Nonlinear System Analysis
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Lecture – 44
Passivity preservation under interconnection

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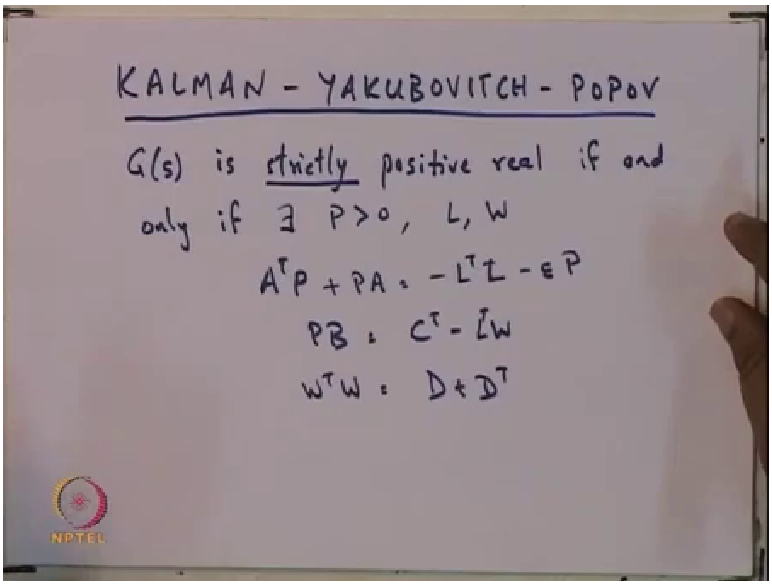

So, let me now revisit and look at what we have been talking about earlier. And why we started looking at this positive real transfer functions and so on. So, the reason why we started looking at this positive real transfer functions is, first of all there was a Aizerman's conjecture. And from the Aizerman conjecture, a certain guess was taken; that you know if you have a non-linearity in some certain sector. And you have a feedback connection of that non-linearity with a linear system.

Then if that linear system with those particular gains gives yours stable close loop system. Then the linear system with the non-linearity in that loop will give you a stable system. And then we also saw that counter examples were given so, Aizerman's conjecture is not correct. Now, after that we came into this passive systems and what these passive systems are and, we have a lot of results with respect to passive systems.

Now, the important thing about passive systems is that, if you interconnect two passive systems. I mean if you have a feedback connection of two passive systems. Then the resulting system is also passive. And the this makes things very good because a what one is really saying is if you start off with some system, which is passive and you have another system which is passive and you interconnect the two. The new system that you get which is the interconnection of the two systems also passive.

And this I mean especially if you think about this passivity in terms of the energy. That means, the passive system is something where the total amount of energy supplied is either dissipated or it goes to increase the stored energy, then this seems very natural.

But what we will now do is, we will formally show that when you interconnect two passive systems the resulting system turns out to be passive. As a result it turns out that this concept of passivity is something, that goes a long way in answering the question raised by Aizerman. And in fact, providing an answer which is similar to what Aizerman guessed ok.

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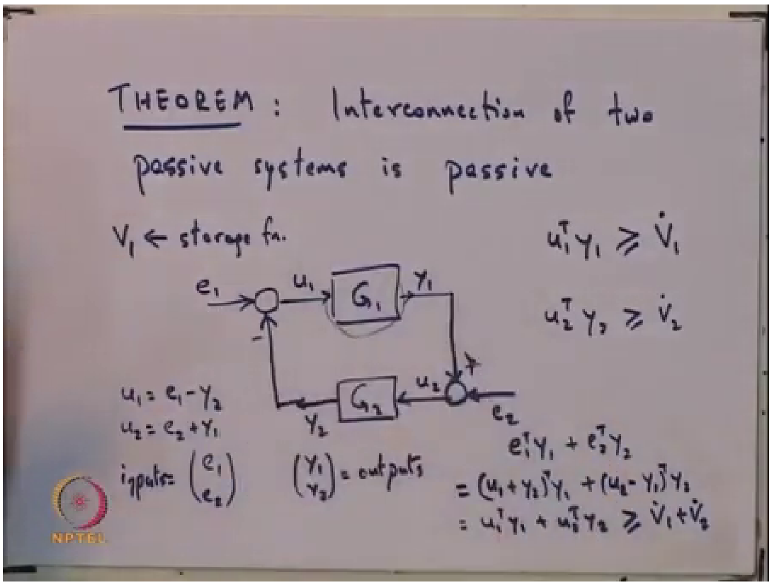

So, let me begin by first talking about the lemma or the theorem ok; might as well call it a theorem. Interconnection of two passive systems is passive ok. So, what do I mean by this interconnection ok. So, let me assume this is system one can I call it G 1. So, let me call the input u 1 and the output y 1. And let me have a second system G 2. So, input you see when you are talking about input and output one needs to probably draw an arrow. So, that its clear what is the input and what is the output.

So, u 1 is the input and y 1 is the output. And let us have another system G 2 and this system has u 2 as the input now and Y 2 as the output. Now, G 1 is passive what does it mean to say G 1 is passive? Well.

One thing that from all the discussion that we had about passivity is that u 1 Y 1; let me say u 1 transpose. So, rather than think of it a single input single output I could think of it as multi input multi output.

So, I am saying u 1 transpose Y 1 is greater than equal to V 1 dot; where this V 1 is the storage function of the first system ok. So, we had said that for the passive systems, the product of the input and the output is greater than equal to the rate of change of the storage function.

So, V 1 is like the amount of energy stored in G 1 roughly and so, u 1 transpose Y 1 is like the amount of energy supplied and the energy supplied is greater than equal to the rate of change.

Or the power supplied is greater than equal to the rate of change of the stored energy in the first system. Now this one being passive essentially you have a similar statement u transpose Y 2, must be greater than equal to V 2 dot. And this V 2 dot is the storage function of the second transfer function ok. Now, let us look at what we mean by interconnection. So, let us interconnect it in the following way.

So, I put in this thing. And I will assume that there is some input e 1 coming into the next system and maybe I subtract this. So, what this essentially tells me is that u 1 equal to e 1 minus y 2 and I do a same kind of thing here. So, there is some input here which let me call it e 2. Now, of this system this is the interconnected system and in this interconnected system, I can think of the vector e 1, e 2.

The vector e 1, e 2 as being the set of inputs ok. And I can continue to think of Y 1 and Y 2 as a set of outputs. So, then input times output is essentially e 1 Y 1 plus e 2 Y 2. Now for e 1 if I substitute u 1 plus y 2. So, I get u 1 plus Y 2 times Y 1 plus. Now, I have not written the equation for this, here this I would continue to call it positive.

So, what I have is u 2 is equal to e 2 plus y 1 ok. So, this e 1 Y 1 plus e 2 Y 2 is equal to u 1 plus Y 2 times Y 1 and for e 2 I can substitute e 2 is u 2 minus Y 1. So, u 2 minus Y 1 times Y 2; of course, there are these transposes, but that really does not matter. Now, you see you the you have a Y 2 transpose Y 1 and you have a Y 1 transpose Y 2, but with negative signs. So, they sort of cancel.

So, what you are left with is, u 1 transpose Y 1 plus u 2 transpose Y 2, which from these two inequalities you know its greater than equal to V 1 dot plus V 2 dot. Now what does this mean? This means that if you take this e 1 and e 2 as the inputs for the interconnected system and the Y 1 and Y 2 as the output.

So, for this interconnected system when you look at the inputs multiplying the outputs. This is greater than equal to the rate of change of a storage function which is in fact, the sum of the storage function of the first one and the second one.

So, in physical systems if this was a physical system and it had some elements, which stored energy and there is another system which has some elements which is store which is storing energy. Then the complete storage function is the sum of this storage function plus this storage function. So, now, this is an extremely powerful sort of result. And therefore, I mean what we can say is if you have two systems which are passive and you interconnected then the interconnected system continues to be passive.

Now, if I mean how this theorem becomes really powerful is the is by the following means. You see suppose you think of this G 1 this system G 1 as a linear system which is passive. G 2 is some system which is let us say a non-linear system, but you can sort of by some method means show that this is passive. Then if you interconnect these two then the interconnected system is also passive. So, if this was a non-linear system and you managed to find some storage function for this non-linear system.

Then in some sense you have found a storage function for the complete non-linear system. Now if you are talking about for example, Lyapunov theory and so, you do not think of these inputs. Earlier we had discussed how given a general non-linear equation you can split it up into a linear part and the non-linear part.

And now, if you can show that this linear part is passive and this non-linear part is passive independently. And for this non-linear part you can find some storage function for the linear part of course, we already have the positive real lemma and the (Refer Time: 11:39) which (Refer Time: 11:40) lemma, by which you can find storage function.

Then the sum of these two storage functions act like the storage function for the net system. But the 0 input that some of the two storage functions would act like the Lyapunov function. And therefore, this is in fact, a way to construct a Lyapunov function for that particular system.