Nonlinear System Analysis
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Lecture - 36
PR conditions for passivity of SISO system

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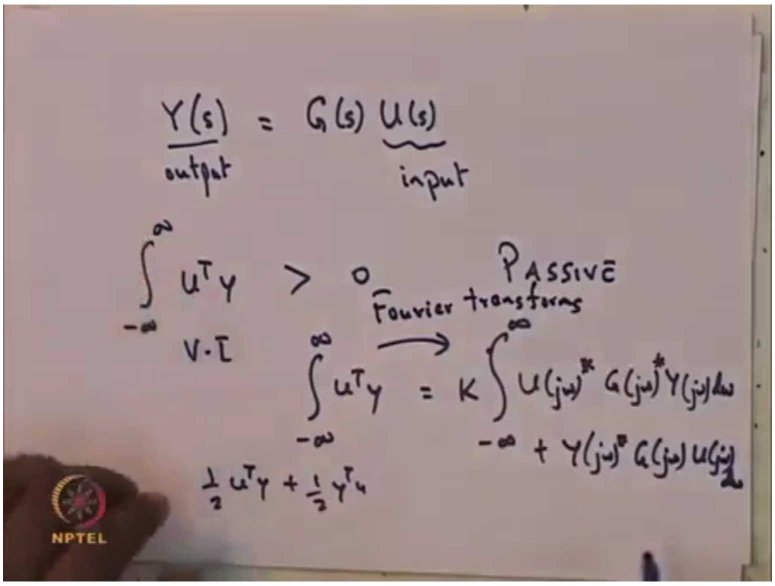

Let me just constrain myself to a single input single output case and let us say the single input single output situation is given using a transfer function as Y s is equal to some G s u s. Where Y is the output and u is the input. Now earlier what we had said was that u transpose y, this is the same in the case of the electrical circuit v dot I is exactly the same as u transpose y.

And we are saying that u transpose y integral from minus infinity to plus infinity. This particular quantity should be greater than 0, then the system is passive ok. Now if you are going to take the integral from minus infinity to plus infinity of u transpose y, one could take the Fourier transforms. And, if one takes the Fourier transforms this is the same as there would be some scaling factor k and minus infinity to plus infinity of u j omega star times G j omega well G j omega star yj omega.

But of course, this u transpose y I could really write it down as half of u transpose y plus half of y transpose u so, sort of making it symmetric. Therefore, along with this expression there will be this other expression also which is y j omega star G j omega u j omega ok. So, this integral of course, this is over d omega. So, this integral would be equivalent to this integral after having taken the Fourier transform. So, we have taken Fourier transforms ok. Now if you look at this expression, this is the same as saying well, I think I have made a mistake sorry. So, let me do it again.

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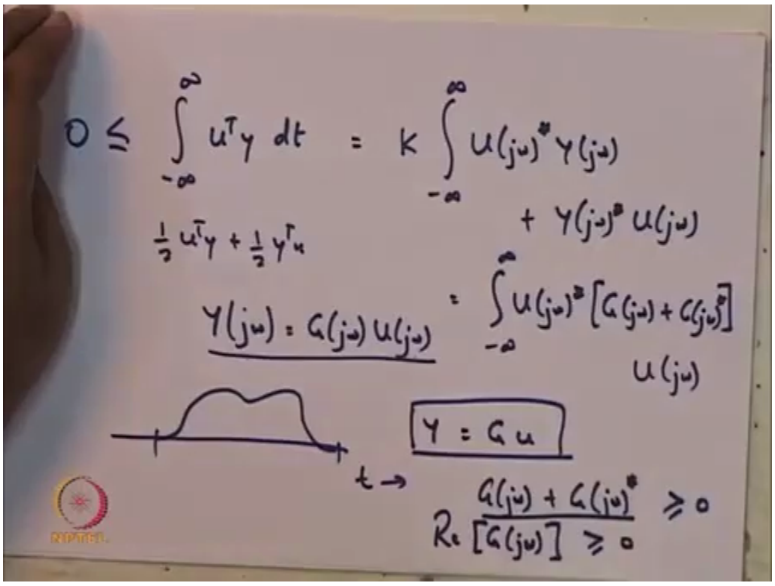

So, if you looking at this from minus infinity to infinity of u transpose y dt. This is in the time domain, this is equal to some constant of proportionality of course, is u transpose y u transpose y, I could write it out as half u transpose y plus half y transpose u. Now, I could now pass over using Fourier transforms into the j omega into the omega domain and I will have again integral from minus infinity to infinity of uj omega star y j omega plus y j omega star uj omega. So, last time the mistake I made was that I put this gj omega in between.

Now, I can bring in the gj omega by substituting you see once you have taken the transforms y j omega is really G j omega times uj omega. And so, substituting this and there we would end up with integral minus infinity to plus infinity I am just forgetting the proportionality constant you have uj omega star multiplying G j omega plus G j omega star multiplying u j omega ok.

Now of course, when I write this down there are several assumptions that are going in the assumptions that are going in are the following. For example, we will assume that u is a compactly supported trajectory; that means, if this is the time axis u is non-zero only over a compact set.

Now, if u is non-zero over a compact set, then it is easy to take the Fourier transform it makes sense to take the Fourier transform. Now, if u is compactly supported because the transfer function tells us that Y is equal to G times u this is the equation therefore, the Fourier transform of Y also makes sense and if u is compactly supported in the circuit we would we can expect that y is also compactly supported. And so, then it makes sense to take the Fourier transforms.

And, now if you take the Fourier transforms, the Fourier transform of Y and the Fourier transform of u is related in this way and. So, once you have this expression you substitute and you get this. And now this is this integral is greater than equal to 0. So, if you look at this last expression whatever u you choose this expression must be positive. And now, one can show that this expression would only be positive if G j omega plus G j omega star is greater than equal to 0.

But this here is really the real part of G j omega. And so, the real part of G j omega must be greater than equal to 0. So, this condition is a condition for passivity and the condition for passivity translates to and here of course, we have only considering single input single output case. So, single input single output case the y and the u the output and the input are related to through this transfer function G of s.

And then what this translates to by this set of manipulations is that the real part of G j omega must be greater than equal to 0. So, let us look at situations where the real part of G j omega is greater than equal to 0 ok. What does that mean?

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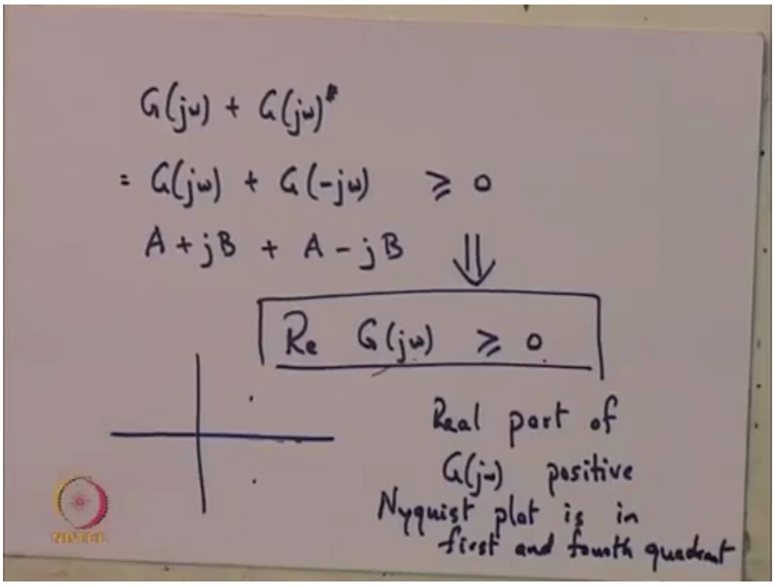

Now, you see the expression that we had is G j omega plus G j omega star, but this is the same as saying G j omega plus G minus j omega. Now if G j omega can be written as A plus jB then, this will turn out to be A minus jB. So, the imaginary part gets cancelled out and you are just left with the real part.

So, this being greater than equal to 0 is the same as saying that the real part of the transfer function G j omega must be greater than equal to 0. How to translate this condition into something more meaningful for us ok. One way we can translate this condition into something more meaningful is you see the image of G j omega is the Nyquist plot.

So, saying that the real part of G j omega is greater than equal to 0 is the same as saying that the Nyquist plot of the transfer function should lie in the first or the second quadrant because then the real part of G j omega is going to be positive. So, what this translates to or whatever we have been doing till now the this result that we have got. What it translates to is that the real part of the transfer function G j omega should be positive yeah.

And what that means, is that the Nyquist plot is in first and fourth quadrant, first and fourth quadrant ok. But as it turns out this Nyquist plot being in the first and the fourth quadrant is not a complete characterization of passivity ok. That is because mathematically the Nyquist plot lying in the first and the fourth quadrant may not necessarily translate into that particular condition that we had where you have a storage function and supply function and that interpretation that you have.