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> Lecture – 43 KYP Theorem

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G(s) positive real G(s) strictly positive real G(s-E) is positive real small G(s-c) 66) 6

Now, of course, just like the positive real lemma, there is also a famous lemma which instead of talking about the equivalence of positive semi definite p and G of s being; G of s being positive real. It talks about the equivalence of strictly positive real and this particular theorem is attributed to three famous people Kalman, Yakubovitch, yeah Yakubovitch and Popov.

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KALMAN - YAKUBOVITCH - POPOV
G(s) is strictly positive real if and
only if
$$\exists P > 0$$
, L, W
 $A^TP + PA + -L^TL - \epsilon P$
 $PB + C^T - EW$
 $W^TW = D + D^T$

So, this lemma is attributed to all three of them and the lemma is exactly the same as the positive real lemma. The only difference is that G s is strictly positive real. So, instead of positive real earlier, you had only positive real now you have the additional strictly if and only if ok.

And so now, you have the equations which are A transpose if and only if there exists P positive definite and other two matrix is L and W such that a transpose P plus P A is equal to minus L transpose L L transpose L minus epsilon times P and P B is equal to C transpose minus L L transpose W and W transpose W is equal to D plus D transpose.

Of course, here just like in the earlier strictly in the positive real lemma, here also all the assumptions are that this A B C and D come from a minimal state representation of G s. So,

as you see between the positive real lemma and this lemma, the only difference is the G s on one side; we are saying as strictly positive real.

And on the other side in this Lyapunov equation instead of A transpose A P plus P A being equal to minus L transpose L. There is this additional epsilon P and this sort of guarantees the strictly positive real situation and it guarantees the positive storage function.