**Image Signal Processing**
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**Lecture No 15**
**Rotational Homography**

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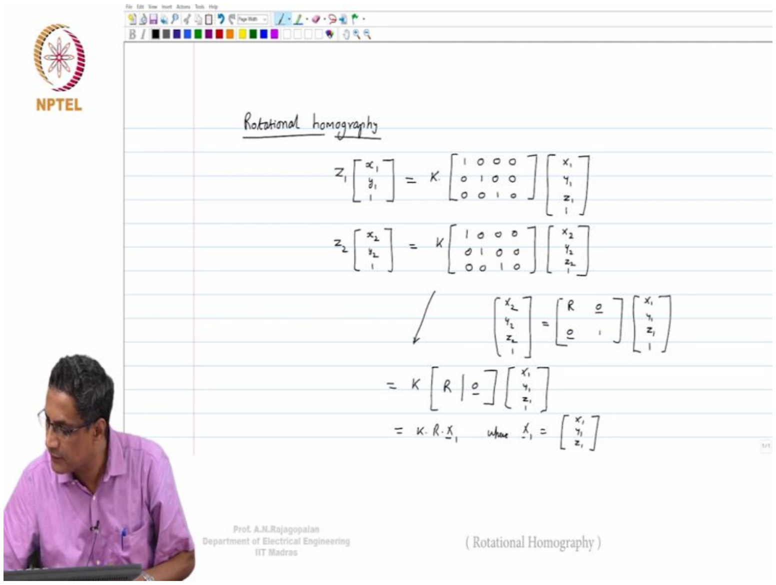


Rotational homography, so I will, I will try to keep this short because since we have already gone through the steps before this was I keep writing I think we will be able to figure out what I am what I am saying it but the main thing is that when I say rotational homography what I really mean is that unlike the earlier case, where we allowed this camera to also translate it we will not translate it. So, it is equivalent to saying that wherever the camera is, it can rotate about its center, but in it should not move. That is why we call it a rotational homography, only a rotational motion is allowed.

It can be a 3D rotation, it can go through Rx Ry Rz but it should not translate. The camera is not allowed to translate, which is why this is a special case, where were we looking at the case when the camera is allowed only to, only to say rotate, so that is a rotational homography. So if you again, go back to the same figure and all, so I mean the nice thing about this one is that it does not impose a constraint that you are scene should be a planar scene, that is a nice thing about this, but then it will also have its own drawbacks.

What those things are, we will see along the way. So there is the way we will, we will look at the same thing, there is a no assume that, assume that there is a point P in the world, but no longer do we have to assume that P has to lie on some plane and so on, or the scene should be a planar scene on which P is sitting.

So, you have some point P and then it has a coordinate whatever a capital X1 Y1 Z1 with respect to the camera's first, first position and then you kind of rotate the camera and then you have, another set of coordinates and then you want to be able to relate the right images that you get from the first few with respect to the second the same problem, whatever have we did.

So, all these notations and all that I would not explain again, since we have already seen them before. So as I keep writing you know, hopefully you will just be able to follow what I am saying. So you have like Z1 which is the Z coordinate of the point and then we have X1 Y1 1. So this we know is there is a camera intrinsic parameter K. And these equations we did already so I am just, I am not going to again go this I just write them and not bother to explain because you know what these things are 0 0 1 0 and then you have this point in the 3D world, which coordinates X1 Y1 Z1 1.

And we are saying that after you actually rotate the camera, then the coordinates become some, this should be small. So we have like is X2 Y2 1 and then you have K 1 0 0 0, 0 1 0 0, 0 0 1 0. Then you can have X2 Y2 Z2 1 this we know. So, now, we also know that we can actually relate the two through X2 Y2 Z2 1 we know this all we wrote earlier also, this we can relate as R and then T of course, in this case since we are not translating so I mean, if you try to go back we had a T here, and then we had written a 0 and then a 1 here.

If you remember, this how you can come from X1 Y1 Z1, you can, you can relate these two but right now we know that this T is 0, because, because we are not translating, so we will just keep this as 0 which is, which is a 3 cross 1 vector that is your TXT by TZ. The translation the camera which is not going to happen in this case.

And therefore, we can go back and we should come, if you kind of, if use the fact that X2, Y2, Z2 1 in turn is this then you can relate the Z2 times X2 Y2 1 to be equal to K times and then again if you multiply this matrix with this, then you will end up with what we had earlier, except that we will write it in terms of a sort of a compartment R and then here this will be a 0 vector earlier you had a translation there, now that is a 0 vector and then you have X1 Y1 Z1 1.

And this, this I can in turn write this as K times K into R into X1 where again, where X1 is again simply X1 Y1 Z1. So all this, we did earlier, at the time you had RX plus T coming out, now there is no T, T is simply 0. So X1Y1 Z1, no 1 there just, it has only three elements. And then we also, and then we also know this is something again that we saw when we did the Planar Homography.

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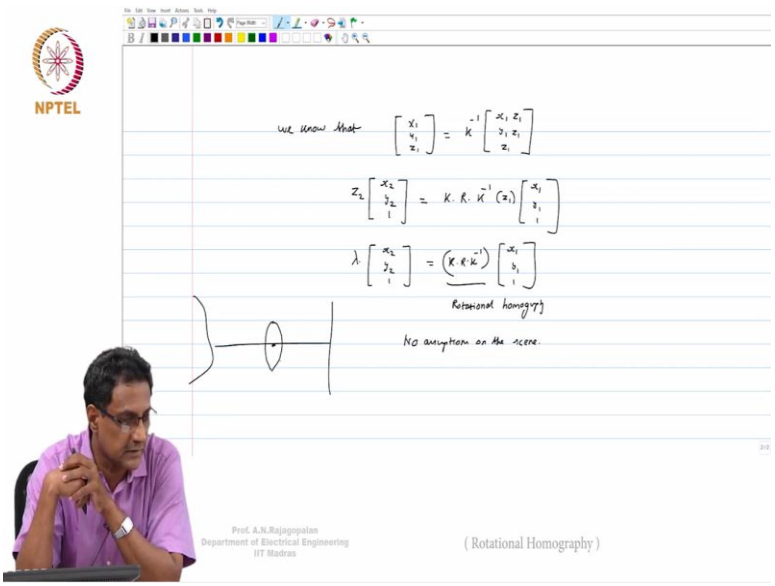


So and then right we know that, you also know this. So, your X1 which is X1, Y1, Z1 that we know that is related to, to the image coordinates as K inverse X1 Y1 Z1, K inverse, Z1 X1 now we get Z1 Z1 Z1 and this again I had explained last time that you cannot get depth directly from simply an image coordinate you cannot do this inverse, but here we are assuming that Z1 is small so, so we can do all this.

Therefore, all this you just go back if you have any doubts just go back and check what we had done earlier and then what we have is Z2 therefore, X2 Y2 1. Now we can now replace this is R times X1. So, we had K R, and then we had we had our X1 guy, so K R X1 is now K inverse and then you have let us say Z1 you can pull out Z1 if you wish and then we have X1 Y1 1.

And therefore it again we end up with this equation as lambda X2 Y2 1 is equal to K R K inverse into X1 Y1 1. And again I mean all of that holds so again it is up to so this is your, this is your rotational homography, rotational homography and, and as you can see, because there was no translation, I did not have to write them, nowhere did I have to assume that, that it is a planar surface or anything I did not make any, any such assumptions.

And, I (()) (6:47) this sort of some arbitrary point P. So the same thing holds, holds for at any point. And we are mainly right we did not make any, any assumptions on the scene. So no assumptions on the scene. So what this also means is that and they know what actually it is, it should be obvious to us because that only when you move there is supposed to be a parallax if you do not move, you do not see the parallax at all.

So if you are sitting there and you are not translating and simply say rotating and doing something like that without moving from the center that means there is no parallax. Therefore, even as the world is 3D, it should not really matter, that is also the reason why, why let us say, the human eyes are the way they are, it is not like one overlapping on top of the other.

There is a separation and the separation is the one which actually tells you that something in the front and something anyway, stereo or something that that I will at least introduce you to, introduce to you sometime later. But the whole idea is that when you translate, you have a depth information coming in.

But then it can also be an issue for you. Because I mean if the scene is 3D, if it is a planar scene, then it is okay, planar scene you can handle even if you translate, but then if a scene is full 3D as let us say it has multiple planes and all of that then we know that not no longer easy to handle. But then what like they say, there is no, there is no free lunch.

So if you actually move, then on the one hand, there is a problem because of the motion, you may not be able to work with all kinds of scenes, you have to restrict yourself to planar scenes but what you are losing out is the fact that the parallax effect. If it is a 3D world, it will bring the parallax effect to you. But then, you do not want it now, that you are saying that, Oh, I cannot handle it.

Whereas that is exactly the one that our eyes and all use. In fact, every stereo camera uses that effect only to, to its advantage. So it all it is all a function of what situation we are in. So as long as we are in the situation where you want to kind of find out computer homography that will relate images in the simplest form. That would typically mean that you either restrict the scene to be a planar in which case your camera can have a general motion or you can allow the scene to be general.

The sense that the scene can be 3D but now you say that the camera itself cannot now do whatever it likes, it can only do a rotation. It cannot, cannot translate it. So you have these, these two, two kind of dichotomy that either, you put a constraint on the scene or you actually put a constraint on the on the camera motion.

So either way, in both cases we can actually work with homography. So the only thing is only you can take home is that no, it is not true that if there is a 3D world you cannot perform homographies. You can still perform homographies provided you restrict the camera motion. Then you can still do, do stitching, only thing is right one wonders what you get, so it is like saying that I am rotating like that.

So you still be able to stitch but typically will be like the pan because that is when, but then there are there are situations right where somebody might just want to do this, in which case, he can still stitch he or she can still stitch images together. And irrespective of what the world looks like, it could be a complete 3D, full blown 3D world it is really will not matter.

Now one of the things that I wanted to do was introduce today kind of so all the while what we are talking about is really a pinhole model till now what we did was all a pinhole and we all know that the cameras that we carry with us they are not really pinhole. They all have lens not just one lens, they have compound lenses and so on. So the whole idea is that how does, this is image formation then what happens?

Are there still ideas from let us say the, the kind of pinhole model that you can still carry forward that are still valid, when you introduce the lens and the other thing is when you introduce a lens what changes will happen? How does one interpret the image now also how does one interpret an image captured through a pinhole imagine that, imagine that that let say that I mean I have a scene sitting out here and suppose I have actually a pinhole here and then I have a sensor my, my sensor plane is sitting here I see some image, correct if it is a pinhole now so suppose I assume that I have the ability to kind of introduce a lens here now. I suppose, I suppose introduce a lens that where there is a pinhole now I would want to know what really will happen to these rays that are coming in how does my image formation change what will happen and so on.

So, so basically that will be the, that will be the thing that will that we will want to move on next to what is called a real aperture camera. Pinhole on the one hand is a kind of a very idealistic sort of, sort of a camera, camera situation. The other thing is really a real aperture, by real aperture means that the aperture is not really a pinhole, it is real in the sense that it is, it is a fair, it is like any aperture that you might want to have.

It is like a circular aperture through which you want you want light to come in. And then and then, then you have a lens that will then gather all that light and try to focus on so. But that is, but then before that, I wanted to just, just kind of walk through this, this other slide now that that had just opened. Now let me just walk through you walk you through that.