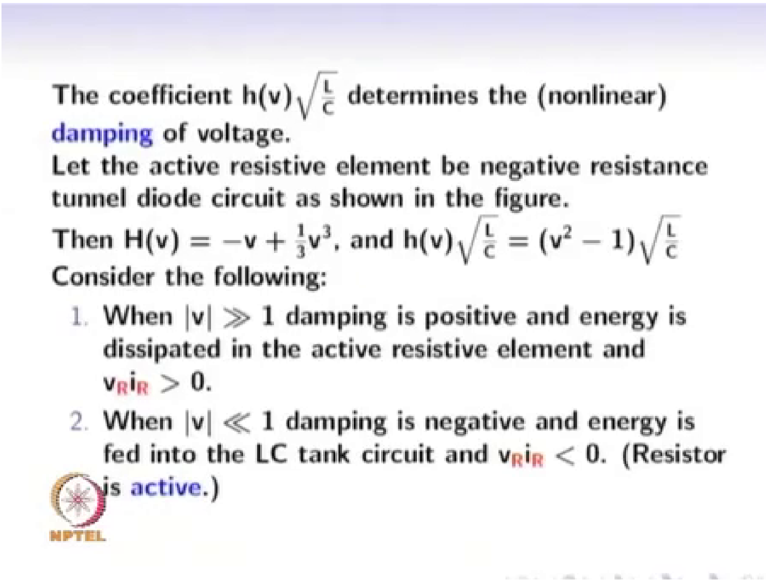

Here we can see that root of LC is epsilon is greater than 0. Now, next we will consider the coefficient of v dot which determines the damping of the system. So, the coefficient of v dot is h of v into root LC and this determines the damping of the voltage. The damping here is a non-linear damping. We can practically implement active resistance element in the form of tunnel diodes. So, it will act as a negative resistance for some value of v and a positive resistance for other values of v.

Now, we will define capital H of v as minus v plus v cube by 3 and small h of v into root LC is equal to v square minus 1 into root LC. So, let us analyze the system. Consider v much greater than 1. So, v have much greater than 1. The damping coefficient is positive. So, the we can say that the damping is positive and since the damping is positive energy is dissipated in the active resistance element.

The transaction of energy is from LC circuit to active resistive element and it is getting dissipated in the resistive element. The resistive element in this case will be positive. The value of the resistor will be positive and we can say that since the energy is getting dissipated v R into i R is strictly greater than 0.

Now, in the second case we will consider v value of v a voltage is much less than 1. So, in that case the damping will be negative as the damping constant coefficient v is negative and the energy will be fed into the LC tank circuit. So, that the transaction energy is from active resistive element to the LC circuit. So, we can say that the for the active resistance element we have v R into i R less than 0 and here that since the damping is negative the resistance is actually an active element. It is acting as a active element or we can also say the resistance is negative in this case.

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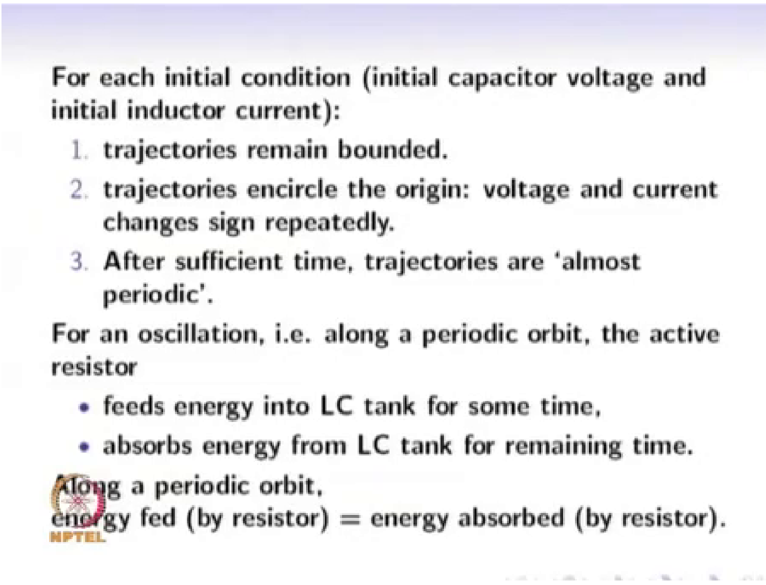

Let us look at the behavior of the system for given a initial condition. Now, that initial condition may be a voltage across the capacitor a initial voltage or it may be a initial current of the inductor. Now, in this case the trajectories will remain bounded. Now, how can we say that as we look before for v greater than 1 and v less than 1 we have damping positive and negative. So, we can say the trajectories are remaining bounded. When damping is positive the trajectories approaching towards the orbit and for the when damping is negative the trajectories in that case also the trajectories are approaching the orbit.

So, we can say that the trajectories are remaining bounded. Now, the second point we can say that trajectories encircle the origin. Now, for this case we will consider when for a initial given condition as said before the initial voltage the damping is positive and negative depending on v is remaining and v greater than 1 region or v less than 1 region. So, the value of v and i are repeatedly changing the sign. So, in that case we can say that since v and I are repeatedly changing the sign. So, the trajectories are actually encircling the origin.

Now, we are concluded the 2 points that the trajectory remain bounded and they are also encircling the origin. The next we will consider after sufficient time that is for any given initial condition if you have a sufficient time we can say that the trajectories are almost periodic. Since for a given initial condition the trajectories are approaching the periodic orbit. So, after much sufficient time we can say that they are almost periodic. They cannot be periodic because the 2 trajectories cannot intersect. So, they will be almost periodic.

Next we will consider oscillation along the periodic orbit. When we have a oscillation along a periodic orbit, the active resistor feeds the energy into LC circuit for some time that is when the active resistor is negative the damping is negative, it feeds the energy into LC circuit and it also absorbs the energy in the LC from the LC circuit when it is positive in that case damping is positive. So, we can say that during the periodic orbit active resistance element is a feeding energy and also absorbing energy for some time.

Now, let us see how can we say that the periodic orbit is stable or not. Now, suppose along a periodic orbit energy feed by the resistance is equal to the energy absorbed by the end resistance then we can say that periodic orbit is stable. We can say this because when the energy is fed is equal to the energy absorbed net energy extend is equal to 0. So, though periodic orbit in that case will be a stable one. So, we will have a stable oscillation for van der Pol oscillator.

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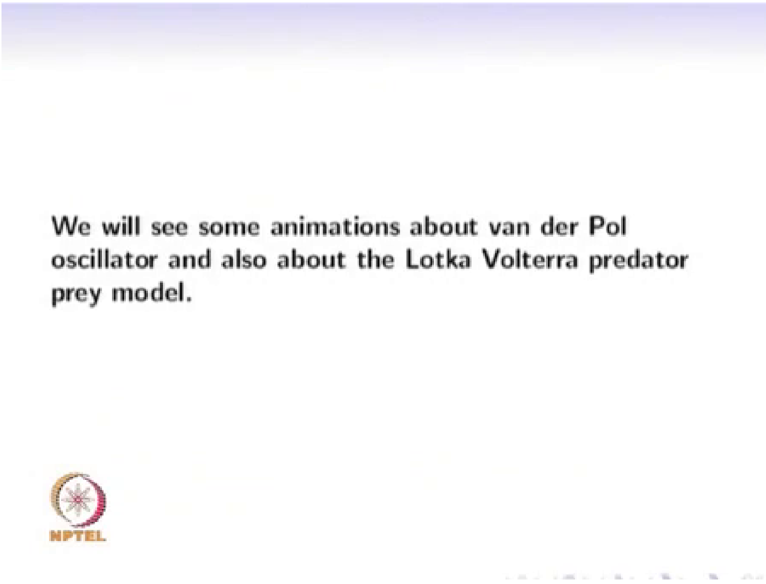

Further we will see some animations about van der Pol oscillator and also about the Lotka Volterra predator prey model.

Thank you.