Nonlinear System Analysis
Prof. Harish K Pillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 37
Examples of PR transfer functions

First, let us look at some Examples of transfer functions which are positive real ok sorry. So, I did not mention that. So, those transfer functions whose Nyquist plots lie in the first and the fourth quadrant well one definition for that is positive real.

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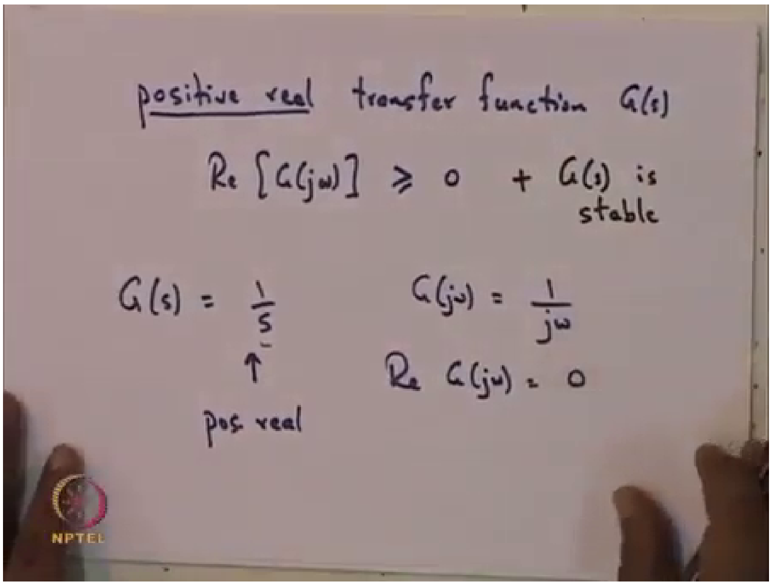

So, positive real transfer function G s is such that the real part of G j omega is greater than equal to 0 ok. I cannot claim that this is the definition of positive real because depending upon various books, the definition of positive real changes ok.

Now, the earlier interpretation that we had for passivity the one would like to say that positive real transfer functions is equivalent to passive. But that is actually strictly not true because if you give the definition of positive real to be that the real part that the Nyquist plot lies in the first and the fourth quadrant. Then one can show that there are some transfer functions which will satisfy this condition of positive reality, but are not actually passive.

So, it turns out that in many books they use the definition of positive reality to be the fact that the Nyquist plot lies in the first and the fourth quadrant. Whereas in many other books the definition for positive reality says that the real part of the transfer function evaluate; that means, the Nyquist plot lies in the first and the fourth quadrant, but in addition the transfer function is stable ok.

So, now I would give some examples to try and tell you what exactly is the difference between defining positive reality in this way and defining positive reality in the sense of this plus G s being stable ok. So, let us look at some examples. So, suppose you look at the transfer function G s equal to 1 by S. Now, this transfer function.

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Is it positive real? Well if you just take this definition, then G of j omega is 1 upon j omega this therefore, the real part of G of j omega is equal to 0. And so, the real part is greater than equal to 0. So, one can say that this transfer function is positive real ok.

If one uses the definition that positive real means this plus G s is stable. If one uses that one's; if one uses this definition then again this particular pause this particular transfer function is positive real because the real part is actually 0. And this is also stable means S being; S being a pole at 0, if you think of that a stability or marginal stability or whatever goes, then one can still continue to call this positive real ok.

So, the definition of positive reality could be just this or could be this along with G s being stable. Again you know as I said earlier it depends on the books that you follow some books use only this definition, but most books use this definition plus the fact that G s is stable.

So, let me use a few more examples to show why there is a difference between just having this condition which is the condition that we got from the earlier equations that we derived and also having this condition that G s is stable.

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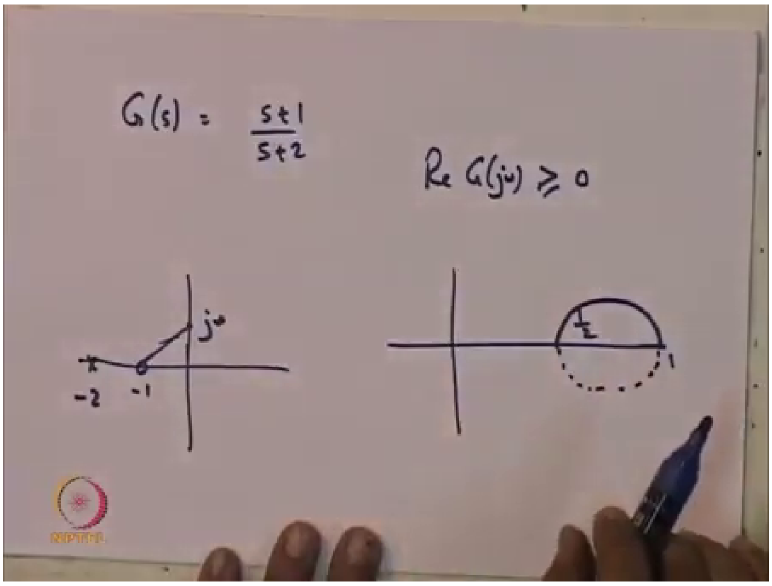

So, let me take another transfer function G s is S plus 1 upon S plus 2 ok. Now, if we look at the Nyquist plot of this, the Nyquist plot of this at S equal to 0 it is at half and then ok. So, this particular transfer function has a 0 at minus 1 and it has a pole at minus 2.

So, as you go up you find at any particular j omega there is this angle which is larger than this angle. And so, it is going to be positive and as j omega tends to infinity this finally, tends to 1. So, you end up with a Nyquist plot which looks like this goes to 1 and then when you look at the rest of it this is what you get.

And so, you it is clear that the real part of G j omega is greater than equal to 0 ok. So, by that definition that the real part of G j omega is greater than equal to 0 this transfer function is positive real. Of course, if you also put in the fact that it should be stable well this transfer function is also stable therefore, by both the definitions this transfer function is positive real.

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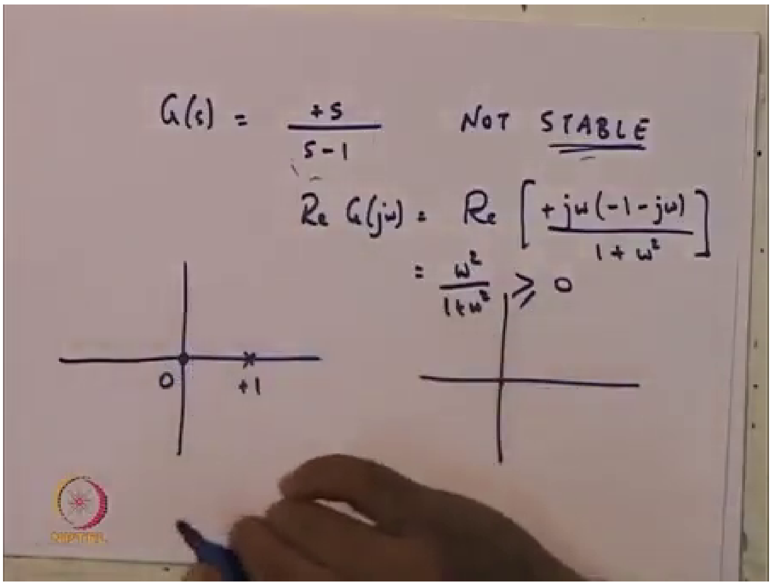

Let me now take another transfer function which is G s is let us say S minus 1. Now, this transfer function of course, is not stable. Now, if we were to draw the Nyquist plot of this it has a pole at minus 1 at plus 1 and it has a 0 at the origin. Now, if you now look if you plot this thing as you go up.

So, at omega equal to 0 this gives you 0 and then as you increase the omega what you would get is the real part of G j omega is equal to the real part of minus j omega multiplying minus 1 minus j omega upon 1 plus omega squared ok and this then turns out to be. So, perhaps I should not use a minus here I should just put plus. So, if I take this transfer function, so S upon S minus 1 this is not stable. And when you calculate this is plus and so, you get 1 plus omega squared yeah.

And so, in the numerator, so, the real part will turn out to be omega square upon 1 plus omega squared. And this quantity here is greater than 0 for all omega; greater than equal to 0 for all omega. So, if you use the definition that the real part of the transfer function is greater than equal to 0, then this thing is positive real this transfer function is positive real. But if in addition you also put in the condition of stability this is not stable therefore, this this transfer function is not positive real ok.

Now, the question is why was this condition of stability brought in to associate these transfer functions with passivity? Now, that is completely dependent on what one would call the storage functions ok. So, it turns out that if you have a transfer function like this you are not going, it would not be possible to synthesize a circuit which has this transfer function using purely passive elements and the reason for that is because this G of s is not stable. As a result the associated storage function that you would get that you can get for this particular transfer function that storage function is not going to be positive definite.