Nonlinear System Analysis
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Lecture - 48
Loop Transformation (Part 1)

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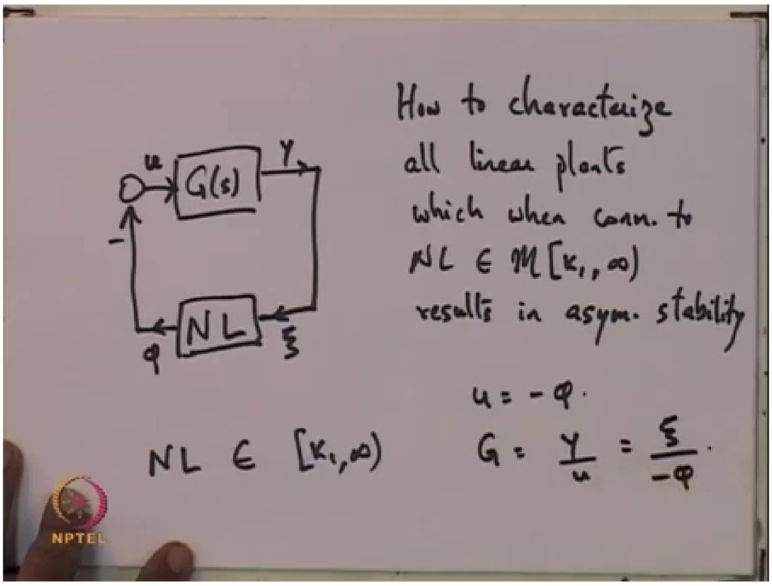

So, what we are going to do is. So, here is a linear plant and here is a non-linearity and we are assuming that this non-linearity is in the k 1 infinity sector ok. And we would like to know that when you do this feedback connection what are the linear plants, ok. So, the question we would ask is the following. How to characterize all linear plants, which when connected to non-linearity in the k 1 infinity sector results in asymptotic stability?

So, now let me call the input to the linear plant u and the output y. And let me call the input to the non-linearity psi and the output phi ok. Then of course, by the feedback connection then you know u is equal to minus phi. And you also know that the transfer function G is really y by u, but that is the same as psi by minus phi ok.

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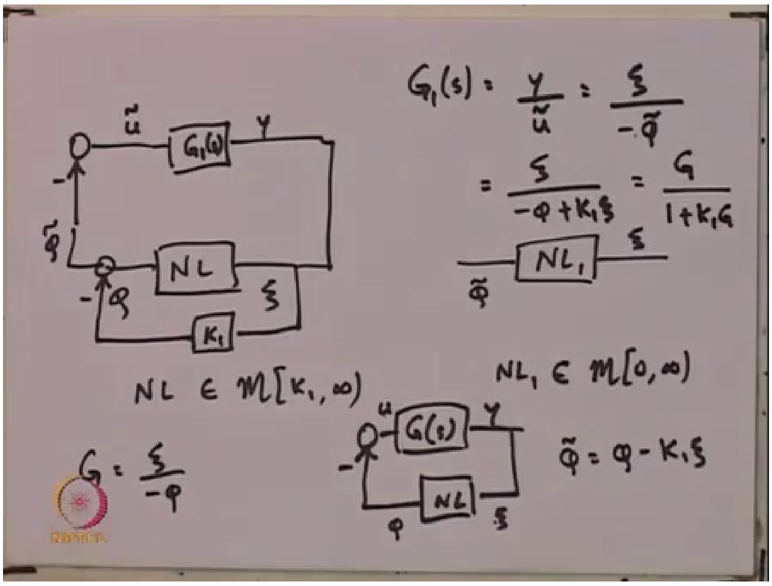

Now, so, earlier we saw that given this non-linearity with input psi output phi such that this non-linearity belongs to the k 1 infinity sector, you can get a new non-linearity. So, let me call it NL 1 such that it has the same input, but now the output is phi tilde such that this NL 1 is really it belongs to the to the 0 infinity sector.

Now, how does one get this NL 1 from this NL we just saw earlier that one way to get this NL 1 from this NL is by using this feed forward where you take this gain k 1 and feed it there. And therefore, now what you have will be phi tilde. Now, you originally had a G of s here which you had interconnected with this NL ok. So, what I am trying to say is that you had this

G s interconnected to the non-linearity NL ok. And we call this u, we call this y, we call the psi, we call this phi ok.

Now, one could modify this transfer function G of s into some new transfer function G 1 of s such that this G 1 of s will take this modified input, but give the same output as the original G of s. So, what we are trying to say is that we modified this G of s in some way such that it takes this u tilde as the input rather than the earlier u as the output, but gives the same output as G of s would give for you. Then we can calculate what does G 1 of s would be. Because the G 1 of s G 1 the transfer function G 1 of s is given by y divided by u tilde. But, this is the same as saying psi divided by because y is equal to psi and u tilde is equal to phi tilde minus phi tilde.

But this minus phi tilde we know is the same as psi this minus phi tilde is the same as minus phi. So, we had taken this was phi tilde to be we had taken this phi tilde to be phi minus k 1 psi. So, I am just substituting for phi tilde. So, I have minus phi plus k 1 psi. Now, if you remember in the last slide I had written that the original transfer function G is really psi by minus phi. So, dividing by minus phi in the numerator and the denominator we get this G 1 is really the original G upon 1 plus k 1 G.

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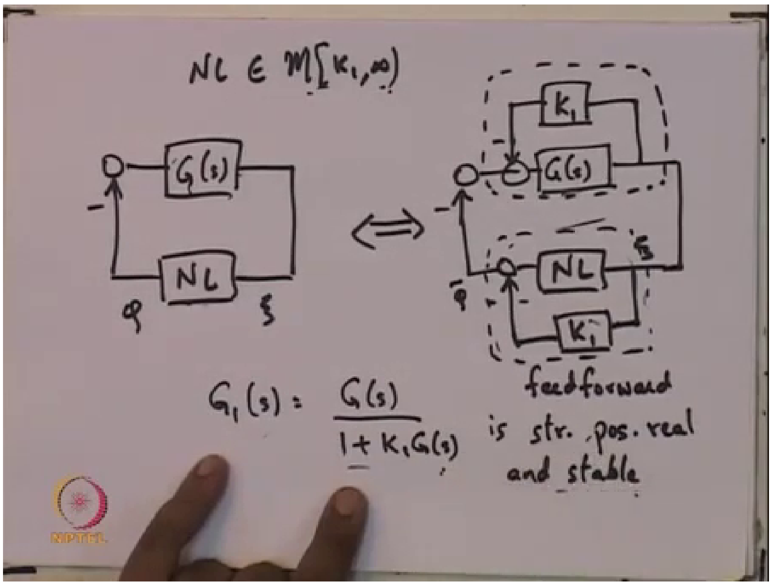

So, what we are trying to say is the following ok. So, what we are trying to say is the following.

So, if you had a non-linearity, psi phi and you had this G of s ok. Then this interconnection is exactly the same as the following interconnection, where you have this non-linearity the same non-linearity at the before you keep the input to the non-linearity exactly the same, but for the output what you do is you put a k 1 here ok. So, you fed forward this k 1 psi and so, now, you have this new input phi tilde. Now this new input phi tilde you are feeding back. Then the resulting transfer function that you would have here which one called G 1 of s as we just derived in the last slide would be G of s upon 1 plus k 1 G of s.

But that is the same as saying you have G of s. And then you have 1 plus k 1 G of s what; that means, is you have this feedback structure G of s with fed back with k 1 s and this resulting system. So, so when you put G s with this non-linearity and this non-linearity this non-linearity, is in the k 1 infinity sector and you have this non-linearity.

Now you can convert this non-linearity into something in the in passive; that means, you can convert it into something in the 0 infinity sector by doing this feed forward. So, you have done a feed forward and what have you done you have forward at the input to the output by this loop.

So, along with the non-linearity you have given a feed forward. But when you do this feed forward on this loop, on the open loop what you do is, you could give a feedback with the same k 1 on the original plant that you had and then this resulting plant along with this new non-linearity. So, the new non-linearity is what is inside this dotted box that I am putting, along with this new linear plant which is what is there in the dotted box there. These two this combination is exactly the same as this combination.

And now, if this combination is exactly the same, now by using this feed forward what we had done is this non-linearity in the k 1 infinity sector you had converted it into a non-linearity in the 0 infinity sector. Therefore, you know that for anything in the 0 infinity sector making use of the positive real lemma and the results about passive systems and so on.

This resulting system here if this is strictly positive real and stable, then that along with this this resulting system is asymptotically stable but that is since this system is equivalent to the system that is the same as saying that this system is asymptotically stable.

Therefore for a non-linearity in the k 1 infinity sector, given a G s, that G s with that non-linearity in a closed loop system, this would result in asymptotic stability. If when you modify G to this thing; that means, when you modify G to G 1 where G 1 looks like this G s upon one plus k 1 G s. And if this is strictly positive real and stable, then the original system is going to be asymptotically stable. So, I hope this is clear. So, what you are really doing is, you are doing some sort of a loop transformation yeah. So, what you did is on the on the non-linearity you gave a feed forward loop. And as a result on the linear plant you get a give a feed feedback loop both these loops have the same gain.

So, there is a feed forward loop with negative gain. Here you have a feedback loop with a negative feedback. Now as a result this modified linear plant along with this modified non-linearity they both together. Now, you invoke the positive real lemma or the Kalman Yakubovich lemma, whichever one is applicable. And then you can talk about the stability of that resulting system and this resulting system is exactly equivalent to this system. So, therefore, you can say something about all the transfer functions, which when interconnected to a non-linearity in this particular sector will give asymptotic stability ok.

So, what is the result? The result is that if you take any plant and you do this transformation of the plant, this transformation of the plant should result in this new plant G 1 of s being strictly positive real and stable. And if that is true then the original plant G of s with this non-linearity in the k 1 infinity sector is going to give you stability ok.

Now, that was one special case, where you took a non-linearity in the k 1 infinity sector.

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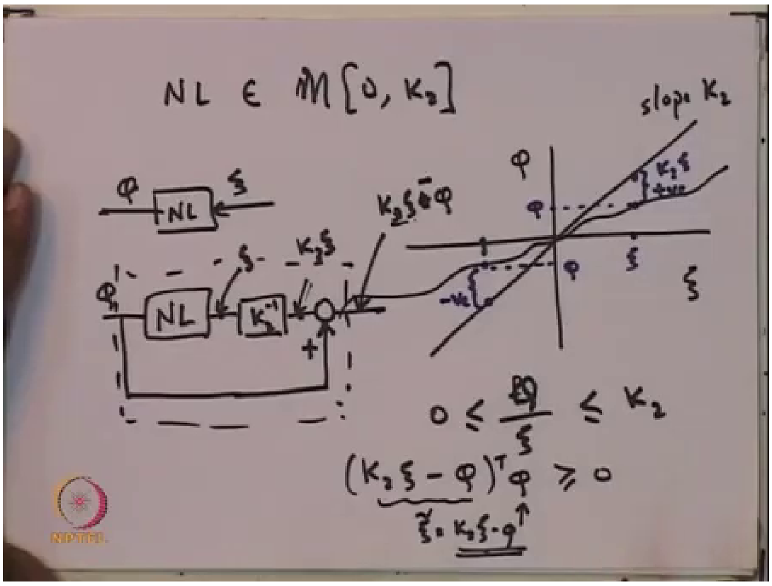

Now, one could also think of taking a non-linearity in the 0 k 2 sector. So, if you take a non-linearity in the 0 k 2 sector. So, what will its characteristics look like? So, if I call this psi the input and this is the output phi and I draw this line which has slope k 2. Since the non-linearity lies in the 0 k 2 sector; that means, non-linearity is something like this ok. Or in other words phi by psi it is less than equal to 0 zero is less than equal to phi by psi, which is less than equal to k 2 ok.

Now, if you have if you have non-linearity of this kind then this particular inequality I could rewrite this in the following way. You see if I take k 2 psi minus phi. So, so suppose I take some suppose I take some point psi. So, this here this point here is k 2 psi and the corresponding phi is this. So, this quantity here is positive and for this particular case psi was positive yeah. Now, similarly if I take psi to be negative I get k 2 psi to be here and I get phi to be here. So, k 2 psi minus phi this is negative when psi is negative. But more importantly k 2 psi minus phi this quantity here is positive.

So, I could write the following. You see, when psi is positive phi the corresponding phi is also positive. And here when psi is negative, the corresponding phi is also negative. So, I could write the following down k 2 psi minus phi transpose multiplying phi the output. When this is positive this is positive and when this is negative this quantity is negative this quantity is negative. So, this product is greater than equal to 0. And so, what I have really done is I have looked at this non-linearity and I have looked at certain quadratic inequality, which gets satisfied by any non-linearity which lies in that sector.

And now what I am going to do is, I am going to use this non-linear this particular quadratic relation to modify my original non-linearity in such a way that now it becomes a non-linearity in the 0 infinity sector ok. So, how do I do that? I do something very similar to what I did the last time. So, here is a non-linearity here let me assume is the input and here is the output phi.

Now what I am going to do, the last time what we did was we kept the input the same and we modified the output. This time round what I am going to do is, I am going to keep the output the same, but I am going to modify the input. So, the new input psi tilde is going to be equal to k 2 psi minus phi ok.

How do I do this? Well, what I could do is, I could take this non-linearity the output is still going to be the same phi, but the input the new input is going to be this psi tilde, but let me do the following. Let me put gain here which is k 2 inverse. So, now, if I give k 2 psi as the input here then because of this k 2 inverse I will get psi here and then I have the original non-linearity. So, what I am trying to do is, I should get psi here such that I have the original non-linearity. So, the original non-linearity I am multiplying before you reach the non-linearity I am multiplying the input by k 2 inverse.

And so, now, I would use this output yeah and I feed this back right. So, now, if I am to gets psi here what I should get here is k 2 times psi. And so, what I should get here, should be k 2 psi plus phi. So, if out here sorry. So, I want k 2 psi minus phi here because that is what I want here. And so, what I will do is I will feed this back that is a positive feedback.

So, if I do k 2 psi minus phi here, if I have k 2 psi minus phi here and I am feeding back this phi this phi will cancel out that phi and I will have k 2 psi here and then k 2 inverse will give me psi here. So, I have the original non-linearity modified in this particular way should give me a non-linearity like this and this non-linearity. So, now, if I box this off this new non-linearity has the same output as the original non-linearity, but as the input it has k 2 psi minus phi. So, it looks like I am out of time. So, we will continue whatever we are doing in the next lecture.