Nonlinear System Analysis
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Lecture - 47
Need for Loop transformation

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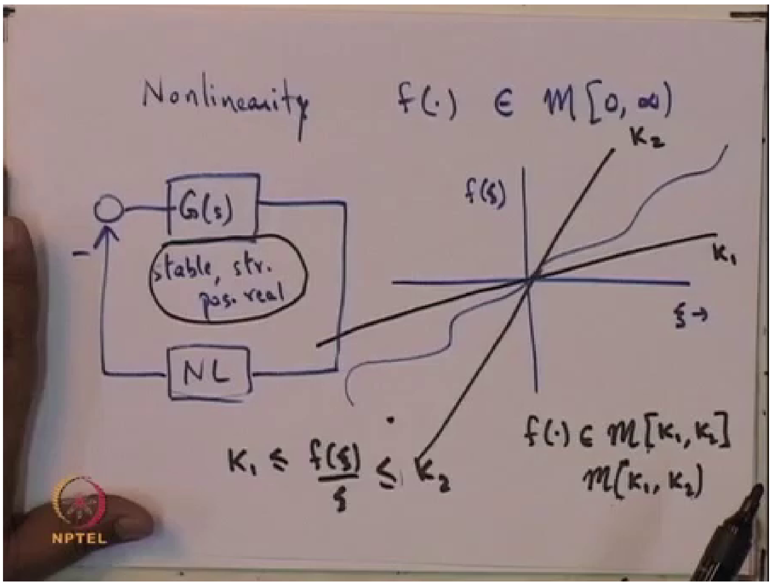

So, now in order to do this what we would do is we will do some characterization of the various kinds of nonlinearities. So, let me begin by a looking at certain classes of these Nonlinearities and I will show that these nonlinearities can be transformed into a non linearity in the 0 infinity sector ok. Yeah let me, make this clear by some examples ok.

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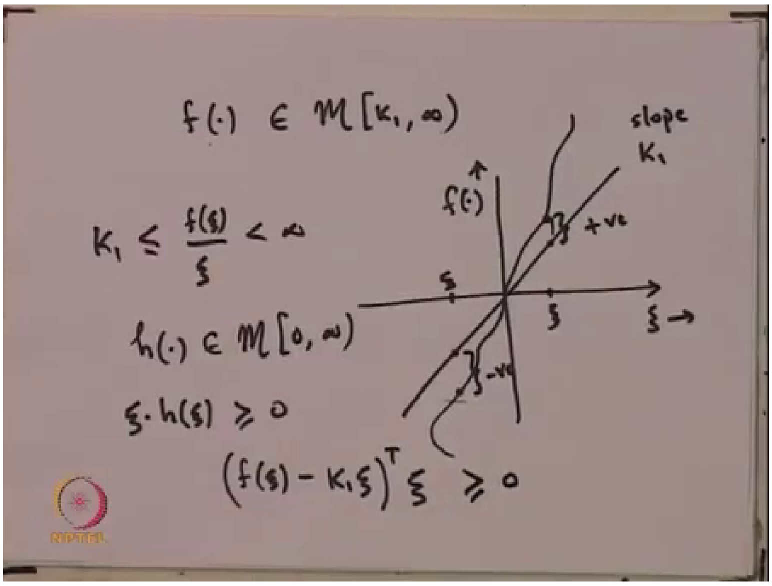

So, suppose you have a non-linearity belonging to the K K 1 infinity sector ok. What do we mean by saying that a non-linearity belongs to the K 1 infinity sector? Well, what we mean is ok. So, here is this oh my god ok, here is this line K 1, let me use a new sheet rather than ok. So, we want to talk about this f which is in the K 1 infinity sector.

So, what we mean by that is. So, this is a line with slope K 1 and if the non-linearity is in the K 1 infinity sector; that means, it lies here and here so it could be something like that yeah. In other words the non-linearity is such that f xi by xi, xi is the input of the non-linearity, and f xi is the output. This is less than infinity, but greater than equal to K 1 ok.

Now, if you have a non-linearity like this, we can convert this into a non-linearity in the 0 infinity sector ok. Now, how does one convert this into a non-linearity in the 0 infinity sector?

Well, the if you have; if you have a non-linearity in the 0 infinity sector, then the following is true xi times h of xi is greater than equal to 0.

Now, for a non-linearity which lies in this sector this is true, but this is true is the same as saying f of xi minus K 1 of xi ok. So, for example, if I look at this particular xi, so this is a f of xi and so f xi minus K 1 xi is this portion here which is positive. On the other hand if I take a xi which is negative, then f of xi is here and this is K 1 xi and therefore, this quantity here is in fact negative, this quantity here is negative.

So, if now this thing, let me put a transpose is multiplied to xi, then here you see you get a positive quantity multiplying positive, so the resulting thing is positive. In the left half you have a negative thing multiplying negative, so the resulting thing is positive. So, for any non-linearity which lies in the K 1 infinity sector we can say that this condition is always satisfied.

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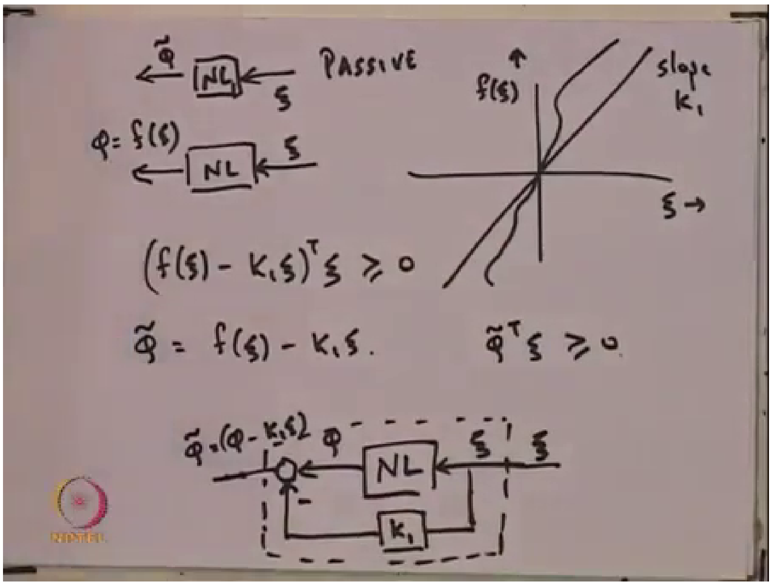

Now, if this condition is satisfied then this is what we do. So, this is slope K 1 and you have a non-linearity like that. And what we had just written was, so the input of the non-linearity is xi and f of xi is the output of the non-linearity. So, if you think of this non-linearity like this. And let me call the input xi and the output is f of xi, then one way we could characterize such a non-linearity is by this quadratic form which is f xi minus K 1 xi transpose xi is greater than equal to 0 ok.

Therefore, now if you think of ok. So, this is the output phi, if you think of a non-linearity some new non-linearity, which has the same xi as the input, but it has phi tilde as the output where, this phi tilde is really f of xi minus K 1 xi. Then this non-linearity this new non-linearity will have the property that phi tilde transpose xi is greater than equal to 0 and therefore, this new non-linearity ah let me call it NL 1 is passive.

Now, how does one obtain NL 1 from NL? Well, one way to obtain NL 1 from NL is the following. So, you have the original non-linearity and it has the input xi and if it has the input xi it will give the output which let me now call it phi. But you want a new non-linearity which does not give phi as the output, but gives phi tilde as the output.

So, how to get phi tilde as the output, you want to use the same xi as the input. Well if I take a K 1 here, so K 1 times xi is what I will get. And I use a feed forward, then what I will have here is phi tilde which is phi minus K 1 xi. And so now if I think of what is in this dotted box as my new non-linearity, it has as its input xi and it has as its output phi tilde and this phi tilde transpose xi is greater than equal to 0. So, this resulting non-linearity is in fact passive.

Now, therefore, this resulting non-linearity if it is now connected in a feedback loop with a plant which is strictly positive real, then the resulting system is asymptotically stable. So, let me draw that.

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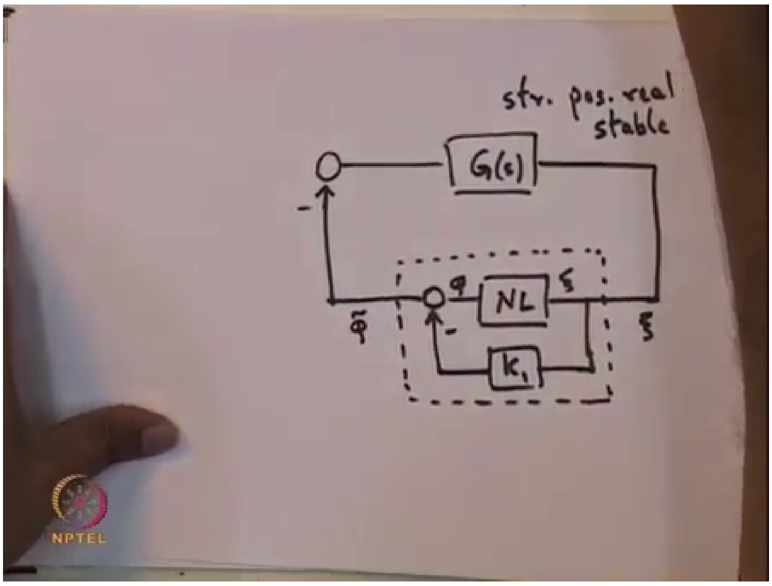

So, you have this non-linearity. So, you have xi and you have K 1 of xi fed forward, so it is still xi here.

So, the original non-linearity had this output phi, but this new non-linearity has the output phi tilde. And now if this phi tilde is connected, to some G s, where this G s is strictly positive real and stable then the resulting system is of course, you I mean using our earlier results whatever we had concluded earlier using the positive real lemma and so on.

We can conclude that this resulting closed loop system is asymptotically stable, but now we are interested in knowing not what can be connected to this new non-linearity, but what can just be connected to this non-linearity, what plants can be connected to this non-linearity and the resulting system would be asymptotically stable.

So, what I am now going to do is, I am going to do a set of transformations and these transformations go under the name of loop transformations. And by loop transformations we can actually conclude or we can find a much larger class of plants which you can connect to the non-linearity and the resulting system would be asymptotically stable.