Nonlinear System Analysis

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Lecture – 34
Passivity inspiration – passive circuits – dissipation equality

So, now at this stage there is there was Aizerman's Conjecture which was promising, but it was shown that this Aizerman's Conjecture is false. So, now, what can we do about analysing general systems and what sort of general theory can be obtained which at least makes use of Aizerman's Conjecture and Aizerman's idea ok.

And so now, we are going to talk about something's which essentially uses Aizerman's idea, but gets past the conjecture and gives an actual answer as to which systems when you interconnect you will end up with something which is asymptotically stable ok. Now the inspiration for this thing comes in this particular technique comes from electrical engineering and it comes from electrical circuits which are called passive ok.

So, what do we mean by passive circuits? Now, you must be knowing very well that if you have a circuit made up of passive elements whatever that passive element is. If you have a circuit made up of passive elements then the resulting, I mean this then the circuit is passive. This really does not give you an answer all it says is that I am using the same word again and again; I am saying if it is made up of passive elements then the resulting circuit is passive. What exactly is passive?

Now, a sort of a very layman kind of definition for a passive system is the following, a passive system is something that does not generate energy. So, what it means is so if you have a passive circuit and you supply energy to the passive circuit then that energy either gets stored in the circuit or it gets dissipated or lost. But a passive circuit does not generate its own energy ok, so maybe we start by looking at some examples to get a generic idea of what passive it is.

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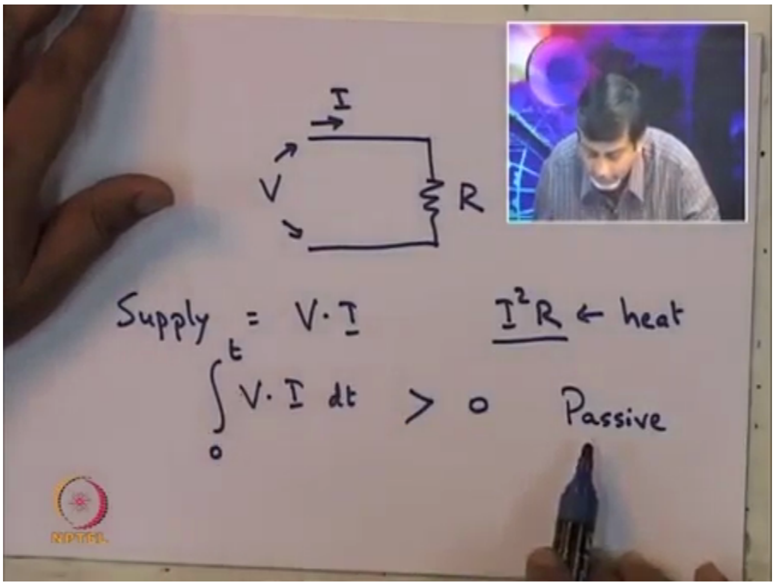

So, suppose we look at a circuit which consists of just a resistance ok. So, the voltage that we apply let me call it V and the current flowing in let me call it I, therefore the power that is fed into the circuit or the supply is V times I ok. Now, when you supply this this current I goes through this resistance and it gets dissipated as I squared R that is heat or something ok.

So, if you look at V dot I in this particular circuit and you look at the over a long period of time; that means, say starting from time t equal to 0 to sometime t, then this resulting thing is always going to be greater than 0. Now, one could use this as the definition for passive. So, when we talk about a passive a system then what we are talking about is that the amount of energy supplied to the system is positive ok.

Of course, this definition of passivity may not I mean this may not always strictly hold the moment we use some element like a capacitance or an inductance yeah ok, so let me try and explain what I mean by that. So, suppose now instead of a resistance you have a capacitance ok.

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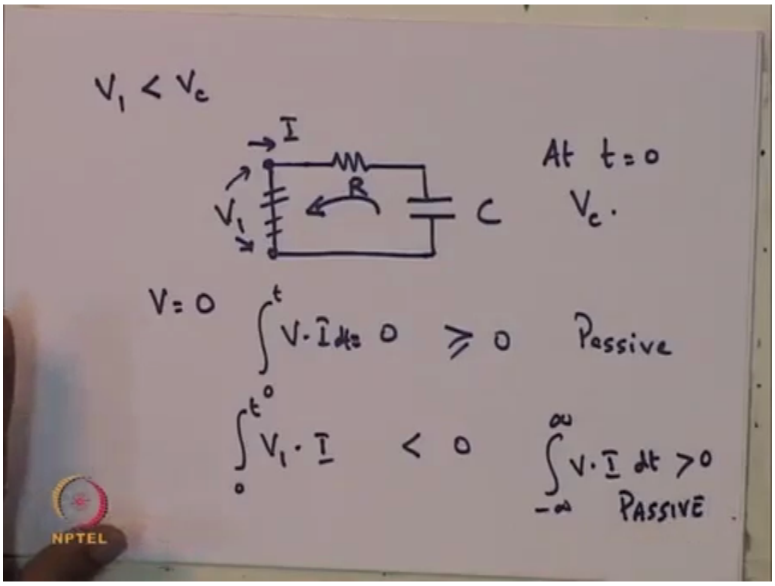

And suppose, you also have a resistance here and then we apply a voltage V and the current going in is let us say I ok. Now, let me also additionally assume that this capacitance is charged and so at time t equal to 0 the charge on the capacitance is V c. And, then let me shot this if I shot this then V; that means, the terminal voltage is 0. Therefore, if you want to talk about supply of energy well the energy that is supplied in is V dot I which is 0. But, of course, in this circuit we know something is happening yeah of course, even in this particular case if you of course, take the integral from 0 to t of V I d t this is 0 which is greater than equal to 0.

So, we could still continue to call it passive yeah, but now suppose we apply a voltage V. So, instead of our shot let us assume that we apply a voltage V 1 where V 1 is less than V c. Now if V 1 is less than V c then what is going to happen is from the capacitor the current would flow this way. And so now if you look at the supply it is going to be V 1 multiplied by I, but I the actual current which is flowing is in the opposite direction to the convention that you use for the I and so this quantity here is negative.

And so when you take this integral you are going to end up with something less than 0. Yeah but of course, you would already know that we call a capacitance a passive element because the capacitance does not generate energy of its own. But what is happening in this particular situation is that this capacitance at time t equal to 0 was already charged to some voltage V c and so the stored energy is been dissipated ok.

So, if we want to also incorporate the stored energy one way we could do this is instead of taking this integral from 0 to t we take the integral from minus infinity to plus infinity. And if you take them integrate from minus infinity to plus infinity of V dot I d t, then any circuit for which this integral V dot I this is the power supplied to the circuit from minus infinity to plus infinity, if this is greater than 0 then we can call this circuit passive alright.

So, so how do we how do we capture this this particular notion of passivity so the basic idea is this. So, suppose we start off with a circuit and we assume that the circuit is at rest ok, what do I mean by the circuit is at rest. So, the circuit could have several energy storing devices like capacitors and inductors the circuit is at rest by saying that the circuit is at rest, what I mean is that all the capacitors are discharged and similarly all the inductors have no current flowing through them. So, there is no stored energy in the circuit that is what I mean when I say a circuit is at rest.

Now, suppose to that circuit you now supply a certain amount of energy, now if you supply certain amount of energy what do you think would happen in the circuit. Well this energy which comes into the into the circuit the part of it of course, will get dissipated part of it will get dissipated in the form of heat in the resistances. But a part of it might get stored either as electrical energy in a capacitance or as magnetic energy in an inductance ok.

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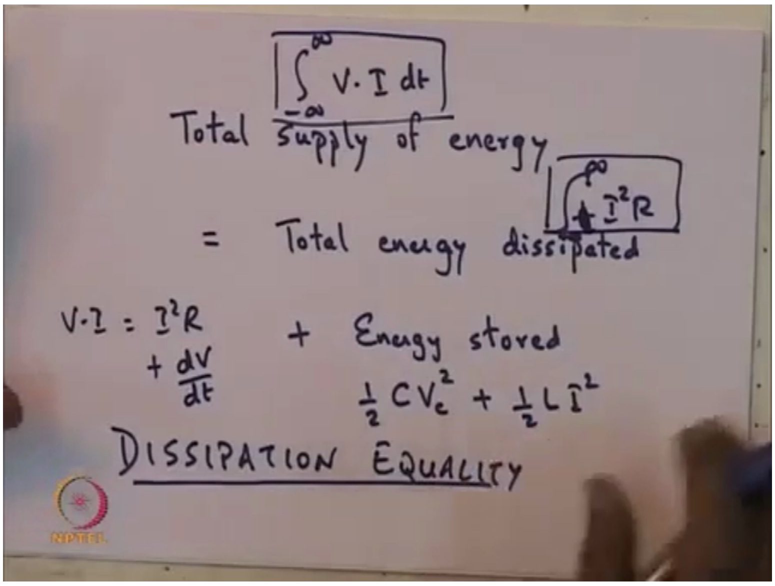

So, what in very broad terms what we can say is that the total supply of energy this would be equal to total energy dissipated plus energy stored all right. So, now the total energy the total supply of energy of course, the expression for this would be V dot I. So, V dot I is the power d t if you integrate this from minus infinity to infinity. So, this is the mathematical expression for the total supply of energy the total energy dissipated well the energy dissipated.

Now, or the power that is dissipated is given by half is given by I squared R and so if you integrate this from minus infinity to infinity that gives you the total ok. I guess there is no half here it is just I squared R I squared R is the total amount of energy that is dissipated. And, you put this integral from minus infinity to infinity this is the mathematical expression for the total energy dissipated and what about the total energy that is stored.

Well it could be stored in a capacitance and the energy stored in the capacitance will be something like this and there could be energy stored in the inductors it will be something like that. So, mathematically this is the expression that we will get that the total supply so you have integral is equal to this integral which is the total energy dissipated plus the energy stored which is this ok. Of course, this is an integral form so this this equation that I have written down can be called dissipation equality ok. And, I can state the same dissipation equality this is the integral form I can state it in the differential form which means I just take that derivatives in each of these cases.

And I would end up with an expression like V dot I is equal to I squared R plus d v d t where d v d t is the rate of change of energy stored in the circuit.

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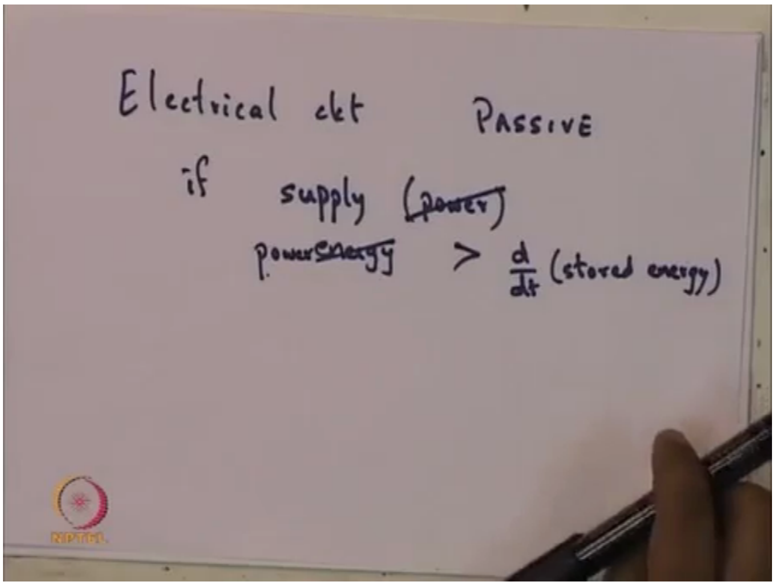

So, then coming back to an electrical circuit we call an electrical circuit passive if the supply or power is ok. Now, the supply in terms of energy of course, is the integrated part power is the derivative part. So, perhaps let me talk about it as the supply in in terms of energy.

The energy supplied is greater than the rate of change of stored energy ok. So, of course, it is the rate of change of stored energy so actually I am talking about the power. So, the supply power is greater than the rate of change of stored energy then such a electrical circuit is called passive ok. Now this stored energy is actually quite important of course, electrical circuits are circuits which have an input and an output.

And, so when you talk about the stored energy this stored energy is something that you have in a in a system with an input and an output and the stored energy plays exactly the same role as a Lyapunov function place in a system without inputs. So, let me use a electrical circuit example to show that the stored energy plays exactly the same role as a Lyapunov function in a system without inputs so let us consider an R L C circuit.

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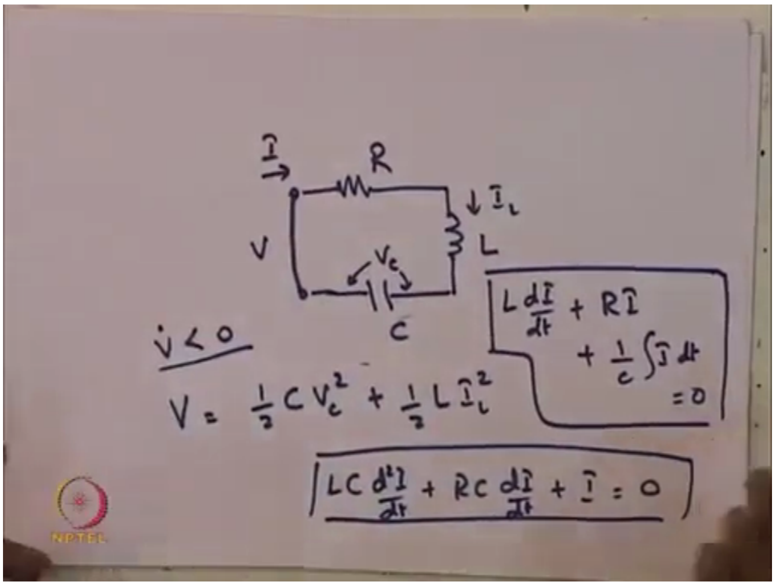

So this is R this is L this is C and let us assume that there is some stored energy which is half C V c squared plus a half L I L squared. Now of course, this is a system which has an input and an output of course, one could think of the voltage as the point one could say that voltage is the input and this is the voltage that we are applying and the output is the current. One could also think of the current I mean one could think of a current source here on the current pushing in would being pushed in and the voltage that is generated as the output.

So, it really does not matter whether you think of the voltage as the input or the current as the input, but here let us assume that the voltage is input on the current is the output. So, this now is a circuit with an input and an output and there is a stored energy which depends on the current I L and the voltage across the capacitor V c ok. Now, here the input is V if you set the input to 0 if you set the input to 0; that means, you just shot here then we have an autonomous system in the sense this is a system without any inputs.

Now, when this is a system without any inputs what is going to happen well what is going to happen is there is going to be oscillation setup in this circuit. And during this oscillations the electrical energy which is stored in the capacitor gets converted into magnetic energy in the inductor through half the cycle. But of course, the current this transfer is taking place through the current because of which there is some energy being dissipated.

And then the magnetic energy in the inductor is going to return that magnetic energy to convert that magnetic energy back to electrical energy stored in the capacitance and again some more dissipation is going to take place. And, in this way this keeps oscillating until all the energy which were stored in the capacitor and all the magnetic energy which are stored in the inductor gets dissipated and then you would have no current in the circuit ok.

But, now if you write out these equations the equations for this this particular oscillation that would take place, well one way to write out is you just think of the current in the circuit and you can write down the write down the drops in terms of the current. So, you will get you will get the voltage drop here to be L d I d t plus the voltage drop here to be R times I plus the voltage drop here to be 1 by c integral I d t this is equal to 0 well this is a second order differential equation.

And the second order differential equation I mean so take derivative once and you will end up with L c. The second derivative of I plus R c times the derivative of I plus I equal to 0. Now for this particular differential equation if you use this V as the Lyapunov function then this Lyapunov function is positive definite. And, if you take the derivative of this Lyapunov function you would find that the derivative of this Lyapunov function is negative which essentially proves that this circuit is asymptotically stable when you convert it into a system without inputs ok.

So, let me stop here right now and we will carry on about this in the next lecture.