Nonlinear System Analysis
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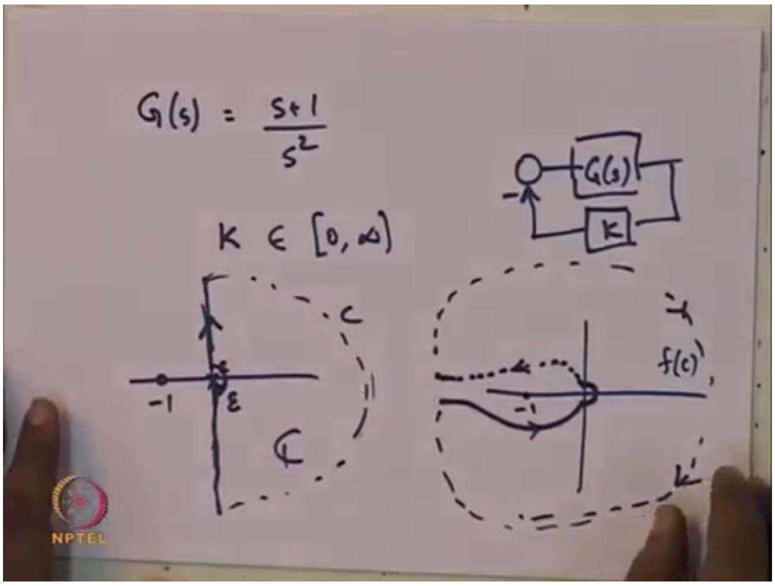
Lecture - 33
Counter example for Aizermann's conjecture



So, for this system I consider the linear plant G s to be S plus 1 upon S squared. So, if you take this linear plant in a feedback structure then, it should be clear that for all K for all feedback K in 0 to infinity this given feedback system is stable; that means, if you take G s and you give a feedback which is a linear gain K then the resulting system is stable ok.

We can see this in several ways. I mean, we have already looked at this Nyquist criterion. So, perhaps we can draw the Nyquist plot of this G s. So, if this is the complex plane and we want to draw the Nyquist plot here then as you start moving up, let us say from a small value epsilon you start moving up, for the small value epsilon you will get something here and you will have a curve like that.

And so, this goes right up to the top and then you have this infinitely large circle and corresponding to that you will get something here like this. And then you come up the negative slope and when you come up the negative slope you essentially have the reflection of the original 1 and of course, this particular thing has double pole at the origin and has a 0 at minus 1.

So, because of that to avoid this double pole we can draw a small circle here of radius epsilon and when you draw the small circle when you look at its image here you will end up getting something like this ok. So, now this contour C be traversed in this way yeah; that means, in the clockwise direction and as a result the contour that we got here using the Nyquist plot was something like this ok.

And then the critical point is the point minus 1 and we find that this Nyquist plot this contour the image of the contour f of c this does not enclose minus 1 and therefore, the resulting system is stable for all gains; yeah all gains from 0 to infinity.

Now, now what I am going to do is I am going to demonstrate a particular non-linearity which lies in the sector 0 to infinity. Now when what do we mean by saying that a non-linearity lies in the sector from 0 to infinity. So, earlier i talked about the slope, but it is not really the slope that we are looking at because the slope could very well be negative.

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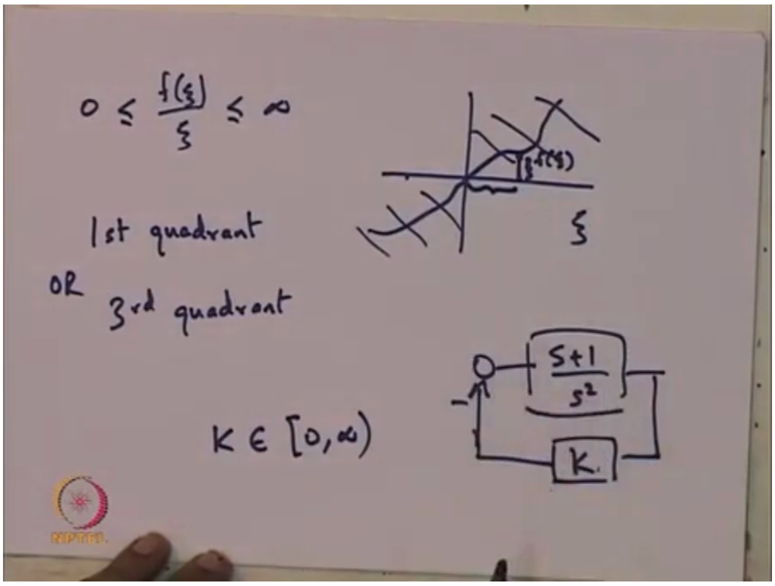

But, what when we when we are looking at f xi by xi and this is xi and let us say the non-linearity is like that. So, let us say this was f xi then when we are looking at this f xi by xi we are really looking at this divided by this.

And, so when you say f xi by xi lies between 0 and infinity what you are really saying is that the characteristics of this non-linearity should lie in the first quadrant; that means, when the input is positive the output is positive, or the third quadrant; that means, when the input is negative the output is negative.

So, the first quadrant or third quadrant ok. Now, that the linear system that we looked at S plus 1 upon S squared we saw that this particular linear system is stable when you give when you put this in a feedback connection when you put this in a feedback connection with this K with this K lying between 0 and infinity. Therefore, if now because this is true and if

Aizerman's conjecture were true, if we put in a non-linearity here whose characteristics lie in the first quadrant and the third quadrant then the resulting system should be asymptotically stable ok.

So, now what I am going to do is I am going to demonstrate a particular non-linearity which lies in the first quadrant and the third quadrant, but the resulting system is not stable ok. So, here is the non-linearity. So, the non-linearity is given by the following equation.

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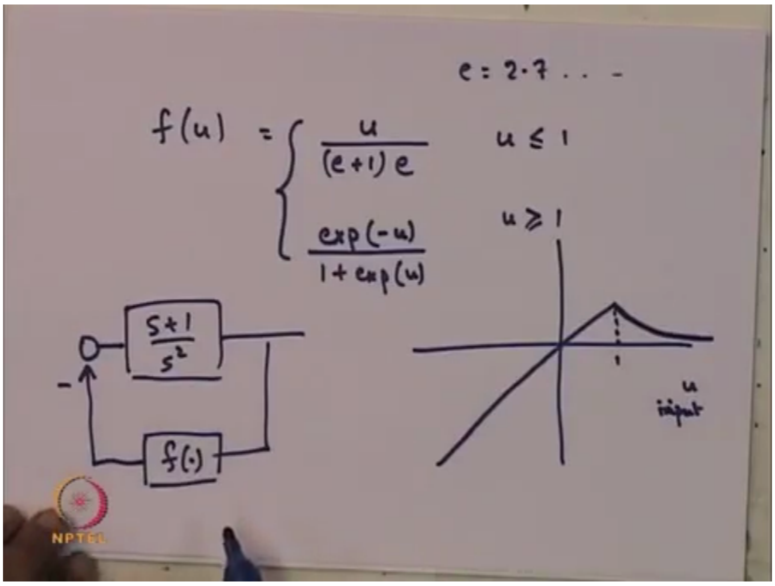

So, f of u; so, if u is the input to the non-linearity, f of u is given to be u upon e plus 1 times e; here e of course is the number e yeah. So, e is that 2.7 something ok. So, this is the equation for f of u for u less than equal to 1, but this if you look at this, this is like really linear.

So, if you are going to draw the characteristics you going to get a line with slope 1 upon e plus 1 by e and this goes right up to 1. So, this being the input u, input ok. So, this is the first part and now the second part for u greater than equal to 1 what you have is this is exponential of minus u divided by 1 plus exponential of u. So, here the characteristic that you are going to get is like that. So, now if you look at this non-linearity it lies in the third quadrant and the first quadrant.

So, this non-linearity given by this f of u in feedback connection with that plant. So, S plus 1 upon S squared and here you have the non-linearity f well. By, Aizerman's conjecture because S plus 1 by S squared a stable when you put any feedback K lying in the range from 0 to infinity. Therefore, this linear plant with this non-linearity defined in this way should result in asymptotic stability ok. Now, now what I am going to show is that this resulting system is not asymptotically stable. I am going to demonstrate a particular solution to this particular system which is actually growing with time ok.

And if you have a trajectory which is perpetually growing with time; obviously, that system cannot be asymptotically stable ok. So, here is the particular solution that I am going to give you.

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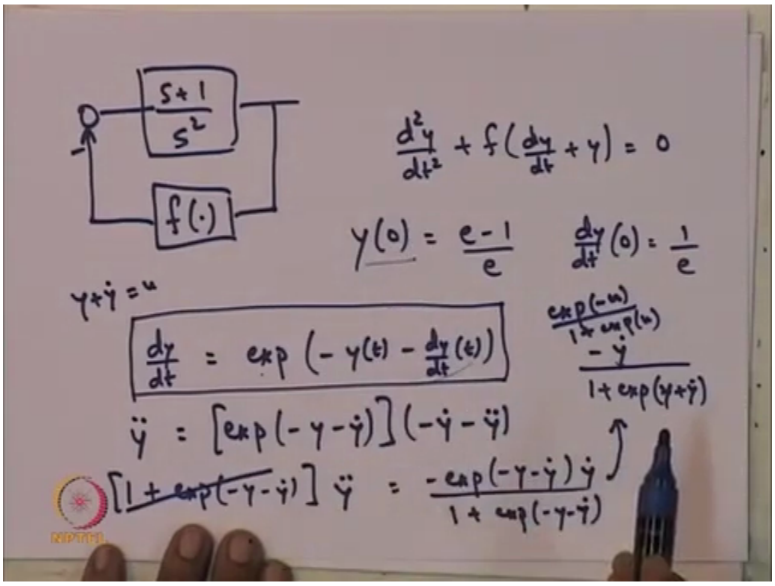

You see, when you put this S plus 1 upon S squared and you feed back through this non-linearity, then you know I reverse the earlier trick this is the same as looking at a system with the equation d 2 y d t squared plus f of d y d t plus y equal to 0. So, looking at this system is the same as looking at this system ok.

Now, for this system I look at a particular solution which is given by. So, y of 0 is specified as e minus 1 upon e and d y d t at 0 is specified as 1 by e ok. So, one wants to solve this differential equation with initial conditions y at 0 is e minus 1 by e and d y d t at 0 is 1 by e ok.

So, I claim that for these initial conditions the solution for this is given by the following. So, d y d t is equal to exponential of minus y t minus d y d t ok. So, I am claiming that this particular thing this particular equation gives us a solution to this original system with initial conditions y at 0 being this and d y d t at 0 being this ok.

So, how do we show that this is a solution? Well, let us differentiate it. So, y double dot that is the second derivative this is equal to exponential minus y minus y dot and then I have to take the derivatives of these. So, the derivatives of these they will give me minus y dot minus y double dot. So, this y double dot I take to the other side and I end up with 1 plus exponential minus y minus y dot, the whole thing multiplying y double dot is equal to exponential minus y minus y dot times ok. So, there is a minus times y dot.

And so, y double dot is going to be I take this below 1 plus exponential minus y minus y dot, but now if you think about this I sort of this exponential of some negative quantity is 1 upon exponential of the positive quantity. So, if I normalize this I I mean or rather I if I simplify this, I will end up with minus y dot upon 1 plus exponential minus sorry y plus y dot ok, and for y dot we had said that this is the solution. So, for y dot we can substitute this and if you now think of y plus y dot as u then this expression is the same as; yeah is the same as exponential of minus u upon 1 plus exponential of u ok.

This minus sign is appearing essentially because when you take the f to the other side there is a negative ok. So, this particular solution is indeed a solution to those differential equation with initial conditions given by y at 0 is e minus 1 by e and d y d t at 0 is 1 by e ok.

Now, if you look at this solution then it should be clear that once this and these are the initial conditions this exponential of some quantity. So, d y d t is going to increase. So, when d y d t increases I mean d y d t is positive. So, d y d t is positive means y increases. So, y increases d y d t increases and this is exponential. So, it will continue to have d y d t to be positive and therefore, y would continue to increase as time goes on which, essentially means that this system is not asymptotically stable.

So, this particular example is an example which tells us that the Aizerman's conjecture is wrong.