

Image Signal Processing
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Lecture 19
Circle of Confusion

(Refer Slide Time: 00:16)

(not a pin-hole) (P.P) Deposits light on pin-hole
 ↳ will always yield a focused image. ↳ RA

Image formation in a RA camera:
 Draw upon pin-hole camera applets.

$$\frac{f}{u_0 - u} = \frac{v}{u} \quad f = \left(\frac{u_0 - u}{u} \right) v$$

$$\frac{v_0 + f}{u_0 - u} = \frac{v_0 + v}{u} \quad v_0 = \left(\frac{u_0 - u}{u} \right) (v_0 + f) - f$$

$$v_0 = \left(\frac{u_0 - u}{u} \right) (v_0 + f) + f = \left(\frac{u_0 - u}{u} \right) v_0$$

$$= \left(\frac{u_0 - u}{u} \right) v_0 - \left(\frac{u_0 - u}{u} \right) v_0 + f = v_0$$

$$= \left(\frac{u_0 - u}{u} \right) v_0$$

$$\therefore v_0 = v_0 = r_c \text{ (Radius of the blur circle)}$$

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(Circle of confusion)

Alright so now let us go ahead and draw this. So when I draw this so you should imagine that that we want to replace this guy this aperture with a lens and we want to see the effects. The first thing we going do is draw this figure. Let us say that I am going to let me try to draw it properly. Let us say that I have a lens, you have to be I think you are be good at drawing things, this is still not good maybe I will draw a smaller one.

(()) (00:58) to small then not sure how it will, this is not bad, something like that. And here is where the where the see pinhole was and now that will be the centre of the aperture of the lens. And you know that a lens has an aperture through which and that aperture is something that allows you to control how much light should come in, just as your eyes have an aperture.

Now imagine that I have a point, now let me draw it with some other colour, I have a point here, which is in the you see 3D world I am right for a specific reason I am not taking, let me decrease the for a specific reason I am not taking this point on the optical axis. This your optical axis. This is your image plane, this is your image plane this your lens and I am not taking point at deliberately here I am taking it here, I could also take an point on the optical as such that would

simplify my equations. Instead of that I will take it away from the optical as some point that is sitting. Now, let me just mark if you now, what will happen is the following. So if you really examine, now assume that my focal length of this lens is f , this is f . Let me draw here that is my focal length.

Then what will happen. So we know that there are a lot of rays that are going to come out of this guy and one ray, which is called the central ray or the say principle ray is the 1, this is a guy that is going to go there and then hit the plane somewhere. Then in order to know where things are going to avoid get focused.

Let us kind of draw this parallel ray which we know will pass through the through f and hits somewhere. So I think so the point is this ray I drew only to sort of indicate where the where these rays will meet. So these rays will meet here and therefore I will kind of remove this parallel ray. Because I do not actually need it. Let it be there it is not coming off. This is a problem. So remove one something else goes off.

Because that ray I just took to indicate where things will come into focus then what will happen you will have another ray, there is going all the way hitting the boundary of the lens and that should also come through the same point. Because that is the whole job of a lens. And then you have another guy that is coming from here. And that will also pass through this. Now here is your image plane let us just extend it. Now some point as so it forms, now the point is this. So this is called a point light source.

So it is as though I am segregating the whole scene. Now I just want analyse one point at a time, I do not want to look at the whole scene and all, just want to see that if there was a way that we could just place one point light source there in the world what will happen. So if you see, now let me just indicate certain things here.

Now, the first thing which you actually notice is that had the image plane be input here then of course, what would have happened is that this point which is with this point would have actually appeared in focus because then a point would have kind a map to a point. So had this image plane be there.

Now what we have so since we do not know and we just started with some with some image plane in sort of a distance from the lens. So in this case, what is happening is so this point is

mapping into this into some kind of you do not know what the shape is but this is called a circle of confusion.

So, what that means is that a point is spreading now, it is no longer a point. It is mapping a some kind of a confusion circle. Whether it is circle or not we will see. And we also know that there is something called the lens law. According to which in this case this is f and let me call this as what is this u , I am calling this as u and this am going to call as u naught.

Let me write a few things here from here to here is D . So the first thing is this lens law is such that 1 by f is equal to 1 by D plus 1 by u . What this actually means is that, that you know that point a point at D would come into focus at u , if the image plane was at u and not at u naught of course. It or see effectively you can have another equation that says 1 by f is equal to 1 by ωd plus 1 by u naught this is the other way to interpreted it.

What is means is that because we image plane is at u naught and because the focal length is f there is some distance called ωd and this ωd means a working distance. It is called the working distance, because if you place any object at that at ωd from the centre of the lens, then that will come in focus.

Because it will mean that it will be there instead of converging here they will all converge here on the sensor plane. So it actually means that ideally your point should have been at ωd in order to have been able to see it in focus. But since you are not, you have not kept it at ωd , you kept it at some other d .

So what is happening it is coming out is blurred. So this lens law you can write it with respect to f , u naught, D sorry f , u , D or you can write in terms f , u naught, and some other ωd . So it means that you should have brought it somewhere to the front or to the back and then it would have come in focus.

So you can already see that it depends upon how these lens parameters, the interplay of these lens parameters with respect to where the scene point is. Now let us just add in a few more things, I am going to draw, the nice thing about these tablets is that you can draw these colours and all. So this is a parallel, a line get a parallel to the optical axis, and we are going to call this as ϵ .

Student: (()) (08:05)

Professor: Very good. And we are going to call this as delta maybe write this you will know that I am referring to this is delta and I am going to call from here to here see this is the central ray. Now the central ray something unique by the way. It goes undeviated see everything else gets kind of zero fractured and then they all converge. The central ray to is going to the, going through is actually undeviated and go straight and hits. So let us call the point where the central ray hits this at distance let us call this as rb_1 just to denote some radius and let us call.

So this is with respect to the outer most ray this rb_1 with respect to the outer most ray that is hitting the lens and then converge and then coming and appearing somewhere in the plane. And similarly down below you have rb_2 we do not know whether rb_1 is equal to rb_2 maybe if I kept it into optical axis to be in more you know, then you can maybe immediately guess what might happen but not when it is here.

And then there is something else also that we should know which is this one. This guy is r naught this the aperture radius, radius of the aperture they have written D , I have written delta all of this is fine. Now using some simple similarity of triangles, let us go do the first one. Let us look at let us kind of look at look at this angle.

Now this is epsilon that is delta. So what do you have delta, what do you have we have delta delta divided by u naught minus u should be equal to. Now this angle should be equal to what is the other angle. So this angle should be call epsilon by u or in other words delta is equal to u naught minus u by u into epsilon.

Now the other angle right that we can look at is, what is the other one, rb_1 minus delta. Let us kind of look at this angle. So what is that, the total is rb_1 from here to here is delta. So it is rb_1 minus delta, rb_1 minus delta should be equal to what is the other angle. So this guy is going here, rb_1 minus wait a minute rb_1 is delta by something I did not write u naught minus u should be equal to now r naught minus epsilon. Because you are looking at here to here. Is that correct?

Student: Yes, sir.

Professor: R naught because this whole thing, this aperture is r naught radius, so r naught minus epsilon by u , correct, or in other words rb_1 will be equal to u naught minus u by u into r naught

minus epsilon plus delta this is equal to $u \text{ naught} - u \text{ by } u \text{ into } r \text{ naught}$ this $u \text{ naught} - u \text{ by } u \text{ into } \epsilon + \delta$. But then if you look at δ δ is exactly this one this is δ know from here $u \text{ naught} - u \text{ by } u \text{ into } \epsilon$ and therefore this whole thing. So these two will cancel off and you will get $u \text{ naught} - u \text{ by } u \text{ into } r \text{ naught}$ this is rb_1 .

Now, let us look at the other one. Let us say let us go here and look at this angle now, which is $rb_2 + \delta$ because from here to here is $rb_2 + \delta$. So what is that now, where do I write now, let me write here itself, $rb_2 + \delta$ so that is again of course $u \text{ naught} - u \text{ divided by } u \text{ naught} - u$ is equal to, now what is the other angle that we should be looking at? This one. So that will be $r \text{ naught} + \epsilon \text{ by } u$, $\epsilon \text{ by } u$.

So, rb_2 is equal to $u \text{ naught} - u \text{ by } u \text{ into } r \text{ naught} + \epsilon - \delta$ and therefore clearly this is again $u \text{ naught} - u \text{ by } u \text{ naught} - u \text{ by } u \text{ into } \epsilon$ as δ and that cancels with $-\delta$ that leaves you with $u \text{ naught} - u \text{ by } u \text{ times } r \text{ naught}$. But that is the same as what you got here. Therefore, this is also equal to rb_1 . Therefore, we can say rb_1 is equal to rb_2 is equal to let say the blur radius rb .

See I mean had I taken this point here it would have been very obvious that it not will be a circle but then what I wanted to show you was irrespective was what this actually means is that if you had a plane like this and then from wherever you are imaging the point could be anywhere of the plane does not have to be on the optical axis and all it will still map into a circle a blur circle that is why we call this a confusion, circle of confusion.

Now, the blur circle radius, this is a radius of the of this blur circle so you can think of this as some kind of a blur circle that is getting formed and this is the radius of the blur circle. If you had if this thing was all in focus, then of course, this radius would had been 0 because then you would had simply the point coming into focus.