Nonlinear System Analysis
Prof. Harish K Pillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 42
PSD Storage function in PR Lemma and how to make it PD (strictly PR)

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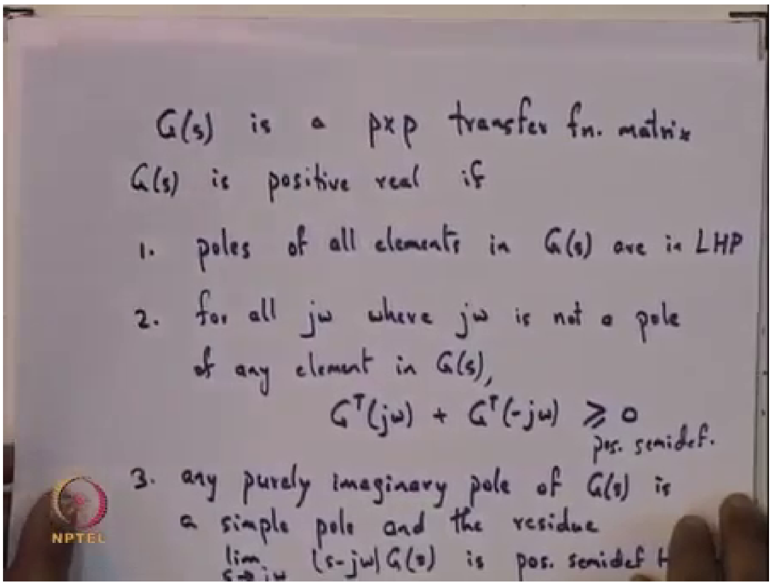

So, now since we have also already done the Positive Real Lemma. There are variation of the positive real lemma. If you remember the positive real lemma ultimately in the statement about the matrix is; there is the specific matrix p which is a positive definite matrix positive semi definite matrix. Now, this positive semi definite matrix during the proof of the positive real lemma or in fact, when I showed that the existence I mean; the existence of those equations are equivalent to the system being passive. I had made use of the fact that in this matrix P defines the storage function.

Now this if P is a positive definite matrix, then the storage function is positive definite. If p is a positive semi definite function, then the storage is positive semi definite ok. Now between the situation when the storage function is positive definite and the storage function is positive semi definite. There is a slight problem and this problem is very much similar to the kind of problem that you would get when you use systems with inputs no outputs and you are using Lyapunov theory.

Now in Lyapunov theory when you take when you take function, which is positive definite then that actually guarantees whatever is the conclusions that you can draw where from using Lyapunov theory. Whereas if you take something which is positive semi definite, you cannot utilize it to the full power of the Lyapunov theory yeah. So, you cannot as a Lyapunov function candidate, you have to always take something which is positive definite and you hope that its derivative is negative definite.

But if the derivative is negative semi definite you cannot draw that is strong a conclusion yeah I mean your conclusions that you can draw is weaker and so on so forth. So, in the same way as is in in exactly in same way as far as storage functions are concerned when you have positive definite storage functions its good and when you have positive semi definite storage function it is not that good. And the statement of the positive real lemma, only guarantees that the storage function is positive semi definite not positive definite.

Now, in order to guarantee the positive definiteness of the storage function, one brings in this additional thing. So, there we had shown that positive semi definite storage function I mean; positive semi definite P and those other matrix conditions are equivalent to the transfer function being positive real. Now, one can give an additional thing which is additional definition kind of thing which guarantees that the storage function is strictly positive definite. Now, the storage function being strictly positive definite is equivalent to the transfer function being strictly proper real.

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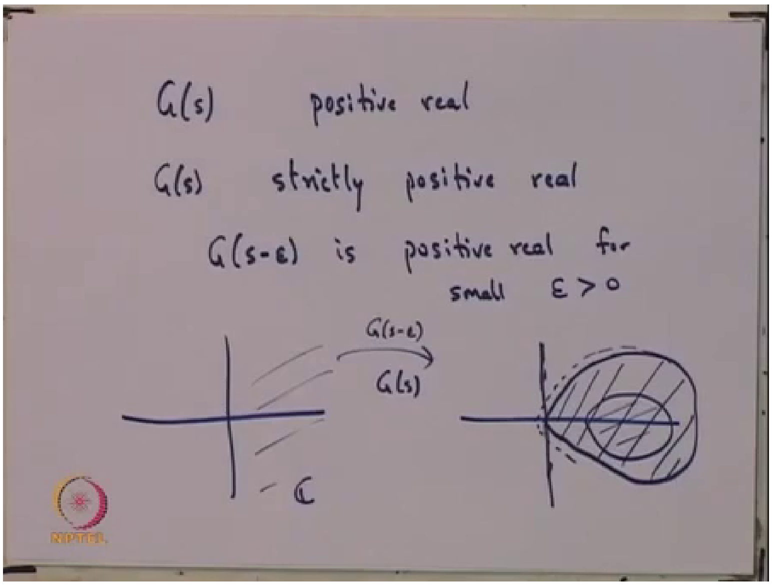

So, we have already given some definition for G s being positive real yeah. So, there is a Nyquist condition and so on so forth. Now we say G of s is strictly positive real when G of s minus epsilon is positive real yeah for epsilon greater than 0. I mean epsilon greater than 0 small for small epsilon greater than 0.

What we mean by this is you see earlier when I was talking about positive realness, I would said that of course, this is a map G of s. So, any point s goes to the corresponding point here and the imaginary axis mapping to something and then the right half mapping to the inside of course, is the good thing that can happen and that is the definition of positive real ok.

Now, in this particular situation where which I have drawn whether imaginary axis maps to this and the right half maps to the inside. Clearly, if instead of G of s, we take the map G of s minus epsilon that also about map to you know some neighbourhood of this. And so, in fact, this particular this particular situation G of s is strictly positive real. It might happen that, you have some plot which looks like that. The imaginary access maps to something like that and then the right half plane perhaps maps to something like that.

And now, if you look at G of s minus epsilon then the perturbation might be such that this gets move like that. And because it gets move like that G of s minus epsilon does not satisfy the positive realness condition and so, this will not be strictly positive real. And this touching in the imaginary access of this area, this region it map to some other region here and that region how it touches the imaginary access that in some sense defines the positive definiteness and the positive semi definiteness of the storage function.

And so, when you do this G of s minus epsilon; that means, you perturb by epsilon in some sense what you are doing is you have moving this imaginary axis. And so, if there are these places where it touches the imaginary axis, this image then, when you shift the imaginary axis, then those things get mapped to the left half. And this G of s minus epsilon does not does not remain positive real. So, such things which are on the boundary they are not strictly positive real anything else strictly positive real.