Nonlinear System Analysis
Prof. Harish K Pillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 51
Examples based on circle criterion and stability under circle transformations

So, in the last class what we were talking about was the circle criterion. So, the general idea was that we know if you have a passive system and you interconnect it a passive linear system and you have a nonlinearity which is passive; that means, something which lies in the 0 infinity sector then when you interconnect the two you get asymptotic stability.

Now, one could think of a class of nonlinearities which is not really the 0 infinity sector but, some other let us say K 1, K 2 sector. So, in the last lecture we went through these various transformations that you can do and these transformations change the given nonlinearity which is in some K 1, K 2 sector into a non linearity in the 0 infinity sector.

Now, when you have a linear plant interconnected with this given nonlinearity and we want to talk about the stability of this closed loop system, we could change the nonlinearity from the given K 1, K 2 sector to the 0, infinity sector. But, you see that is on the feedback loop; so, then appropriate changes need to be done on the linear plant and when you do the appropriate changes on the linear plant, then the interconnection between this new linear plant and the new nonlinearity is completely equivalent to the interconnection between the old linear plant and the old nonlinearity.

The only difference in this whole process is that now the new nonlinearity is in the 0 infinity sector which means it is passive and because that is passive, the corresponding new linear system if that is passive then, we know that in this new system with the new linearity and the new nonlinearity that is asymptotically stable and because the two systems are equivalent therefore, the old linear system interconnected with the old nonlinearity stable ok. So, this was the essential idea that was used.

Now, transforming a given nonlinearity in the K 1, K 2 sector to a nonlinearity in the 0 infinity sector we can do that using these loop transformations and, but then, if one does not want to do this loop transformation, but you are given a linear plant you are given linear plant the old linear plant and the nonlinearity; then, you know by loop transformation the linear plant is converted to some new linear plant which must be passive; but, without doing this conversion of the linear plant if one can predict whether the new linear plant along with becomes passive or not by looking at the Niquist plot of the old linear plant then that there is some advantage in this.

And the circle criterion is one thing which let us do that ok. So, perhaps I will just repeat a bit about what we have already discussed earlier. So, perhaps the situation where you are looking at a nonlinearity in the K 1, K 2 sector and we will see what the linear plant changes to and so on ok.

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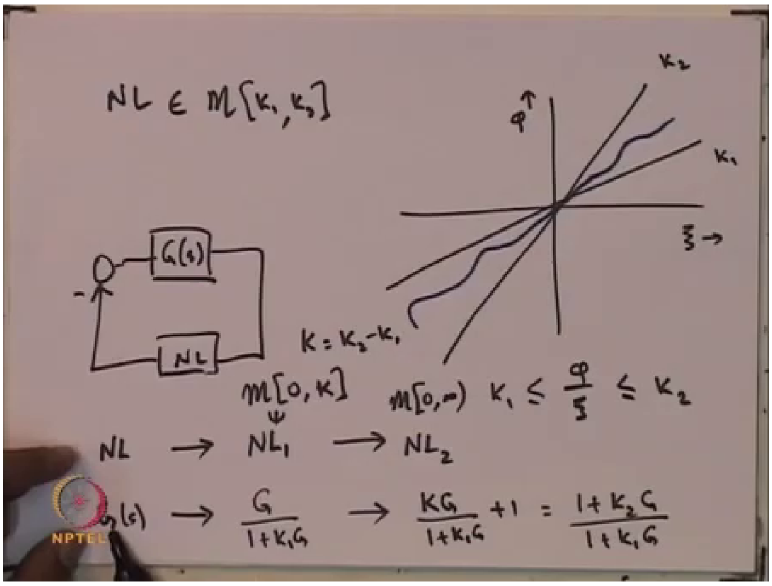

So, suppose we consider a non linearity; so, suppose we consider nonlinearity which is in the K 1, K 2 sector ok. So, what we mean by that is if you think of xi as we input to the nonlinearity and phi as the output to the nonlinearity well, there is this line with slope K 1; and there is this other line with slope K 2; and what we are saying is that the nonlinearity is such that it lies in the K 1, K 2 sector and of course, the other way to talk about this is that phi by xi is this thing is greater then K 1 and it is less than K 2. So, this is another way that you can rewrite this you know characterize this nonlinearity.

Now, if one is looking at a linear plant G of s and interconnected with this nonlinearity NL in this feedback form ok. So, suppose we have this particular situation then we want to talk about the asymptotic stability I mean under what conditions on G s I mean what should be the characteristics of G s such that when G s is interconnected with a nonlinearity in the K 1, K 2 sector the resulting system is asymptotically stable.

And then, what we had discussed in the last class is that this nonlinearity in the NL I mean this nonlinearity NL this can be converted into nonlinearity now so, we can go through it in two steps; so, first you have this NL and you first convert it into let me call it NL 1 which is something in the so, NL 1 belongs to so, this is a nonlinearity in the 0, K sector where this K is K 2 minus K 1 ok.

So, you can do one transformation like this and then this can be followed by another transformation the second transformation is when you convert something in the 0, K sector to this second nonlinearity which is a passive nonlinearity; that means, in the 0, infinity sector ok.

Now, when one does this then the linear plant which we had here G or G of s that also gets transformed in a certain way and we have talked about it earlier. So, the way it gets transformed is when you take the nonlinearity NL to NL 1 then the linearity gets transformed to G upon 1 plus K 1 G ok.

So, this becomes the new linear plant with this nonlinearity. So, the interconnection of these two is equivalent to the original intersection that we were interested in and then this conversion from the 0, K sector to the 0, infinity sector makes a conversion here which makes this KG upon 1 plus K 1 G plus 1; but then, we saw that this is equivalent to 1 plus K 2 G upon 1 plus K 1 G ok.

So, now this nonlinearity with this plant which is 1 plus K 2 G upon 1 plus K 1 G this interconnection feedback, interconnection between this linear plant and this nonlinearity is exactly the same as the interconnection between this original plant and this nonlinearity. And because this nonlinearity is in the 0, infinity sector, we now can use the passivity theorem; and so, if this resulting plant from G given G and use K 1 and K 2 and make this new plant and if this new plant is passive and stable or in other words the Niquist plot of this new thing lies in the right half plane and it is stable. Then, this interconnection is asymptotically stable and that translates to this original interconnection being asymptotically stable ok.

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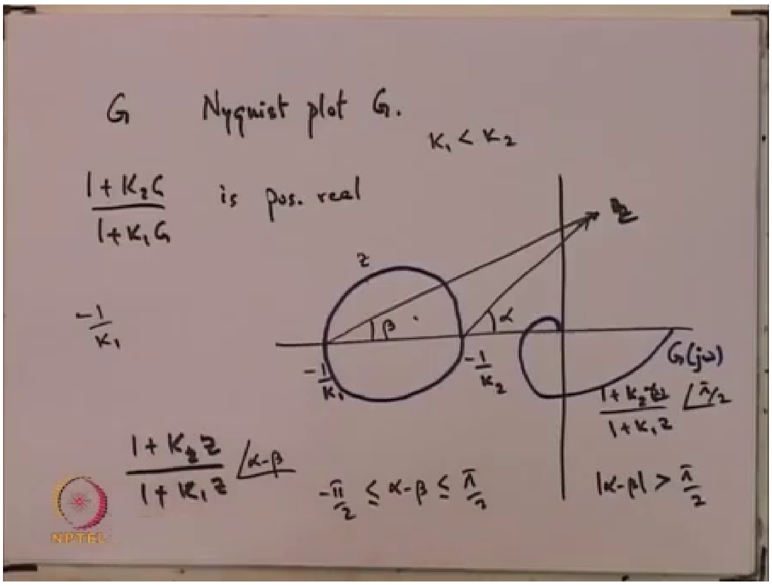

But, one would like to check that so, what do you want to check is the following; so, given G and therefore the Niquist plot of G we want to check whether this given 1 plus K 2 G upon 1 plus K 1 G whether this is positive real ok.

And then, in the last class I sort of demonstrated how we do this checking; so, one thing you do is if you look at the denominator this gives I mean this gives that the pole of the system means like when G is equal to minus 1 by K 1. So, of course K 1 of course, K 1 is less than K 2 and since K 1 is less than K 2. So, therefore, minus 1 by K 1 let us say is this point; and minus 1 by K 2 is this point; and when you trying to evaluate this transfer function given G j omega.

So, we said that suppose, you have any point Z here; and let me call it Z Z and you want to evaluate 1 plus K 1 or the K 2 Z upon 1 minus 1 plus K 1 Z; then the angle of this is essentially you draw these vectors I want here and here to here and look at these angles alpha and beta and the angle of this transfer function is going to be alpha minus beta.

Now, asking for this transfer functions Niquist plot to lie in the right half plane is the same as asking for alpha minus beta to be in this range between minus pi by 2 and pi by 2 ok.

And then, we made the main statement of the circle criterion which was that you look at this circle ok; now, inside the circle if you have any point Z I mean along the boundary of the circle if you take any point Z, then 1 plus K 2 Z upon 1 plus K 1 Z this has an angle which is precisely pi by 2 pi by 2 if it is up there and minus pi by 2 if it is in the lower semi circle; if it is in the upper semi circle it is plus pi by 2; in the lower semi circle minus pi by 2.

If the point Z is outside then this alpha minus beta satisfies this condition and if the point Z is inside the circle then alpha minus beta in fact, turns out to be I mean the modulus value of alpha minus beta turns out to be larger than pi by 2 and this is where we stopped the last time ok.

So, what does this mean? This means, so, suppose you have given this G and you are given this Niquist plot of G and the Niquist plot of G lies completely outside this circle ok. So, maybe this is the Niquist plot of this circle this is a Niquist plot of the plants so, G j omega.

Now, this lies completely outside the circle. So, for every point along the Niquist plot because of the argument that we had given earlier alpha minus beta this angle is going to be between pi by 2 and minus pi by 2. So, what it means is for this particular plant G if you calculate 1 plus K 2 G upon 1 plus K 1 G and plot the Niquist plot of this new transfer function then that Niquist plot is lie going to lie completely in the right half plane.

But, in the discussion that we had earlier we had said that given a linear plant if the Niquist plot lies in the right half plane; that means, the real part of the of every point on the Niquist plot is positive that does not necessarily mean that the transfer function in the given transfer function is positive real. I had mentioned that if one also insists that not just the imaginary axis, but all of the right half plane maps into the right half plane, then that transfer function is certainly positive real.

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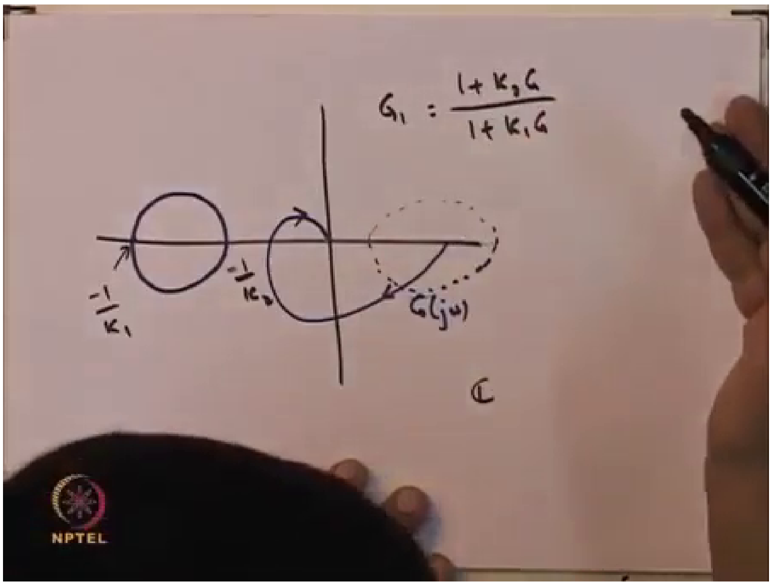

Now, how does one guarantee that given such a situation where so, what we were looking at earlier. So, let us say; let us say we have the circle here and let us say this is minus 1 by K 2 and this is this here is minus 1 by K 1 and you have a Niquist plot and let me so, let me think of the Niquist plot like that. So, G j omega of course, this G j omega does not enter into the circle and therefore, when you transform G j omega into 1 plus K 2 G upon 1 plus K 1 G so, let me call it G 1.

So, if you draw G 1 of j omega then that will lie completely in the right half of the complex plane ok; but when you do this mapping of G 1 into this maybe in this particular case, I would not know what the Niquist plot of G 1 looks like but, let us suppose that it looks let us say something like this ok. Now of course, this would be the other half of the Niquist plot.

Now, whether the right half plane under this map G 1 maps inside or outside how does one decide that because that will decide whether the resulting transfer function that you have got apart from being positive real it should also be stable; or apart from being I mean apart from being positive real, it should also be stable or another equivalent definition is that apart from the Niquist plot being on the right hand side all of the right half should map into the interior or rather into the right half plane.

So, how can we now check that with respect to this original the you know the Niquist plot of the original plant which is G j omega. Now it turns out that the way one does this is very similar to the to the Niquist plot criterion that one uses for linear plants ok. So, let me now try and motivate this interpretation ok.

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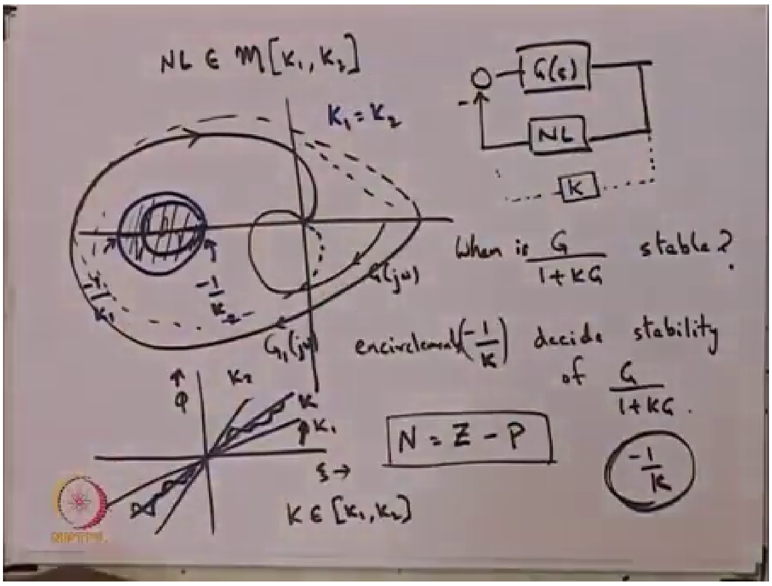

So, here we go let me draw that thing once more; so let us say this here is the Niquist plot that we have and let me suppose that this here is the circle that we had obtained earlier. So, this is minus 1 by K 2 and this is minus 1 by K 1.

Now, we are in this particular situation analysing this closed loop system which has the linear plant and with a nonlinearity in feedback loop and this nonlinearity this nonlinearity is a nonlinearity that lies in the K 1, K 2 sector ok. Now, what do we mean by this K 1, K 2 sector well, this is also clear this is a line with K 1; this is a line with K 2; this being the input to the nonlinearity, this being the output. So, what we mean when we say nonlinearity is lying in the K 1, K 2 sector is that the nonlinearity something like this.

Now, instead of thinking of a nonlinearity let us think of linearity I mean let us think of a linear element that lies in the sector. So, that says something like this. So, this has slope K where K is in the interval. So, K is in the interval from K 1 to K 2; so, the slope of this blue line here is K and so instead of the nonlinearity, let us assume the feedback instead of the nonlinearity is really a linear feedback with value K ok. So, let us assume that this is the portion which is connected and not the nonlinearity.

Now, if K is connected instead of the nonlinearity then the resulting transfer function is going to be G upon 1 plus KG ok. When can we say that this G plus 1 G upon 1 plus KG; when can we say when is G upon 1 plus KG is stable ok. The way we decide when this transfer function is stable is again by looking at the Niquist plot and what we should have is that the Niquist plot should not encircle the point minus 1 by K ok.

So, encirclements of minus 1 by K decide stability of G upon 1 plus KG and how is that done that is done by using the Niquist you know the Niquist criterion which is that suppose the original transfer function G was stable, then the number of encirclements of this point minus 1 by K must be 0; on the other hand, if G was unstable.

If you recall that there was this theorem if 0 if the number of zeros in the right half plane of the transfer function is given by Z; and the number of poles in the right half plane of G j omega was given by P; then we had something like N is equal to Z minus P; this kind of a formula in the Niquist criterion.

And what that translates to is depending upon the number of right half zeros or right half poles of G of s one can specify that this G of j omega should encircle this particular point minus 1 by K; the appropriate number of times in the clockwise or the anticlockwise direction depending upon you know whether the number of zeros is larger or the number of zeros the number of zeros in the right half plane is larger or the number of poles in the right half plane is larger ok.

So, if one makes the assumption that G is a stable plant for example, then in that case this G j omega should not encircle the point minus 1 by K and notice that this minus 1 by K is going to be some point here because the slope is between K 1 and K 2. So, now if you look at all these linear plants which can lie between K 1 and K 2 each time you will get some you know for stability.

So, suppose we start with G of s which is stable, then for the resulting closed loop system to be stable, you would say that this G of j omega should not intersect some point minus 1 by K and this point minus 1 by K will vary here between the point minus 1 by K 2 which is what you will have if you take the slope of the linear part to be K 2 or minus minus 1 by K 1 if you take the slope to be K 1.

So, as you vary this K you get all these points in the real part of inside this circle and it says that G j omega should not encircle any of them that is of course, if you start off with a G of s which is stable then this Niquist plot should not encircle ok.

On the other hand, if you start off with some G s which is not stable; that means, it has poles or zeros in the right half plane then what you would get is that each of the times it should encircle the point minus 1 by K the appropriate number of times in the clockwise or the anticlockwise. So, you could very well have a G of j omega which looks like that ok; and then for each one of these points this guy might result in so, let me call this G 1 j omega; and this is such that for each of these points minus one by K it encloses it an appropriate number of times.

So, if I also draw it is reflection I am sorry it should go something like that which means that any point here minus 1 by K gets enclosed once and twice in the clockwise direction and so, twice in the clockwise direction means the original transfer function suppose the original transfer function had had 2 poles in the right half plane then, if because you have this two encirclements therefore, the resulting transfer function is going to be stable ok.

Now, that was for the Niquist criterion told us. Now, that is when you have a linear feedback and for each one of these linear feedback what we are claiming is these points; but now, we do not have a linear feedback, but we have a nonlinearity and this nonlinearity you can think of as like think of a as like a linear feedback with a linearity lying in between these slopes K 1 and K 2 with some perturbation ok. So, you could think of this nonlinearity as something linear like this K but with some perturbations.

Now, one way to view the circle is that this perturbations from this K are captured here within the circle and so, the any nonlinearity in this K 1, K 2 sector can be thought of as a linearity with perturbations and that linearity with perturbations well for the linear parts you get this thing and the perturbation is the rest of the circle.

So, if you avoid any point in this rest of the circle, and you have a G of j omega which favoids that point, but then the original G of s suppose it is stable then in fact, this whole circle should not be encircled ok. But, if G of s had unstable zero unstable poles then this whole thing should be encircled the appropriate number of times.

Now, here is a very interesting way to think of it is; suppose, you think of this K 1 going up; that means, this interval is such that it lies between K 1 and K 2 and this K 1 is allowed to go up. Now, as K 1 is allowed to go up therefore, the value of K 1 changes and say therefore, this becomes so, K 2 is kept constant. So therefore, this becomes another circle a smaller circle and then as it is allowed to go up and up finally let us say this K 1 is made larger and larger until finally, K 1 is equal to K 2. Then, what would have happened is this circle would have shrunk until it becomes just this point minus 1 by K 2.

Now, if this interval is shrunk from K 1, K 2 to K 2, K 2; that means, K 1 has become K 2 then there cannot be a nonlinearity the only feedback that you have is in fact, the linear feedback with linearity being K 2. But, what that would have meant is that this circle has shrunk down to this to this one point minus 1 by K 2 and then by the Niquist criterion for linear plants, we know that the number of encirclements of that minus 1 by K 2 by this G j omega would depend upon the open loop G j omegas I mean the open loop plant G s whether it is stable or not.

If it is stable for example, then you should not have any encirclements of minus 1 by K 2. If it is not, if it is unstable, it has poles in the right half plane then there should be an appropriate number of encirclements of the point 1 minus 1 by K 2.

So, in some sense all those nonlinearity lying between K 1 and K 2 is captured by this the circle and that circle shrinks down to a point when you shrink this interval down to making it a linear gain. And conversely if you start from a linear gain and you expand it out, then as you expand it out, the uncertainty comes out in the form of this circle here; you know it expands out into that circle with the appropriate size and if the transfer function does the correct number of encirclements for that for that circle, then the resulting system is asymptotically stable.

So, in a sense it is the generalization of the Niquist criterion that one uses for the linear plants ok. So, now you could have the various circle criterion for various different nonlinearities and so let us now look at what happens as you change the nonlinearity or the sector in which the nonlinearity is present.

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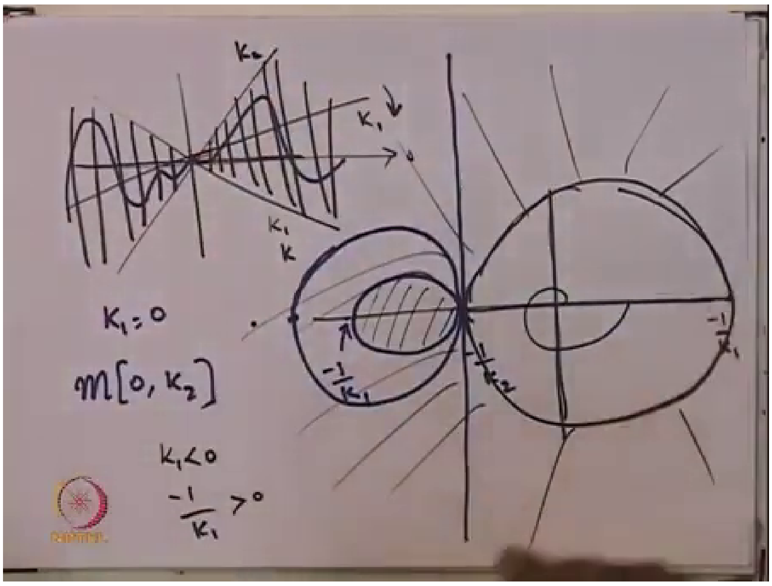

So, suppose you take this nonlinearity and let us suppose this is slope K 2; and this is slope K 1; now, as a result of this what you are going to get here the circle that you are going to get; however, circle you are going to get is something like that like so well that might not look like a circle, but let us just assume that this is the circle. So, this is minus 1 by K 2 and this is minus 1 by K 1 and it is in the K 1, K 2 sector and this is the circle criterion; that means, the Niquist plot should not enter the circle.

So, in some sense the forbidden region. So, the inside of this circle is the forbidden region. So, long as the Niquist plot lies outside you find the transform Niquist plot they will lie on the right half ok. Now, let us do one thing let this K 1 so, the nonlinearity is lying in the K 1, K 2 sector let us move this K 1 downwards; that means, the K 1, K 2 the lower limit is made even lower. So, this is 1 by K 1. So, as K 1 is made smaller minus 1 by K 1 becomes a larger thing and therefore, the resulting circle is larger.

So, as you are made it smaller so, suppose you made it is this small, then you will end up with a circle which is larger ok; so sorry this might not look like a circle, but you have to imagine this is a circle and as you keep; as you keep lowering K 1 further and further, this circle becomes larger and larger until you lowered this K 1 so much that K 1 became equal to 0; that means, now you are thinking of the nonlinearity in the sector 0, K 2.

Now, what is going to happen here this circle that point minus 1 by K 1 is becoming larger and larger until when K 1 becomes 0, this value of minus 1 up minus 1 upon K 1 becomes minus 1 upon 0 that is infinity. So, it is gone real far off and then, the circle criterion essentially tells you that particular circle is everything to the left which means the Niquist plot in this particular case if you are looking at K 1, 0 and K 2; that means, if you are looking at a nonlinearity like this then the Niquist plot should lie through right.

Now, instead of pulling this K 1 down if you keep the K 1 constant and you push K 2 up then, this was the original circle when you push K 2 up this gets extended until when you hit infinity it becomes a circle with minus 1 by K 1 here and 0 here so, some circle like that.

If you expand K 1, if you bring K 1 down to 0 and at the same time take K 2 up to infinity and make it a 0, infinity sector well, as you are taking K 2 up this point keeps expanding until you get a circle like that and as you keep expanding K 1 it goes off to infinity and so finally you have this imaginary axis and everything to the left of this is the forbidden area. So, your Niquist plot should lie completely in the right half plane which is what essentially the result about positive reality is all about.

So, this result is in fact, more general kind of a result of which the positive real condition is a special case. But now, interestingly we can do more things for example keep K 2 like this and K 1 could be extended to such an extent that K 1 in fact becomes negative. So, this is K 1. So, one is looking at a nonlinearity which can lie in this whole sector where the case of this is the zero slope so in this whole sector; so, you could think of a nonlinearity like this ok.

Now, what would happen in this particular case? So, if you go back here you keep in K 2 constant and K 1 you are extending and it keeps going until it reaches infinity and so therefore, you have this whole region is the forbidden region. After that when K 1 becomes negative 1 upon K 1 minus 1 upon K 1 is in fact a positive quantity which is you know close to plus infinity.

So, then what is going to happen is when you are looking at K 1, K 2 interval with K 1 being less than 0 therefore, minus 1 by K 1 is greater than 0 and so this minus 1 by K 1 is probably some point here minus 1 by K 1 and minus 1 by K 2 is here. So, you can take these two points and think off this circle here ok.

So, you get a circle like this, but there is a catch it turns out that now the back portion is the outside of the circle in other words, in other words, the Niquist plot should lie completely within this region and anywhere outside is the forbidden region. So, earlier we had the circle and the inside of the circle was forbidden and anywhere outside was allowed; but now, when this K 1 has become negative it turns out that you again get a circle, but it is the outside which is forbidden and the Niquist plot has to lie inside ok.

Now, now one way to now one way to think of this is in the following way; so, suppose you have the complex plane, so what you can do is you have this complex plane and all points which are the infinite points you can think of folding the complex plane up and all the points which are infinity think of them together as one point therefore, now this complex plane has become like a sphere ok.

Now, this circle that we drew on the complex plane if you now translate it on to the sphere, you end up getting a circle on the sphere, on the sphere on the surface of the sphere somewhere you have drawn this circle.

Now, if you draw a straight line on the complex plane then think about the straight line the straight line if you translate onto the sphere, you will mark all the points in the complex plane the corresponding points on the sphere, but you see all this infinity you collected up and you had the special point and so on. The if you are thinking about this sphere, think about the north pole of the sphere as the special point which is all the infinity is collected together.

So, when you are looking at a straight line this straight line goes to plus infinity and minus infinity which means when you translate it into a curve on the sphere, it will touch the north pole and so straight lines essentially translate into circles on the sphere which pass through the north pole ok. So, if the circles pass through the North Pole, now this is the good part you had you see in this diagram you had the circle and the circle kept expanding; that means, you have the circle in the left half plane which contained the forbidden region and it kept expanding.

Now, it kept expanding until it became a straight line that is when K 1 became zero slope; so, that gave you a circle which pass through the North Pole so, you see you had a small circle on the surface of the sphere and the circle kept expanding and the inside of this I mean on the sphere, you drawing the circle and the inside region of the circle is a bad region; the outside region of the circle is the good region. As far as the surface of the sphere is concerned and you translate that into paper this is what you get.

Now, as the circle keeps growing finally, when it becomes the straight line; that means, the slope K 1 is equal to 0, then the circle has grown in such a way that it now passes through the north pole. Now, when the slope is further reduced from K 1 equal to 0, then what happens is that this circle which pass through the North Pole has got larger and the infinite point is a forbidden point. Now on the sphere if you have a circle, that circle will translate either into a straight line if it passes through the North Pole or into a circle ok.

Now, the point of at infinity will correspond to the infinite region I mean the point you know the outer region when you translate it into the map, it translates to the outer region. So now, when the circle expanded so that it became larger, this point was the forbid I mean the infinite point the North Pole was a forbidden point. So, that is precisely what happens here when this keeps expanding out and comes to the other side, then the point at infinity is a forbidden point; so, the forbidden part is the outer part and this part is the nice part.

Now, if you keep this K 1 constant here and now start moving K 2. So, K 1 has a negative slope and now you start moving K 2; as you bring K 2 down what is going to happen is this nice region this K 2 down means this is going to go that way. So, this nice region is something which is going to keep expanding because this minus 1 by K 1 is constant and minus 1 by K 2 keeps going further and further so, it keeps expanding that way.

So, the nice region keeps expanding, but the outside region is the bad region until when K 2 hits 0. So, when K 2 hits 0, this minus 1 by K 2 has gone off to infinity which means you have a straight line here; and the good portion is this side; and the bad portion is on to the other side.

And then suppose K 2 becomes negative, then minus 1 by K 1 is here and minus 1 by K 2 would be further to the right and then you have would have a circle there and the interior of that circle has to be avoided whereas, the exterior is the good portion ok. So, maybe I would just draw a series of pictures with the circle and I will also draw which is the good region, which is the bad region.

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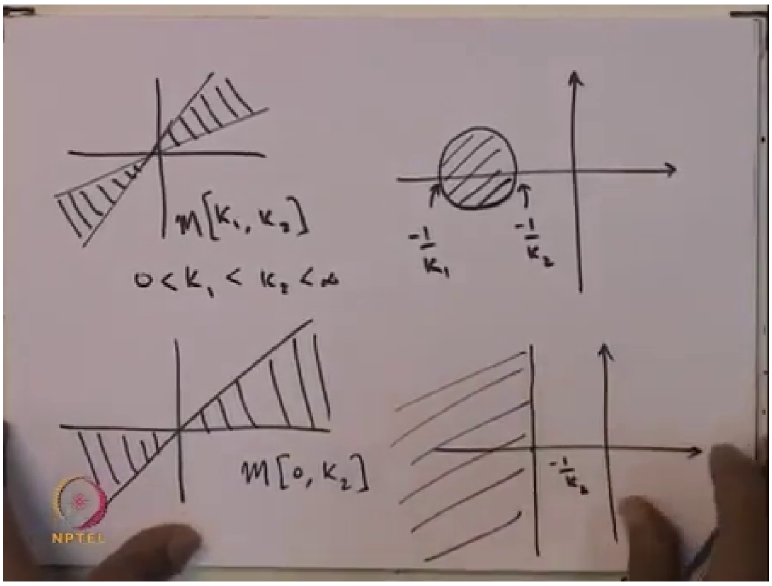

So, suppose you have a nonlinearity like this so, the nonlinearity is in this shaded region. So, the nonlinearity is between K 1, K 2; where K 1 is positive and less than K 2 less than infinity; what this translates to? Is a circle; this is the point minus 1 by K 2; this is the point minus 1 by K 1; and the forbidden region is the shaded part ok; this is what we first showed.

Now, if the nonlinearity is expanded such that the nonlinearity is in this sector like this ok. So, the nonlinearity now is in the 0, K 2 sector then what this translates to here. So, here you have this outside is the good part now, here what you will have is that minus 1 by that minus 1 by K 2 is still there ok.

Let me probably draw these arrows for the original axis. So, minus 1 by K 2, but minus 1 by 0 is infinity. So, what you have is this line like this and this thing that I am shading is the forbidden region; this is the nice region. So, if you had a nonlinearity like this, this circle is there and this circle is the forbidden region if you have the nonlinearity lying between 0, K 2, then it is this thing this whole half plane in some sense is the forbidden region and the G j omega has lie there I mean the Niquist plot has to lie to the right of this particular thing.

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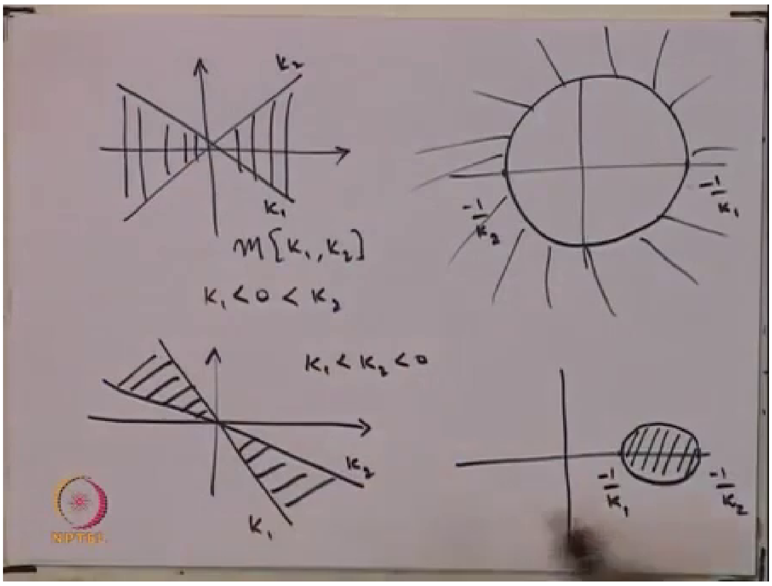

Now, further if you have a non linearity. So, let me draw the axis and you have a non linearity which lies in this area ok. So, the nonlinearity is in the K 1, K 2 sector. So, this is K 1; this is K 2; where K 1 is less than 0 is less than K 2 then what the circle criterion really tells us is because this is less K 1 is negative therefore, minus 1 by K 1 is appear and here somewhere is minus 1 by K 2 and so you will have a circle and the forbidden region is the outside of the circle. So unlike the earlier case, now the forbidden region is the outside of the circle ok.

And then, if you push further and you have a nonlinearity which lies in a sector like that so, this here is K 1; this here is K 2; so, what we have is K 1 is less than K 2 which in turn is less than 0. Now, in this case what you have is so, you will have minus 1 by K 1, and here you have minus 1 by K 2, and here you have a circle and now it turns out that the forbidden region is again the inside of the circle ok.

So, there are all these various interpretations that happen. So, one quick way to see it is if you have a nonlinearity in the range between K 1 and K 2 and therefore, you can plot the points minus 1 by K 1 and minus 1 by K 2 and you can draw the circle which connects these two now just like here or here or in the earlier two cases like here or when one of the slope was 0, so that is infinity; so, these two ok.

Now, if the sign of K 1 and K 2 are both the same like in this case or in this case then whatever is the circle you got the interior of that circle is the bad part, but if the sign of the two are different like in this case then it is the exterior that is the bad part.

So, this sort of sums up the various situations that can happen of course, K 2 could be made equal to 0 or K 1 could be made equal to 0 and you have these special cases K 2 could go off to infinity, which means that in the original axis it will hit 0 there are circle it will hit 0. And the special case of 0, infinity sector being the you know the left half is the forbidden part, and the right half is the nice part ok. So, that gives the complete sort of interpretation for the various aspects of the circle criterion.

Now, the other thing is that of course, we have been till now asking about global asymptotic stability, but sometimes using circle criterion we can talk about local stability and we cannot talk about the global stability ok. Now, what I mean by that is following.

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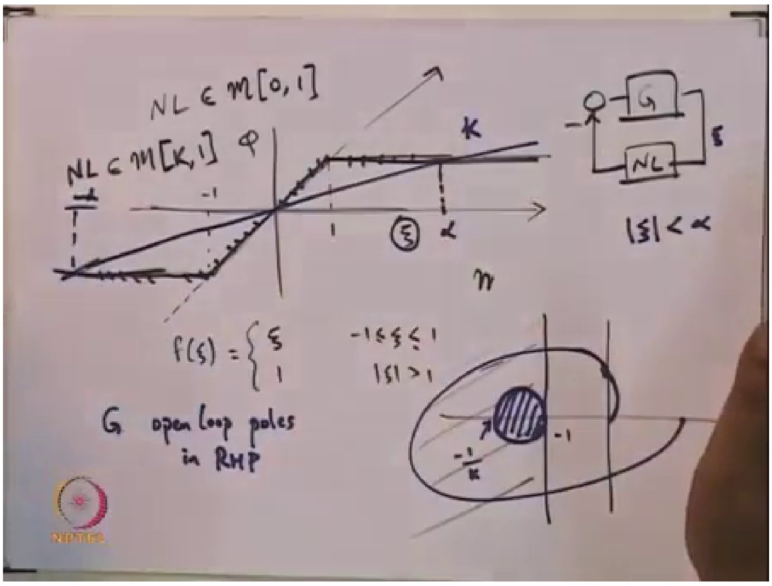

So, it might so happen that the nonlinearity has some characteristics which maybe look like this. So, let us say it has some characteristics like this ok. So, I mean one way that you could write out this characteristics is that the nonlinearity is such that f of xi that is a nonlinearity is equal to xi when let us say xi is between plus 1 and minus 1. So, minus 1 less xi less than plus plus 1 maybe I will make it equal and this is equal to 1; so, this is like a saturation; so, it is like linear and then it saturates is equal to 1 when model xi is greater than 1. So, you have some nonlinearity like this ok.

So, if you have a non linearity like this; now what is the sector under which this nonlinearity lies well, clearly this slope and the other slope being the zero slope; So, we could think of it as lying in that sector. So, you can think of this nonlinearity as a nonlinearity this nonlinearity as a nonlinearity that lies in the 0, 1 sector because this 1, 0 and of course, this nonlinearity lies completely within this particular sector ok.

So, now if you use this circle criterion to translate this; what that means, is so there is 0, and there is 1. So, what that means is if you looking at the Niquist plot. So, corresponding to 1 there is this minus 1 and so the everything to the left of this is forbidden and you can have a Niquist plot lying on the right and then such a plant so, such a G when interconnected with this nonlinearity will give us will give us global asymptotic stability ok. But now, the following could happen maybe you had a G and that G had Niquist plot which looked something like that ok.

And let us say that this G was such that it had open loop poles in the right half plane. Now, by circle criterion it might be that there is this circle here with inside being forbidden, and this Niquist plot is such that for this circle the resulting closed loops stability can be predicted. But now, if this is the case, so here this might be some minus 1 by K and this minus 1 by K might correspond to a slope like K here ok.

Now, if you look at the original nonlinearity the original nonlinearity, is this 1 and the original nonlinearity of course, lies in the 0, infinity sector; but, if you have to think off a nonlinearity in the K 1 sector. So, if you have to think of this original nonlinearity as lying in the K 1 sector then as far as this nonlinearity is concerned, it is only up to here up to this value let me call this value alpha and here it may be minus alpha.

So, it is between minus alpha and alpha; that means, for the input of the nonlinearity lying between minus alpha and alpha can this nonlinearity that we have originally drawn this nonlinearity here; we could think of this nonlinearity as lying in the K 1 sector if you restrict yourself to minus alpha and alpha. Now this xi that means input to the nonlinearity is essentially the value of the signal on this branch here.

And if this branch value is restricted to xi more xi less than alpha then the nonlinearity that we are considering this nonlinearity has characteristics which lie in the K 1 sector ok. And because it lies in the K 1 sector and the G j omega does not intersect this particular circle, we can say for that so long as this xi is restricted to something less than alpha this given system is asymptotically stable.

So, it is not globally asymptotically stable; but it is asymptotically stable so long as you look at only that portion of the phase space where the xi is less than or the model modulus value of xi is less than alpha and then for that restricted region of this phase space of course, this includes the situation when xi is equal to 0; and when xi is equal to 0, the output is also equal to 0; and that in fact, is the equilibrium point that we want to get things into.

So, you have the phase plane, you have the origin of this phase plane, and for xi less than alpha you have an area surrounding it and what we can say from this Niquist plot criterion is that so long as the xi is less than alpha, and you start somewhere for this system with this xi value being less than alpha you guaranteed the also and so, this is like local asymptotic stability as opposed to global asymptotic stability ok.

So, there are various things that you can do with the circle criterion depending upon half how the original Niquist plot looks like. So, anyway so, with that I guess and out of time for this lecture; so let me stop here now, and we will continue with the new topic in the next lecture.

Thank you.