Nonlinear System Analysis
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Lecture – 50
Circle criterion for PR

So, this transformation that we have, this goes under I mean this is a theorem on its own and it goes under the name of the Circle Criterion ok.

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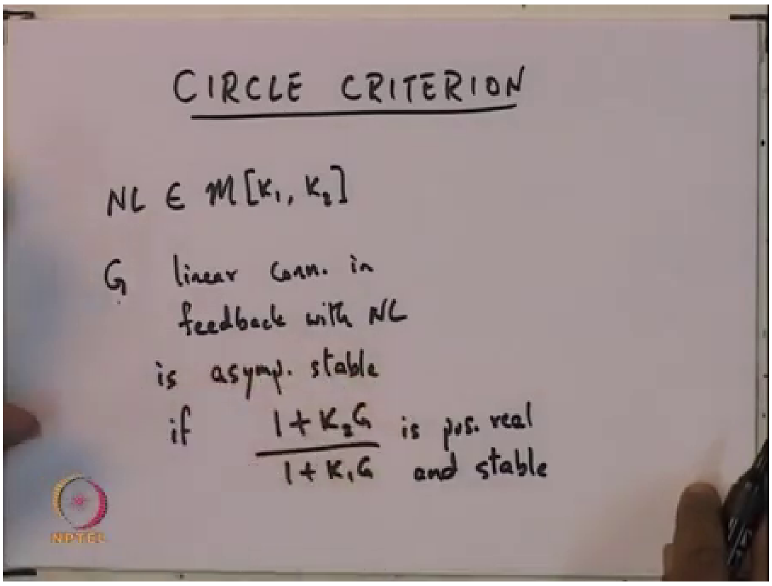

So, let us find out what the circle criterion is all about ok. So, what we had said that if you have a non-linearity in the K 1, K 2 sector and you have a linear plant connected to the non-linearity so, linear plant G linear connected in feedback with non-linearity NL is asymptotically stable if K.

So, this is what we had shown the last time 1 plus K 2 G upon 1 plus K 1 G is positive real and stable ok. So, given a G what one could do is, one could calculate this 1 plus K 2 G upon 1 plus K 1 G and then check whether this transfer function is positive real. But you know checking for positive realness, one way to check for positive realness is by using the Nyquist criterion.

Now, is there a way to check whether 1 plus K 2 G upon 1 plus K 1 G is positive real? But we still want to use the Nyquist plot of the original G. Now, it turns out that this is possible and this way of predicting whether 1 plus K 2 G upon 1 plus K 1 G is positive real and stable by using the Nyquist plot of G, this is what circle criterion is.

So, we will look at this transfer function and we will use the Nyquist plot of G and try to say whether this transfer function is positive real and stable ok. So, this is what we do. So, for that let us look at the complex plane ok.

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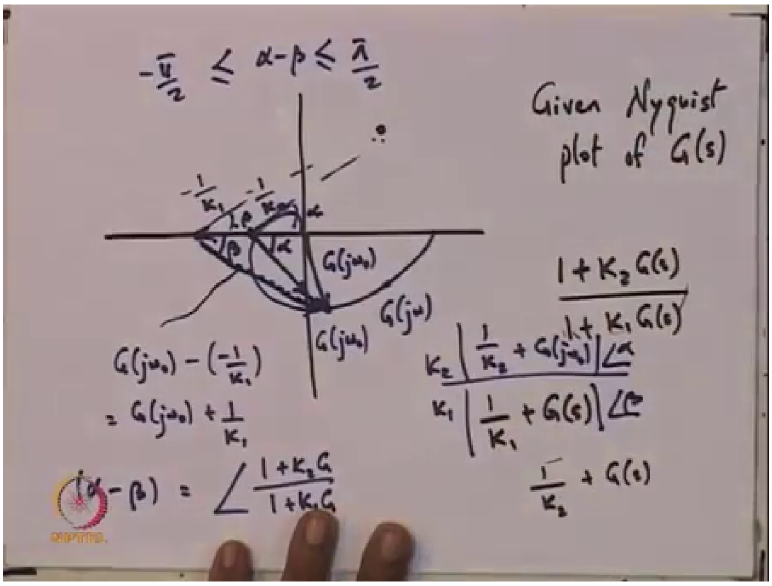

So, give we are given the Nyquist plot of G of s. So, the Nyquist plot of G of s essentially means that so, maybe we have something like that. So, this is G j omega. So, at various omegas we have evaluated G s and we plotted that that is the Nyquist plot. So, let us take some particular point here.

So, this is let us say G at j omega naught ok. Now, we are interested in finding something out about this transfer function which is 1 upon K 2 G s upon 1 plus K 1 G s. So, let us just look at the denominator. So, this denominator I could pull K 1 out and this is the same as 1 upon K 1 plus G s.

Now, if I am going to evaluate this at omega naught, well this vector here is G j omega naught and if I if this point here is minus 1 by K 1, then this vector is minus 1 by K 1. And therefore, this vector here is this vector here is G j omega naught minus minus 1 upon K 1 which means this is really G of j omega naught plus 1 by K 1. So, whatever is in the denominator is obtained by looking at this particular vector.

Now, in the same way, one could also look at the numerator and for the numerator if one pulls out K 2, you have 1 by K 2 plus G s and so, G j omega naught is the same and 1 by K 2 will be again a point here; minus 1 by K 2 will be a point here and the negative axis because we are assuming this K 1 and K 2 are both positive. And of course, K 2 was a larger number than K 1 and so, 1 by K 2 would be a smaller thing. And so, let us say now sorry K 2 is larger. So, 1 by K 2 is going to be smaller. So, this is minus 1 by K 2 and so, you will have a similar vector here ok.

So, now if we wanted to evaluate this at j omega naught, what we are really evaluating is this vector; the magnitude of this vector in the denominator and the magnitude of this other vector 1 upon K 2 with plus G j omega naught in the numerator. And of course, because I pulled out this K 1 and K 2, this will be K 2 by K 1. So, the magnitude of this; magnitude of this and this, but what we wanted to know was this transfer function is positive real. But what would; that would mean is that the resulting Nyquist plot should have an angle which lies between plus 90 degrees and minus 90 degrees.

But this angle of this transfer function is essentially the angle of this which let me call it alpha and the angle of this let me call it beta. So, this angle is alpha; so this is alpha here and this angle is beta. And so, for this transfer function 1 upon K 2 G upon 1 upon K 1 G. The angle of this particular transfer function is really equal to alpha minus beta. Now, if this transfer function were to stand for something which is positive real, then this angle alpha minus beta must be less than equal to ok.

So, the magnitudes will give us a magnitude and this angle must lie between plus 90 and minus 90 or plus pi by 2 and minus pi by 2 if you thinking of this in radians. So, this must be less than equal to pi by 2 and greater than equal to minus pi by 2. Yeah so, now, what; that means, is if you take any point then if that point is to be a point on the on the Nyquist plot of

G s, then to know the angle corresponding to that point; we draw these lines from minus 1 by K 2 and minus 1 by K 1 and look at the angles.

So, you have alpha and beta and if the difference between these two angles alpha minus beta lies in this range. Then when you do the transformation, then the resulting point is going to lie in the right half plane. So, everything essentially depends on these two points minus 1 by K 2 and minus 1 by K 1 and so, let us now look at how those two points get related.

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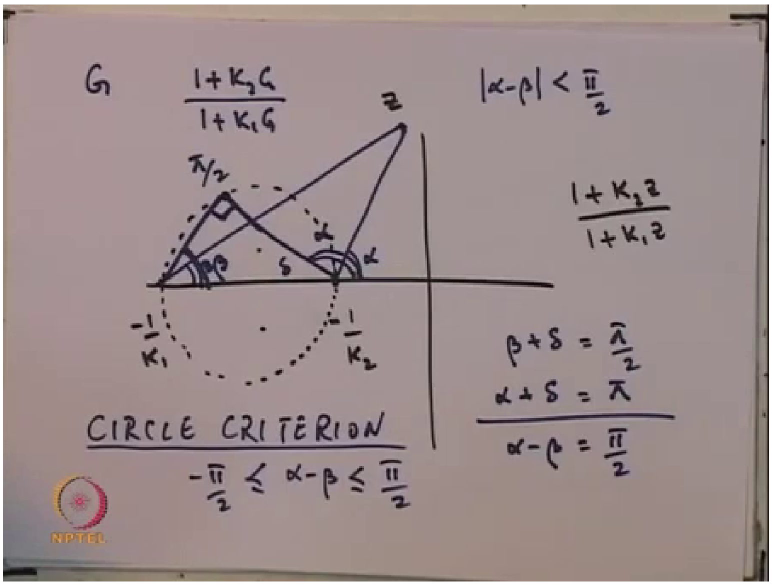

So, let this point be minus 1 by K 2 and let this point be minus 1 by K 1 ok. So, if we are interested in let us say some point here, then what one does is we look at this vector and we look at this vector. Look at this angle alpha look at this angle beta and we are saying that alpha minus beta should be less than equal to pi by 2 and should be greater than equal to minus pi by 2 and this guarantees if alpha minus beta is in this range, then it guarantees that this point z.

So, suppose I call this point z, then it guarantees that 1 plus K 2 z upon 1 plus K 1 z, the point to which z will map under this bilinear transformation lies in the right half plane ok. So, all the points z such that alpha minus beta is in this range are permissible points where the Nyquist plot of the original plant G s could exist. But now how do we find all those points where alpha minus beta satisfies this inequality.

Now, if you recall high school geometry then, you might remember that if you have a circle ok; this might not really look like a circle, but let us assume this is a circle. There is a circle whose diameter is this distance between minus 1 by K 1 and minus 1 by K 2. And if you take any point and if you take any point on the circle and you look at these two lines ok, then in high school you would have learned that the angle subtended by these 2 this angle here is pi by 2 ok.

Now, if this angle is pi by 2, then what can we say about this particular angle alpha minus this particular angle beta? Well we know beta plus this angle; if I call this angle delta. We know beta plus delta is equal to pi by 2, but we also know alpha plus delta is equal to pi.

So, if you subtract the second one from the first one, you get alpha minus beta is precisely equal to pi by 2. So, this is something that we would have learnt in our high school geometry that if you draw the circle then any point on the circle if you subtend, it subtends an angle 90 degrees. As a result this quantity alpha minus beta for any point on this circle is going to be precisely pi by 2. So, then it turns out that if you take any point outside this circle, then the angle will be less than or the modulus of the angle the modulus of the angle would be less than pi by 2.

And if you take any point inside the circle, then the angle that is going to get subtended, its modulus is going to be greater than pi by 2. So, this is really the circle criterion. So, what it says is that for so, given G if the Nyquist plot of G; so, given G. So, suppose you want to find something out about this transfer function 1 plus K 2 G upon 1 plus K 1 G given G, then from one from the information about K 1 and K 2, you can plot these two points minus K 1 by K 1 minus 1 by K 2 and you can look at the circle.

And if the Nyquist plot of G does not enter the circle, then the Nyquist plot of this transformed transfer function is going to lie completely in the right half plane and that is the circle criterion ok. Now, this sort of throws up a lot of very interesting things which one would like to talk about. So, I would talk about what are the various kinds of interpretation that you can get with the circle criterion in my next lecture.